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On the Dimorphism of the English species of Nummulites. By Mr J. J. Lister.

[*Read* 31 October 1904.]

The author gave an account of his examination of the characters of three English species of *Nummulites, N. laevigata* (Brug.), *variolaria* Lam., and *elegans* Sow., with respect to dimorphism.

De Hantken and de la Harpe called attention to the general occurrence of Nummulites *in pairs* in the same strata, and the phenomenon was referred to by them as the 'association of species.'

When in 1880 Munier-Chalmas propounded his view of the dimorphism of the species of *Nummulites* and of other Foraminifera, it was objected by de le Harpe that though association in pairs was the rule, there were several instances of the occurrence of a single form ("privée d'une sœur") in a deposit; a circumstance which, if established, would be difficult to reconcile with the theory of dimorphism. Although that theory is now firmly established the difficulty raised by de la Harpe appears never to have been met. *N. variolaria*, in the Bracklesham Beds of White Cliff Bay, and *N. elegans*, from the Barton Beds in Alum Bay, have been described, as they occur in these beds, as such solitary forms, although in deposits on the Continent they have both been found associated with a companion 'species.' It therefore seemed worth while to examine them to find what support they might have to give to de la Harpe's objection.

In *N. laevigata* the two forms of the species are readily distinguished when fully grown, owing to the fact that the microspheric form attains a much larger size than the megalospheric. A section in the median plane of such large specimens reveals an initial chamber (the microsphere) 17—19 μ in diameter, while sections of the smaller form show that they sprang from an initial chamber nearly ·5 mm. in diameter.

In examples of the two other species from the beds mentioned no such difference in the size of the test is displayed; but when examined in section the individuals of the species are found to be, like most other Foraminifera, dimorphic in respect of the size of the initial chamber.

Among 134 specimens of *N. variolaria* thus examined three were found to be microspheric, the diameter of the microsphere being 16—19 μ. In the remainder (megalospheric) the diameter of the megalosphere varied from 38—96 μ, the average diameter being about 68 μ.

In *N. elegans* among 94 specimens two were found to be microspheric, the diameter of the microsphere being 17—20 μ. In the others the megalosphere varied from 66 to 148 μ, the average size being about 95 μ.

It is therefore clear that these species, far from invalidating the conclusion that the species of *Nummulites* are dimorphic, are in complete accord with it. We have however to recognize that while in many species of the genus the microspheric form attains a much larger size than the megalospheric, in others the two forms attain the same size—a condition which is indeed the rule in the great majority of the Foraminifera.

On a Problem concerning Wood and Lignified cell-walls. By H. MARSHALL WARD, D.Sc., F.R.S., Professor of Botany.

[*Read* 31 October 1904.]

During past years Dr W. J. Russell has published several papers bringing forward the results of a remarkable discovery. He finds that if bodies such as leather, cork, wood and numerous others, are allowed to lie on a sensitive dry photographic plate in the dark, for periods varying from a few hours to some weeks, the plate on undergoing development in the ordinary way may have an image on it, in many cases of extraordinary sharpness and beauty.

I am not here concerned with any of his many interesting and important experiments, excepting those in which wood was employed; and he has just summarised the results of his latest work in a paper in the *Philosophical Transactions of the Royal Society*, Vol. CXCVII., Ser. B, pp. 281—289, entitled "On the Action of Wood on a Photographic plate in the Dark"*. This memoir, illustrated with a series of excellent plates, shows clearly the progress that the author has made towards the solution of the various questions he has raised. Dr Russell finds that almost all the woods he has tried act on the plate in the dark; though they vary as regards intensity of action, and as regards the length of time and the temperature needed for tangible results.

His method is to obtain a smooth face on a block of wood, dry, and place this face in contact with the plate, the whole being left in the dark until development of the plate.

Dr Russell had previously shown the probability that hydrogen peroxide is the active agent, and thinks that resin in the wood is probably the indirect causal agent: he gives proofs that it is not a case of radio-activity.

He gives a fairly long list of British and Foreign woods used, and summarises the results; and he also tests the question of the action of ordinary resin (colophony), various other resins, turpentine and a number of gums, with the result that while the bodies of more gum-like nature are inactive, those of more resinous nature are active.

---

* See also *Nature*, Sept. 22, 1904, p. 521.

Dr Russell also shows that oak a century old, and oak from a bog, and even decayed wood are still active.

Among his most interesting discoveries are, that light of various kinds stimulates the activity of a piece of wood exposed to it; that the same is true of leather, cork, resin, paper, &c.; and that certain bodies—*e.g.* glass—are opaque to the action, whereas others—*e.g.* porous bodies, gelatine, &c., transmit it. Moreover the block of wood need not be in actual contact with the plate; it will act if an appreciable layer of air intervenes.

For the further understanding of these and numerous other interesting facts concerning the physical and chemical points concerned, I refer the reader to Dr Russell's fascinating and important papers.

My position in what follows is this. About a year ago Dr Russell was so kind as to show me some of his experimental results, and I was naturally much interested in his work, because, apart from the more purely physical and chemical aspects of the phenomena, they would appear at once to have a direct bearing on the problem of the constitution of wood cell-walls.

Beyond proposing to apply Dr Russell's results in particular ways to this botanical question, and perhaps having obtained certain modifications of his process and a few results of additional value, I have no claims to originality in the matter, and certainly no credit is due to me as anything but a humble follower of his discoveries. With these disclaimers, then, I pass to a summary of my own experiments.

I was struck at the outset with the difficulty of referring Dr Russell's results to the resins, because, in the first place the Larch and the Scots Pine behave so differently.

In the Larch the resin-canals and, presumably, the bulk of the resin, are in the Autumn wood zones, and it is these zones which induce the strongest image; but in the Pine, although the resin is distributed in the Autumn zones as in the Larch, the most marked effect is in the Spring wood. Russell finds the Larch less active on the whole than the Pine, but from my knowledge of the two I should have thought them equally resinous, and their contrary behaviour seems anomalous.

But the second point strikes one as even more extraordinary if resin is the active body. The Oak gives very good figures, Russell placing it in the same group with the Pine as "very active." Now, without saying that there is no resin in Oak, it is well known that tannin and tannin-like bodies are the characteristic substances in Oak, and it at once occurred to me that this series of constituents ought to be tried.

Nevertheless if we examine Russell's lists of "very active," "active," and "slightly active" woods, it cannot be said off-hand that either resin or tannin is at once suggested; for instance, the Sycamore is "very active," while the Beech is only "active," and the Alder and Elm are but "slightly active."

It appears to me, therefore, that while the evidence is in favour of some substance emanating from the cell-walls being the active agent, this substance is either not yet recognised, or it differs in different cases. My reason for agreeing so readily with the conclusion that it is some substance—perhaps, but not certainly, volatile—is this. Russell found that "a glass or mica screen of one-thousandth of an inch in thickness entirely protects the plate from being acted on"; and he concludes from this that it is not a case of radio-activity.

Whether his conclusion be sound or not, the efficacy of such a screen—and the fact that a screen of porous substance, or of gelatine, or of wood does not prevent the action—points to some such conclusion as the above.

Having confirmed Russell's results, I proceeded to the enquiry: (1) Can we obtain similar photographs with thin sections of wood? For, if so, it might be possible to obtain micro-photographs and trace the action—by its relative sharpness—to the solid lignified walls or to the cavities. (2) Can we, by streaking such sections with solutions of substances to be tested—*e.g.* Tannin, Tannic Acid, Gallic Acid, Xylol, Turpenes, Oils, or any other bodies derived from wood—obtain local intensification of the results? In this way it might be possible to run down the active bodies, and compare them with the various woods in their natural conditions.

The first question was soon answered. The results of all the experiments it is proposed to discuss here are summarised in the following Table, in which the experimental running number, the species of wood employed in thin sections, the condition of the wood-section used, and any treatment to which it was subjected before placing it on the plate in the dark-room and wrapping it up, are given in the first four columns.

The time during which the contact with the plate continued is given in column v.; and the temperature during the period of contact in column VI.

In the last column I have summarised the results in a few words, sufficient to show the principal features of any figures obtained.

Analysis of this table shows that in most cases a considerable incubation-period is necessary to obtain any picture beyond a "mere ghost," so faint that one can only say that some "activity" has been displayed.

Whether this is because my thin sections contain very little of the active substance in the small amount of cell-wall present (as

No.	SPECIES OF WOOD	CONDITION	TREATMENT	EXPOSURE	TEMPERATURE	RESULT
1	Aristolochia Sipho	Old and dry secs.	None	8 days	20–22° C.	Mere ghost
2	Berberis vulgaris	,, ,,	,,	,,	,,	Merest possible ghost
3	Brownea grandiceps	,, ,,	,,	12 days	,,	Distinct but faint image
4	Hemitelia integrifolia	,, ,,	,,	,,	,,	,, ,, ,,
5	Ulmus campestris	,, ,,	,,	,,	,,	Merest trace of ghost
6	Vitis vinifera	,, ,,	,,	,,	,,	Distinct, but very faint image
7	V. labrusca	,, ,,	,,	,,	,,	No trace discoverable
8	Quercus Robur	,, ,,	,,	,,	,,	Very faint image: also oval of paper
9	Rhamnus catharticus	,, ,,	,,	18 days	,,	Distinct faint image: traces of detail
10	Salix daphnoides	,, ,,	,,	,,	,,	Distinct but faint: sap and heart distinguishable
11	Castanea vesca	,, ,,	,,	,,	,,	,, ,, rings distinguishable and ? sap-wood
12	Quercus Robur (pedunculata)	,, ,,	,,	,,	,,	,, ,, no distinct details: mottled
13	Libocedrus tetragona	,, ,,	,,	,,	,,	,, ,, ,, ,, ,,
14	Cedrela Toona	,, ,,	,,	,,	,,	Distinct image: can recognise rings, M. R. and vess.*
15	Ceratopetalum apetalum	,, ,,	,,	,,	,,	,, ,, ,, ,, rings and M. R. only
16	Avicennia tomentosa	,, ,,	,,	,,	,,	Mere ghost
17	Brownea grandiceps	,, ,,	Clove oil	6 days	,,	No trace, even after long development
18	Deal shaving	Carpenter's wood	Moist	8 days	35° Incub.	Good image, but fuzzy edges, rings and resin canals
18′	,,	,, ,,	,,	,,	,,	Longer development. Resin canals better
19	,,	,, ,,	,,	,,	,,	Good image, traces of detail but fuzzy
19′	,,	,, ,,	,,	,,	,,	Longer development, traces of detail but fuzzy
20	,,	,, ,,	,,	,,	,,	Good image and traces of detail
20′	,,	,, ,,	,,	,,	,,	Longer development. Hardly better
21	Pine shaving	,, ,,	,,	,,	,,	Best image yet obtained: excellent rings, &c.
21′	,,	,, ,,	,,	,,	,,	Longer development: deeper image
22	Hakea dactylodes	Old and dry secs.	None	20 days	20–22° C.	Faintest possible, if any trace?
23	Acacia umbrosa	,, ,,	,,	,,	,,	Very faint, but trace heart and oval
24	Porana volubilis	,, ,,	,,	21 days	,,	Faint but distinct details
25	Muhlenbeckia saggittæfolia	,, ,,	,,	,,	,,	,, ,, ,, ,,
26	Hemiteleia integrifolia	,, ,,	Clove oil	11 days	,,	Deep image and stain round, due to effusion on paper
27	Aristolochia Sipho	,, ,,	Tannin	,,	,,	Good image. Faint M. R. Blotched (with tannin?)

28	Quercus Robur	" "	Pyro gallol (part)	"	"	Deep image. . M. R., rings and even vess. seen. Blurred and deeper where treated. One of the best yet
29	Sophora microphylla	" "	Xylol (part)	12 days	"	Faint ghost of wood, deeper where it lies on paper. Deepest stain on the paper
30	Tectona grandis	" "	Xylol	"	"	Faint image, but shows rings and M. R. especially where on the paper. But deep blotch where soaked paper
31	Ulmus campestris	" "	Pyro acid (part)	"	"	Good clear image, but faint except where streaked. Rings and M. R. visible, but little else of detail
32	Vitis Labrusca	" "	Tannin	"	"	Sharp image, but only rings and M. R. distinct
33	Berberis vulgaris	" "	Gallic acid	"	"	Fair image, but blurred. Faint rings and M. R.
34	Vitis vinifera	" "	"	"	"	Fair image, blotched and little or no detail
35	Pinus sylvestris	" "	None	29 days	"	No trace of image
36	Larix europaeus	" "	"	"	"	" "
37	Pinus Pinaster	" "	"	30 days	20–22°C. for twenty days and then at 35° C.	Faint, but sharp, good detail
38	Ficus Australis	" "	"	"		Faint, no detail, fairly sharp
39	Menispernum fenestratum	" "	"	"		Faint, sharp and some detail
40	Xylomelum pyriforme	" "	"	"		Faint, but sharp and beautiful detail
41	Elaeocarpus grandis	Fresh shavings	Wetted and Dried	10 days	35° C.	Good image and lots of detail, but fuzzy
42	Cedrela australis	" "	" "	"	"	" " " "
43	Grevillea robusta	" "	" "	"	"	Good image, some beautiful detail, some fuzzy
44	Pitch Pine (joiner)	" "	" "	"	"	Good image and lots of detail, but some fuzzy
45	" " "	" "	" "	"	"	Good and dark, and lots of detail, but fuzzy edges
46	Oak (joiner)	" "	" "	"	"	Dark image and some detail, some sharp, but some fuzzy
47	" "	" "	" "	"	20–22° C.	Good image and details, but very fuzzy edges
48	Pitch Pine (joiner)	" "	" "	"	"	Fuzzy, but lots of detail and good image
49	Abies pectinata (joiner)	" "	" "	72 days	35° C.	Fuzzy, but good images
†50	Pinus sylvestris	Old dry secs.	None	"	"	Sharp and detailed image, though not very deep
51	Larix europaea	" "	"	"	"	" " " "

---

* In Nos. 14 and 15, etc., column of results; "rings," refers to annual rings; "M. R." to medullary rays; and "vess." to the large xylem vessels. "Heart" refers to heart-wood; "sap" to sap-wood.

† Nos. 50 and 51 are the same sections as were used in experiments Nos. 35 and 36, and therefore show well the effects of longer exposure at a higher temperature with more sensitive plates.

contrasted with the large blocks used by Dr Russell), or whether the fact of these thin sections being pressed flat between glass plates, wrapped up closely in several layers of black and other papers, prevents the access of air, are matters for further experiment. Both might be causal factors.

It should be noticed that in experiments Nos. 35 and 36 I used Scots Pine and Larch, on "Ordinary" Ilford plates; and that 29 days at 20—22° C. failed to give an image. In experiments Nos. 50 and 51, the *same* sections were again used, but on Ilford "Special Rapid" plates, and at 35° C., and good images were obtained from both in 72 days.

Dr Russell had already found that the "Ordinary" plates will not do, and that raising the temperature of incubation and lengthening the exposure increase the effects, so that this experiment confirms his results so far, and in one case (Scots Pine) with a wood known to be very active.

But my experiments, Nos. 44, 45 and 48, with Pitch Pine, another very active wood, where the period of incubation was only 10 days, indicate that temperature is not always an important factor, for the image obtained in experiment 48, at 20—22° C., is but little if any fainter than that of No. 44 or 45 incubated for the same period at 35° C.

The determining factor of greatest importance appears to be the freshness or otherwise of the wood, as shown by the truly excellent photograph obtained by eight days' exposure at 35° C. of a shaving of moist Pine—*e.g.*, in experiments No. 21 and 21′. Russell had also observed that fresh wood is more active than old dry wood.

It is clear, then, that the action on the photographic plate can be exerted by thin sections, though the time required is longer. Unfortunately for my purposes the sharpness of the image is as a rule deficient, and consequently it is as yet impossible to discover the exact seat of the local action of the wood cell-walls.

Before passing to some experiments where other factors are introduced, let us now examine the table with reference to woods other than the resinous Coniferæ.

Experiments Nos. 8, 12, 28, 46 and 47 were made with Oak, a wood which abounds in Tannins, but in which there is no recognisable Resin.

As No. 8 shows, a thin section of Oak, old and dry, gave but a poor image in eight days at 20—22° C.; and, as No. 12 shows, little if any better results were obtained in 18 days at the same temperature.

Fresh shavings of Joiners' Oak, incubated for 10 days, however, whether at 20—22° C. (experiment 47) or at 35° (experiment 46) gave far better results; but it must be noted that these shavings were moistened, to facilitate the cutting, and then dried, and I attribute the fuzzy outlines to their being moist—for the ordinary drying alone afforded to the shavings, left them by no means really dry.

In these cases, then, it seemed impossible to suppose that the active body could be a Resin; and, not unnaturally with such wood as Oak, one thought of Tannin or some such body as Gallic or Tannic acid as a possible factor.

In experiment No. 28, *the same piece of wood* was used as in No. 8, but before putting it on the plate half the piece was painted over with a camel-hair pencil dipped in a *weak solution of pyrogallol*, and a streak of the same was drawn across the middle of the untreated half, and the section then dried between filter-paper.

After incubation for 11 days at 20—22° C., a sharp image was obtained *particularly dark at the parts impregnated with the pyrogallol*, rendering it more than probable that my surmise was right.

I then instituted a set of trials with various bodies, more or less directly derived from wood, with a view to seeing if they gave similar results.

Clove oil was used on *Brownea grandiceps*, on a section which (experiment No. 3) had given no good results at 20—22° for 12 days. The treated section at 20—22° for six days yielded *no trace at all*: it seemed as if the clove oil had stopped the action, but, since the incubation-period was shortened, the trial was not conclusive in this respect.

Hemitelia integrifolia, which gave similar results to the foregoing when not treated (experiment No. 4), was similarly tested with clove oil, and incubated for nearly the same period (11 days) at 20—22° (experiment 26), and the effects of the clove oil in deepening the action were at once apparent, the effect showing *even on the paper into which the oil had diffused.*

Here, then, we must conclude that clove oil is also an active agent, though the experiments do not decide whether it is so alone, or only in contact with some other body in the wood.

The next substance tried was tannin. As Exp. Nos. 1 and 7 showed, *Aristolochia Sipho* was very inactive in eight days at 20—22° C., while *Vitis Labrusca* seemed utterly without action even after 12 days' incubation. After being washed in a weak tannin solution and then dried, however, *the same sections* gave good images in 11 days and 12 days respectively, at the same temperature, the *Vitis* being especially active.

Tannin would therefore appear to be a markedly efficient agent, and the fact suggests an explanation why Oak is such a "very active" wood.

I then tried pyro-gallol. As already shown, this was decidedly successful as evinced by streaking the Oak section (Exp. No. 28); but its activity was even more clearly evident with *Ulmus cam-*

pestris, the Elm. As Exp. No. 5 shows, the Elm gave the merest traces of activity in 12 days at 20—22° C., and Russell had also found the Elm but slightly active; but *the same section when streaked with pyro-gallol and incubated for the same period at the same temperature* (Exp. No. 31) gave an extremely good image, *particularly black where the pyro-gallol was streaked across it.*

The next substance tested was Xylol. As experiments Nos. 29 and 30, with *Sophora microphylla* and Teak, *Tectona grandis*, show, Xylol is an effective agent, because the paper into which it soaked gives a dark image; but since I have not as yet tested these woods alone it is impossible to say whether this agent alone is responsible for the action, or whether the results are due to the co-operation of something in the wood.

One thing seems fairly clear. I thought it possible that the water used for the solutions of tannin, pyro-gallol &c. might play an important part, because, as Russell had also shown, damp wood acts more rapidly than the same wood when old and dry. But in the cases of xylol and clove-oil this seems impossible: these reagents were used pure, and the surplus merely soaked off by filter-paper.

I have also obtained positive results with gallic-acid. *Berberis vulgaris*, the Barberry, showed the merest traces of activity in 8 days at 20—22° C., as shown in Exp. No. 2; and although the Vine (*Vitis vinifera*, Exp. No. 6) gave better results in 12 days, the image was too faint to show on a screen. But *the same sections* after washing in gallic-acid (Experiments Nos. 33 and 34) gave *fairly dark images* after 12 days at 20—22°, proving that this reagent also is effective.

Enough has now been stated to show that whatever the activity of wood on the photographic plate may be due to, it is not merely Resin or Resin-like bodies; but that Tannin and Tannin-like bodies as well as some others may be responsible.

My knowledge of the chemistry and physics of the subject is too slight to enable me to attack the problem of the mode of action on the plate; moreover the question is in sufficiently able hands with Dr Russell.

It seemed, however, a sufficient reason for this sketch, to point out that we have here a Botanical problem of great interest—it may be of fundamental importance.

It is clear that some body or bodies in the wood cell-walls—*i.e.* lignified cell-walls—reduce silver-salts in the plate. That such bodies are either shot off, as if volatile, or diffuse readily, seems clear from the want of sharpness in the microscopic details.

I attribute the blurring of the figures to the facts that the activity is not confined to the actual planes of contact of the cut solid cell-walls with the film of the plate, but manifests itself in the cavities, suggesting that particles are thrown off from the walls standing off at right angles &c.

It seems to me that we botanists have here a definite problem —or rather a series of problems. First, to see what woods, if any, are *totally without activity*. Secondly, to find out what substances in lignified cell-walls are *most active*. Thirdly, to try and co-ordinate the results with *the various degrees of lignification of the walls*. Further, to see if the effect is due to *substances in the body of the lignified walls*, or to secretions or excreta left behind in the cells.

Some of the discoveries of Dr Russell appear to be of promising significance—*e.g.* Cork is active, but pure cellulose is not; decaying wood, and wood soaked in peat (Bog Oak) may be active; and, especially, the curious intensifying action of light, for it has long been known to Forest botanists that after a tree is felled, the exposed heart-wood may change colour.

Indeed, one of the first tasks I would suggest is to compare the effects of fresh and old heart- and sap-woods; of wood cut in the dark and exposed to light—in or out of contact with air—and of substances extracted from both heart- and sap-wood both in contact with the wood itself and with pure inactive cellulose.

Moreover these and other experiments should be made on the same wood, in various conditions, as well as on species known to be very active—*e.g.* Oak and Pine—as well as on species found to be most inactive.

It will be obvious to all that such experiments promise to stimulate suggestions for other modes of attack on what has hitherto proved a veritable Port Arthur to the botanist,—viz. the constitution of the lignified cell-wall.

The Pine-apple Gall of the Spruce: a Note on the early stages of its development[*]. By E. R. BURDON, B.A., Sidney Sussex College.

[*Read* 31 October 1904.]

The galls are caused by the hibernating generation of certain Aphidae belonging to the genus Chermes, which lives exclusively on coniferous trees.

Several species of Chermes have been described by continental writers, two of which (*C. abietis* and *C. strobilobius*) are certainly present in Britain, and further search will probably result in others being found.

The insects have a most complicated life-cycle which has been worked out by Dreyfus, Blochmann and others in Germany, and Cholodkovsky in Russia. This cycle exhibits a heterœcism similar to that which obtains in Flatworms, Uredineae, etc. The main facts are briefly set forth in the following table, in which the successive generations are denoted by biologic names[†].

Of these various generations it is worthy of note that only one of them—the Fundatrices on the Spruce—induces the formation of a gall, though all are equally engaged in sucking either on the stem or needles.

The galls caused by the different species of *Chermes* all follow the same general plan of development, and, although there are slight differences in detail, the following account, which refers specially to the galls of *C. abietis*, probably represents what occurs in every case.

The life of the gall is naturally divided into three distinct stages:

(1) its development whilst still enclosed in the bud-scales;

(2) its development after emerging from the bud until the opening of the chambers and exit of the inhabitants;

(3) its death, decay and subsequent fate.

In the present paper my remarks are limited to the events which occur during the first stage.

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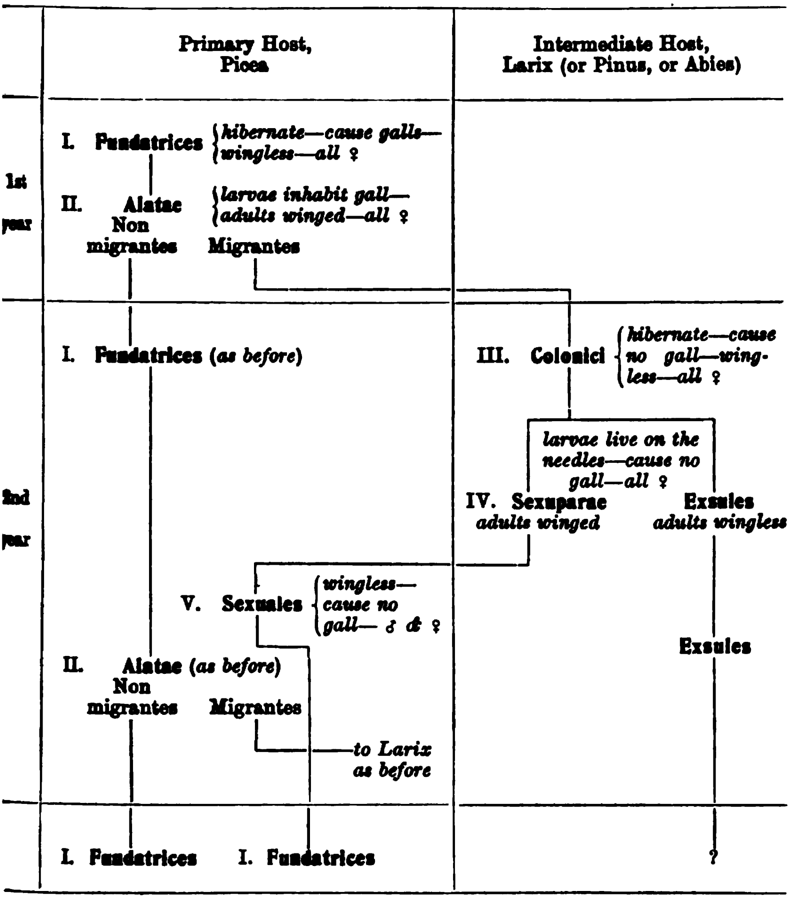
[*] Figures and full details will appear in a later paper. The present communication merely summarises the most important points and conclusions.

[†] For fuller details see Prof. Cholodkovsky's "Beiträge zu einer Monographie der Coniferen-Läuse," *Horae Soc. Ent. Ross.* xxx. pp. 1—102, and xxxi. pp. 1—61.


Primary Host, Picea
Intermediate Host, Larix (or Pinus, or Abies)
1st year
I. Fundatrices { hibernate—cause galls— wingless—all ♀
II. Alatae { larvae inhabit gall— adults winged—all ♀
Non migrantes
Migrantes
2nd year
I. Fundatrices (as before)
III. Colonici { hibernate—cause no gall—wingless—all ♀
larvae live on the needles—cause no gall—all ♀
IV. Sexuparae adults winged
Exsules adults wingless
V. Sexuales { wingless— cause no gall— ♂ & ♀
Exsules
II. Alatae (as before)
Non migrantes
Migrantes
—to Larix as before
I. Fundatrices
I. Fundatrices
?

The gall arises in the following manner:—

In the autumn a *Chermes* larva drives its long proboscis either into the stem of the Spruce just below a bud or into the bud itself, and thus securely anchored, passes into a hibernating condition. The insects at this stage are very minute, their total length barely exceeding 0·5 mm., but they are provided with an extremely long proboscis which is about three times as long as the whole body. This proboscis is driven right through the cortex and the apex lies in the neighbourhood of the cambium. Once the insect has established itself in this position she never again quits the spot, but here remains until her death in the following June.

Before passing on to the development of the gall it will be well to recall briefly the structure of the normal winter bud of the Spruce.

If the scales are removed from a winter bud before it begins to develope in spring, the entire shoot for the following year is found already laid down in miniature, with the rudimentary needles closely arranged in a spiral around the abbreviated axis, which they altogether conceal.

A longitudinal section shews the stem of the young shoot to be already differentiated into pith, primary vascular bundles and cortex. The pith and cortical cells have square ends and are densely filled with protoplasm which shews no signs of vacuolation, rendering it evident that active growth has not yet set in.

Chlorophyll is present in every part of the young shoot except in the vascular bundles.

Resin canals generally are absent at this stage, though here and there indications that they are about to be formed may be found.

In the outer cortex short chains of secretory cells or solitary tannin sacs occur and may be recognised by the abundant deposit of tannin or resin which they contain.

The cells of the pith, and the mesophyll cells of the leaves contain a copious reserve of a substance which I believe to be tannin. My examination of this substance is not complete, but in the meantime I will refer to it provisionally as tannin.

Minute specks of starch are found in the chlorophyll corpuscles, but the quantity is so small as to be negligible, and there is nothing that can be regarded as a regular starch reserve. This fact is important in view of the abundant store of starch which makes its appearance at the very outset of gall formation.

In the spring the insect awakens and begins to suck and secrete a tuft of white wool-like wax from glandular plates on the dorsal side of her body. The time when this happens varies according to the weather. This year (1904) we had a late spring in Cambridge, and it was not till the middle of April that I was able to discern any signs of life.

As soon as the insect begins to suck, the cells in the region of the cambium where the apex of the proboscis lies, are forced into precocious growth. They at once increase rapidly in size and undergo active division. The protoplasm becomes filled with large vacuoles and the nuclei enlarge in about the same proportion. The daughter cells repeat the process and the swelling and growth radiate outwards in every direction from this centre.

At first the formation proceeds symmetrically and both the pith and the cortex are invaded to an equal extent. Very soon however a limitation is imposed on the growth of the pith cells by the lignified vessels of the protoxylem, and the cells of this region do not continue to respond so readily to the stimulus. The growth then takes the line of least resistance, and extending rapidly outwards through the cortex becomes excentric.

One of the first indications of the gall growth is the disappearance of the chlorophyll, and this is at once visible to the naked eye on bisecting an infected bud longitudinally. As the area extends its boundaries the chlorophyll gradually recedes, and at length the whole of the affected portion of the shoot appears, on removal of the bud-scales, as if it had been bleached.

Neither resin canals nor secretory cells of any description are to be found in the gall area, any that existed prior to the commencement of gall formation having been absorbed into the gall parenchyma.

The "tannin" contained in the cells of the pith and mesophyll also disappears. In microtome sections it breaks up first into large granular masses, then into smaller granules, and finally disappears. In cells which are still normal, this deposit of tannin, after fixing with dilute Flemming, is so dense as to entirely conceal all details of the protoplasm and nucleus, but wherever the gall action has penetrated these details are laid bare, and the cells look as if they had been treated with some clearing agent.

One is so accustomed to connect tannin with galls that this statement may at first appear doubtful. It should however be mentioned that at this stage the shoot appears incapable of struggling against the disease, but in the next stage, after it has emerged from the bud-scales and has begun to assimilate, tannin is again found in abundance. I believe however that the tannin found in the later stages is of a different nature and composition to the tannin present in the bud.

The disintegration of the tannin seems to be closely connected with the formation of starch which now makes its appearance in great quantity. The cells round the periphery of the gall area become as closely packed with starch granules as are the endosperm cells of wheat. The quantity decreases as one proceeds inwards towards the centre of the gall, and in the very centre no starch or only a few scattered granules are found.

I think a possible explanation of the origin of this starch may be that the tannin contained in the pith and mesophyll cells is in reality a glucoside, which on breaking down forms considerable quantities of glucose, and this serves the insect for food. The *Chermes* however cannot absorb the glucose as quickly as it is formed and the result is that the glucose solution soon reaches a high degree of concentration. To reduce this concentration starch is precipitated, and a molecule of water is consequently liberated.

This would explain the distribution of the starch in the gall, for in the centre where the apex of the proboscis lies, the glucose is being continuously drawn off and would not therefore become too concentrated. But the effect of this leakage would be less felt at the periphery and it is here consequently that the most abundant precipitation of starch takes place.

The sweet drops of "Honeydew" which, as is well-known, are exuded by these insects, may perhaps prove to have the closest bearing on this point.

In connection with this suggestion I must however state that I have not yet succeeded in demonstrating the presence of glucose in the gall, but hope to carry out some careful tests next spring when the galls are again in season.

I have mentioned above that the nuclei enlarge in about the same proportion as the cells containing them, but the change is not confined merely to enlargement. In the normal resting nucleus the chromatin network is very evenly distributed, and no well marked nucleoli are to be found. In microtome sections one of the first indications that a cell is destined to form part of the gall tissue is given by the chromatin of the nuclei. The network gradually becomes fainter and fades away, while at the same time numerous nucleoli make their appearance. In course of time the network in some cases entirely disappears and the whole of the chromatin becomes aggregated into these nucleoli. Some of them attain a considerable size and project like warts from the surface of the nucleus, which assumes a pallid unhealthy appearance.

So far as I have examined them at present, the mitotic figures seem to be of the usual somatic type, and I have found no indication of heterotypical mitoses, such as one would expect to find were the growth of a cancerous nature.

The net result of these various changes is that almost all previous differentiation of the stem has been obliterated, and in its place a parenchymatous tissue, consisting of abnormally swollen cells with extremely thin walls, has been formed. The cortex on the galled side of the stem has become 2 to 3 times as thick as the cortex on the other side which is still normal, and the symmetry of the stem has thus been destroyed.

The gall growth continues its outward course until it reaches the bases of the needles, and these in their turn begin to swell up. As the growth proceeds up them the needles assume a tapering shape—very much thickened at the base and gradually diminishing in size until about half-way up, where they still remain normal. The gall is now visible to the naked eye as soon as the bud-scales are removed, being rendered evident not only on account of its swollen nature, but also by reason of its bleached appearance due to the absence of chlorophyll.

The needles at first increase evenly in thickness on every side, but before long the swelling becomes confined on the inner (ventral) side, since this faces the main axis, which is also swelling, and the two soon meet. When this happens any further growth goes on chiefly on the dorsal side, and the leaf becomes asymmetrical in transverse section. The outer (dorsal) side of the needles soon comes in contact with the inner surface of the bud-scales, but as these are gradually unfolding all the time, they do not offer much resistance to the swelling, and the dorsal side of each gall needle consequently becomes convex in shape.

Owing to the erect position of the needles at this stage and their crowded spiral arrangement round the main axis, this enlargement of their bases quickly results in adjacent needles coming into contact with each other, and grooves are formed on the surface of each needle, where other needles have pressed upon it. The phyllotaxis is such that the base of any one needle must on swelling come in contact with the swollen bases of no less than four other needles, two belonging to the spiral above it, two to the spiral below.

Each needle-base is consequently marked with four grooves. Of these, the two on the inner side are much deeper than the two on the outer side, owing to the pressure being greater in the former case, and between these grooves a slight ridge is formed which corresponds with the space between the needles of the spiral above, or below as the case may be.

As the stem gradually elongates the needles are carried slightly apart, and the resulting space above each needle forms a chamber which is later on taken possession of by the young offspring of the *Chermes* mother. The deeply grooved inner (ventral) side of the needle forms the floor of this chamber, whilst the two needles of the spiral above each contribute a half towards the formation of the roof. It will thus be seen that each individual leaf participates in the formation of three distinct chambers, its ventral side forming the floor of the chamber above it, and the grooves on its dorsal side each forming one-half of the roofs of the two chambers below it to right and left.

At a later stage these chambers become closed in by the development of thick tumid lips around their mouths.

About the beginning of May the shoots begin to emerge from the bud-scales but the galls are not at once visible since the base of the shoot is still concealed in the persistent basal bud-scales. They soon however make their appearance and the galls then enter on the second stage of their existence, the history of which must be left at present as my examination is not yet completed.

But to return to the insect. Whilst the changes described above have been proceeding in the bud, the *Chermes* larva has been steadily sucking and increasing in size. She has secreted a quantity of white wool-like wax, which, while it entirely conceals her, renders her position most conspicuous. She undergoes three ecdyses before reaching maturity, and the cast skins may be found lying beside her in the "wool." As soon as she reaches maturity she commences to lay eggs, and continues the process through May and on into June, until a heap of some hundreds of eggs accumulates beside her. Each egg is attached to the stem by a delicate hair-like stalk. The first eggs begin to hatch soon after the gall has emerged from the bud-scales, and the minute larvae at once creep up the shoot and establish themselves in the gall-chambers, where they find a fleshy succulent tissue already prepared for them. They at once commence sucking and become entirely enclosed within the chambers by the development of the tumid lips previously referred to, and here they remain until the galls open.

These then constitute the most important events which characterise the development of the galls during the first stage —that is whilst still enclosed in the bud-scales.

One very noticeable feature is the apparent absence of any effort on the part of the plant to resist the attack, for the insect appears to have everything its own way. This is I think chiefly due to the fact that the shoot is enclosed in the bud-scales, which exclude light and air, and thus keep the tissues of the shoot in a plastic condition. Next season I hope to carry on some etiolation experiments which will probably throw considerable light on this point. It is of course also due in part to the insect having, as it were, stolen a march on the plant, forcing the young shoot to develope before it was fully prepared.

Although my examination of the later stages is not yet complete I have seen enough to convince me that the conditions are reversed in the next stage, and a great effort is made by the shoot to overcome the influence of the insect. And this effort is to some extent successful in so far as the shoot is enabled to limit the insect's sphere of influence to the area over which it has already gained sway. But the effort is made too late, and always ends in the death of the galled portion of the shoot, if not of the whole shoot.

With regard to the ultimate cause of the gall-formation there is, I think, good reason to believe that it is due to an injection by the *Chermes* mother. It would be out of place in this paper to give the reasons for this belief in detail, but the behaviour of the chlorophyll, tannin, protoplasm and nuclei, and the gradual radiation of the influence in every direction all seem to point to an injection as the cause. Further, if the insect be removed, say a week after it has commenced to suck, the abnormal growth is *not* brought to an end. The gall continues to develope, and emerges from the bud-scales just as if the insect had never been removed.

The only possible explanation of this posthumous growth appears to me to be, that the poison injected during that week of the insect's life continues to act on the tissues of the plant after the death of the mother.

But this naturally gives rise to another question. If the insects of one generation inject some fluid into the plant, surely the insects of the other generations, the Alatae, the Colonici, the Sexuparae, etc., must also be supposed to inject a poison when they are engaged in sucking. How then is it that they do not cause the formation of a gall? Why does the injection answer in one case and not in another even on the same tree?

The answer to this is, I think, that the injection will only give rise to gall-formation when it acts, not only on embryonic tissues, but on embryonic tissues which are not confined by the presence of other lignified or cuticularised tissues. We have seen how such feebly lignified tissues as the protoxylem strands were sufficient to turn the course of gall-formation from concentric to excentric. Again, when the gall emerges from the bud-scales the gall area is at once confined to the region already affected in consequence of the exposure to light and air, which results in the hardening of all parts which still remain normal.

The behaviour of the Larch needles when attacked by the larvae of Generation IV. also confirms this. The young needles when they first emerge from the buds are exceedingly tender, and if attacked at this stage, they at once respond to the influence. Each needle becomes bent into the shape of a knee at the point where the insect is seated, and very frequently the "knee" is distinctly swollen, shewing that the tissue has responded to a slight extent and formed a minute gall. The needles however quickly harden under the influence of light and the gall developes no further. When the needles are older and harder the attacks produce neither bending nor swelling, but the chlorophyll still responds in exactly the same way as it does in the Spruce gall. It disappears, leaving a yellow spot to mark each place where an insect has been seated. When old Spruce needles are attacked by the Alatae, the Sexuparae or the Sexuales, the same thing is apparent, and I have little doubt that the injection is the same in all cases, but that the hardening of the surrounding tissues prevents the formation of a gall.

Note on some peculiar features in seedlings of Peperomia. By Mr A. W. HILL.

[*Read* 28 November 1904.]

The seedlings of *Peperomia umbilicata* were found in the Andes of Bolivia at about 13,500 feet above sea-level. The species is a geophilous one with small bulbs and peltate leaves.

The peculiarity of the seedlings lies in the fact that, though they are dicotyledonous in structure, only one of the two cotyledons leaves the seed to exercise the functions of an assimilating organ; the other remains permanently in the seed as an absorbent organ.

The other bulbous species from the Andes apparently shew the same features of germination, and several other species from Central America, preserved in the Herbaria of Kew and South Kensington, whilst differing in their vegetative habits, shew a similar type of germination.

*On the dynamical significance of Kundt's law of selective dispersion, in connexion with the transmission of the energy of trains of dispersive waves**. By J. LARMOR, M.A., St John's College, Lucasian Professor of Mathematics.

[*Read* 14 November 1904.]

1. This subject has been treated by the writer, *Phil. Trans.* 1897A, p. 243. If we are content with a naïve theory of an independent vibrator for each line of the spectrum, or even with a conception of a molecule as a connected dynamical system vibrating about a position *of rest*, it is readily inferred that the index of refraction always trends upward with increasing frequency, so as to be abnormally great on the lower side of an absorption band and abnormally small on the upper side of it, as was originally remarked by Kundt to be actually the case in anomalously dispersive media. But when the molecule is considered, as it must be, notwithstanding analytical difficulties, to be a dynamical system vibrating about a permanent state of steady cyclic motion, the gyrostatic terms in the equations of its vibrations render a theoretical discussion of Kundt's law difficult, and it was not then completely effected; though it was easy to see that the law should be connected with the necessarily positive quality of the vibratory energy. A recent paper by Prof. Lamb, in *Proc. Math. Soc.*, consequent upon a remark by Prof. Schuster, seems to afford a key to the matter.

When we speak in optics of wave-length λ_0, we can only mean light of wave-lengths comprehended within a small interval $\delta\lambda$ around λ_0,—that is a train of undulations of wave-length uncertain from point to point within this interval $\delta\lambda$, which is itself a measure of the defect of purity of the beam. There is no such thing as an absolutely homogeneous train of wave-length exactly λ_0. Now Sir George Stokes has explained (Smith's Prize Examination Questions, 1876) how in such an undamped train, nearly homogeneous, the velocity of propagation, considered as the rate at which the actual disturbance travels onwards, is in a dispersive medium not the wave-length λ of the wave-form divided by its periodic time τ, but is

$$d\frac{1}{\tau}\Big/d\frac{1}{\lambda}.$$

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* The first section of this note was communicated to the British Association at the meeting in Cambridge last August.

And Lord Rayleigh has verified (*Proc. Math. Soc.* 1877; 'Theory of Sound,' vol. i, Appendix) by a direct analysis of high generality that the energy of the undulations is in fact conveyed with this velocity. Thus if the medium could be such that the frequency diminished with diminishing wave-length, the energy would travel backwards, namely in a direction opposite to that in which its wave-form is propagated. The question considered by Schuster and Lamb is as to whether this is actually possible. Examples are given by Prof. Lamb of infinitely extended trains in which it occurs. But such a train requires to be fed with energy, so to speak, at both ends. With a train of light advancing across a dispersive medium from one side only, the case cannot occur, for the energy cannot become negative*.

The curve of dispersion is usually constructed with the frequency ν, equal to τ^{-1}, as abscissa, and the index of refraction μ as ordinate. If c is the velocity of light in a vacuum

$$\lambda = \frac{c}{\mu}\Big/\nu.$$

Thus the group velocity U is $c d\nu/d(\mu\nu)$. Hence

$$\frac{c}{U} = \frac{d(\mu\nu)}{d\nu} = \mu + \nu\frac{d\mu}{d\nu}.$$

This must be positive; thus $d\mu/d\nu$ can indeed be negative, but the greatest allowable negative value at any point is $-\mu/\nu$. So far then as this present condition goes, the dispersion curve *may* trend downwards, but to a limited extent, outside a band of absorption, whereas upward trend is unrestricted. The condition that a source of radiation must be emitting, not absorbing, energy thus allows the dispersion curve to be an undulating line in a region not containing absorption bands, but with a limit to the steepness of the downward steps of the curve. In a region of absorption this restriction is not imposed; hence the trend will usually be downward. Thus the character of the curve of dispersion would be in the main that remarked by Kundt, though the present consideration by itself does not require an upward trend in transparent regions to be invariable.

2. The general analytical verification of the rate of propagation of energy by Lord Rayleigh, above mentioned, prompted by a more special discussion by Prof. O. Reynolds, involves use of the proposition that the mean energy, in stationary or progressive

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* This argument would be evaded if we could suppose that the wave-form travels backward: but in the case of a train which is not endless, the wave-form is of necessity propagated forwards, that is, in the direction in which the front of the train advances.

vibratory systems, not affected by viscosity, is on the average half potential and half kinetic. The ultimate foundation of this important general principle seems to be adequately contained in the following remark, and more abstruse considerations are hardly necessary. The characteristic of all kinds of steady standing vibrations or undulations in a non-dissipative system, however complicated in its structure, is that each particle of the system describes an orbit in simple harmonic motion. For each particle therefore the two kinds of energy, kinetic and potential, are equal on the average; and this includes their equality in the aggregate. For uniform trains of progressive waves similar considerations remain applicable. The argument extends also to the most complex types of dispersive media, in cases where absorption is negligible: it is to be noticed that the energy that is 'propagated' with the group-velocity is there the energy in the ether, together with that of the vibratory disturbance of the stationary molecules, —which latter is on the theory of cyclic systems of Kelvin and Routh a perfectly definite quantity added on to the intrinsic kinetic energy of each molecule.

3. The principle involved in propagation by groups may be somewhat generalised, thus affording insight into its essential character. An equation

$$y = Af(nx - pt)$$

represents a disturbance whose profile is of the form $y = Af(nx)$, propagated onward with uniform velocity p/n. When the form is a periodically undulating one the equation represents a progressive wave-train of definite wave-length. The problem is to specify the general features, if any, presented by an aggregate of related progressive forms, all differing but slightly from a central type $y \propto f(n_0 x - p_0 t)$ but otherwise undetermined. We may represent one of these component forms by $\delta y = \delta A \,.\, f\{(n_0 + \delta n)\,x - (p_0 + \delta p)\,t\}$, where δn and δp are treated as infinitesimal. The definiteness of type of the forms under consideration implies that n is given as some function of p, so that $\delta n/\delta p$ is here a function of p_0 and so constant, say U_0. The application of Taylor's theorem allows us in general to express the component form above mentioned as

$$\delta y = \delta A f(n_0 x - p_0 t) + (\delta n . x - \delta p . t)\, \delta A f'(n_0 x - p_0 t).$$

This expansion holds so long as $\delta n . x - \delta p . t$ is small: that is, it is valid for the neighbourhood of a point travelling with the definite velocity $\delta p/\delta n$, say U_0. Under this restriction, the aggregate of a crowd of such closely related progressive forms is therefore represented by an equation

$$y = \phi\,(x - U_0 t)\, F\,(n_0 x - p_0 t).$$

Thus, to an observer travelling with velocity U_0, so that $x - U_0 t$ remains constant, the aggregate disturbance *in his neighbourhood* has the features of a simple disturbance of type $y = CF(n_0 x - p_0 t)$, namely of form $y = CF(n_0 x)$ travelling with velocity p_0/n_0. This resultant form $y = CF(n_0 x)$ is not the same as the original standard form $y = Af(n_0 x)$, belonging to the components which make it up. But if the component form is represented by a sine or cosine, the resultant is also of that type; and an observer travelling with velocity U_0 then keeps in touch with a steady aggregate disturbance of the standard type, which travels as a group in company with him.

The generalised result is the proposition that the local features of a train formed of disturbances of nearly identical types travel with a definite velocity, different from the velocity of propagation of the individual waves, and therefore travel through the waves; but the travelling group is of the same type as the individual disturbances which in the aggregate give rise to it, only when these latter are simple harmonic trains represented by sines or cosines.

Thus, for example, we may for an instant imagine a radiation, the aggregate of an indefinitely great number of similar discrete radiant pulses shot out from the radiating molecules, each pulse being propagated without change of type with a velocity determined by the scale of its dimensions in space of time: in the aggregate they would present the appearance of disturbances travelling onward with the group-velocity determined above, but there would be no similarity between these groups and the original type of pulse. This illustration is however purely ideal, for the dynamics of propagation require that it is only single harmonic wave-trains that can travel unchanged in a dispersive medium.

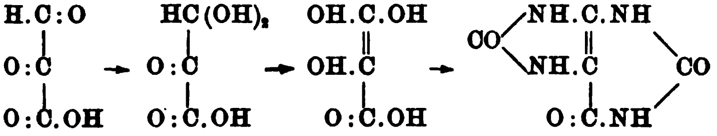
An attempted Synthesis of Uric Acid. By H. J. H. Fenton.

[*Read* 14 November 1904.]

Most of the so-called derivatives of uric acid have been synthetically prepared by condensation of their characteristic parent acids with urea, but in the case of uric acid itself this direct synthesis has not been effected owing to the fact that its parent acid—trihydroxyacrylic acid—is yet unknown.

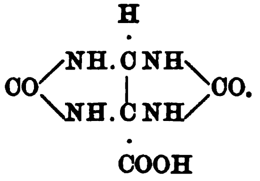
It has been shewn by the author in previous communications that by the oxidation of dihydroxymaleic acid with ferric or mercuric salts it is possible to obtain the semi-aldehyde of mesoxalic acid $CHO.CO.COOH$, and it is evident that the aldehyde hydrate of this might be tautomeric with the desired trihydroxyacrylic acid.

Attempts are now being made to bring about the condensation of mesoxalic semi-aldehyde with urea and it seems not improbable that under favourable conditions the synthesis of uric acid might be accomplished according to the following scheme:

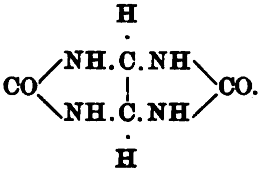


or that, failing this, an acid isomeric with pseudo-uric acid might result, namely,

glyco-uril carboxylic acid



When the condensation is attempted in aqueous solution carbon dioxide is evolved and a very sparingly soluble crystalline substance is obtained, which proves to be glyco-uril



This compound was obtained in the first instance by reduction of allantoin with sodium amalgam (Reineck), and afterwards by condensation of glyoxal with urea (Schiff). In the present case its production probably results from decomposition of glyco-uril carboxylic acid and not from the previous formation of glyoxal, since mesoxalic semi-aldehyde itself is relatively stable under the conditions of the experiment; but this matter requires further investigation.

The properties of glyco-uril have been carefully studied by many investigators, but a remarkable colour reaction, which appears to be very characteristic, has been overlooked. If the substance is evaporated to dryness on a water-bath with strong nitric acid and the residue treated with caustic soda only a slight pink or orange colour is obtained, but on the addition of sodium hypochlorite this changes to an intense purple.

Note on the Diffusion of Hydrogen through Palladium. By O. W. RICHARDSON, M.A., Trinity College.

[*Received* 15 November 1904.]

The rate of diffusion of hydrogen through a palladium tube has been examined quantitatively by Winkelmann[*], who found, contrary to what one might expect *à priori*, that the mass of gas diffusing in unit time at constant temperature was proportional to the gradient of a power of the pressure less than unity. Winkelmann explained this result by supposing that the hydrogen dissociated and that only the atoms had the power of diffusing through the metal. More recently an interesting research on the same subject has been published by G. N. St Schmidt[†], whose conclusions might tend to throw doubt on the advisability of the assumption of dissociation.

The diffusion of hydrogen through platinum was also examined by Winkelmann[‡], who again found that the pressure variation could be explained on the hypothesis that dissociation occurred. A more detailed examination of this subject has been made by the author conjointly with J. Nicol and T. Parnell[§]. The principal results of this investigation were that the rate of diffusion varied, (1) as the gradient of the square root of the pressure, down to pressure differences of 3 mm., and (2) as an exponential function of the temperature of the tube. All the results were found to be accurately expressed by a formula deduced theoretically on the hypothesis that the hydrogen inside the metal was in a state of dissociative equilibrium and that only the dissociated atoms diffused freely. The experiments yielded no evidence of an appreciable amount of dissociated hydrogen outside the metal.

In view of the consistent explanation of the diffusion of hydrogen through platinum furnished by the dissociation theory and the general similarity in the phenomena as regards both metals, it does not seem too much to suggest that a hypothesis which appears to furnish a complete explanation of the one case should not be lightly rejected as regards the other. The object of the present note is to point out that some of the objections to the dissociation hypothesis raised by Schmidt's work are not so great as might appear at first sight.

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[*] *Drude's Ann.* Vol. VI. p. 104.

[†] *Ib.* Vol. XIII. p. 747.

[‡] *Ib.* Vol. VIII. p. 388.

[§] *Phil. Mag.* S. 6, Vol. VIII. p. 1.

Before proceeding it is necessary to remark upon one important difference in the nature of the substances which result when hydrogen is absorbed by platinum and palladium respectively. It seems now to be generally admitted that the balance of evidence is in favour of both cases being manifestations of solution rather than of chemical combination. The work of Hoitsema* indicates that in the case of palladium hydrogen, if the composition lies between certain limits, a mixture of two immiscible solutions is formed: whereas, so far as is known, in the case of platinum this does not occur. Naturally the occurrence of two immiscible solutions together would considerably complicate the diffusion phenomena; but fortunately all Schmidt's experiments seem to have been made at pressures slightly lower, or temperatures slightly higher, than those at which the solution of greater hydrogen concentration begins to form. This probably explains why irregularities were encountered when it was sought to push the experiments to lower temperatures.

The author† has shown that the rate of flow through a slab of solvent of thickness d, of a diatomic gas which dissociates in solution, when the amount of the external gas which is dissociated is negligible, is given, per unit area, by the expression

$$\left[\frac{\mu_1}{A}(P_0 - P_1) + \frac{\mu_2}{2}\left(\frac{k_2}{A}\right)^{\frac{1}{2}}(P_0^{\frac{1}{2}} - P_1^{\frac{1}{2}})\right]\frac{1}{d},$$

where P_0, P_1 are the gas pressures on the sides;

μ_1, μ_2 are the coefficients of diffusion of the molecules and atoms respectively;

A is the partial solubility of the undissociated gas,

and k_2 is the constant of the reaction $H_2 \rightleftarrows 2H$ inside the metal.

This formula applies to any case where the properties of the solution are continuous, and therefore will presumably apply to Schmidt's experiments on palladium if the dissociation theory is true.

The first point is to enquire how the rate of diffusion varies with the pressure at constant temperature. Winkelmann‡, who had zero pressure on one side of his tube and pressures up to atmospheric on the other, found that the rate of diffusion varied in one experiment very nearly as the ·71th power of the pressure, which would come within the range of the above formula. Schmidt*, however, finds that his results agree more nearly with the view that the rate of diffusion is proportional to the pressure difference on the two sides. These statements are not however necessarily contradictory, as the experiments were not made under the same conditions. In Schmidt's case there was a high pressure on both sides of the palladium tube so that the square root term would not be proportional to the square root of the difference of the pressures, but to the difference of the square roots of the pressures. Now the differences of the square roots of a series of numbers like a, $a+b$, $a+c$, etc., where b, c, etc. are small, are approximately proportional to the differences c, b. Hence, unless the experiments of Schmidt were very accurate we should not expect them to be capable of deciding how the rate of diffusion varied with the pressure.

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* *Zeitschrift Physikal. Chemie*, Vol. xvii. p. 1.

† *Phil. Mag.* S. 6, Vol. vii. p. 266; Vol. viii. p. 14.

‡ *Drude's Ann.* Vol. vi. p. 104.

A closer examination of Schmidt's numbers fully substantiates this conclusion. The numbers given in Tables XIV—XXVII of his paper are the times necessary for the volume of hydrogen inside the apparatus to increase by 1 cm. of the barometer tube. If P_1 is the pressure and θ the temperature of the gas in the barometer tube it will be readily seen from the data given that the number of gram molecules which correspond to this increase in volume is given by

$$\cdot 4005 \times 5{\cdot}93 \times 10^{-6} \times \frac{273\,P_1}{273+\theta}.$$

Now $273+\theta$ is constant during the experiments within the total experimental error so that the mass of gas diffusing per second varies as P_1/t, where t is the value given in the third column of the tables cited. Hence if the rate of diffusion were proportional to the pressure difference $(P_0 - P_1)\frac{t}{P_1}$ should be constant, if it were as the square root of the pressure $(P_0^{\frac{1}{2}} - P_1^{\frac{1}{2}})\frac{t}{P_1}$ would be constant, whilst on the general dissociation theory this constant quantity would be one of the above terms plus a constant quantity into the other.

To test the relative agreement of these formulae values of $(P_0 - P_1)\,t/P_1$ and $(P_0^{\frac{1}{2}} - P_1^{\frac{1}{2}})\,t/P_1$ calculated from Schmidt's numbers are given in the following table. The first column gives the table in Schmidt's paper from which the numbers are taken, the second the temperature in degrees centigrade of the palladium tube during the experiment, the third the values of t/P_1, whilst the fourth and fifth contain values of $(P_0 - P_1)\,t/P_1$ and $(P_0^{\frac{1}{2}} - P_1^{\frac{1}{2}})\,t/P_1$ respectively.

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* *loc. cit.* p. 767.

I	II	III	IV	V
Table	Temperature Centigrade	t/P_1	$(P_0-P_1)\,t/P_1$	$(P_0^{\frac{1}{2}}-P_1^{\frac{1}{2}})\,t/P_1$
XXII	300°	·224 ·286 ·366 ·503	6·72 7·15 7·32 7·55	·441 ·454 ·454 ·447
XXIV	290°	·240 ·288 ·366 ·473	7·20 7·20 7·32 7·10	·468 ·458 ·458 ·435
XXV	222°	·408 ·508 ·608 ·887	12·24 12·70 12·16 13·30	·799 ·804 ·748 ·798
XXVI	218°	·433 ·528 ·668 ·860	12·99 13·20 13·36 12·90	·844 ·840 ·836 ·792
XIV	170° 166° 140°	·98 ·84 1·63	29·4 25·2 48·9	1·91 1·61 3·18
XV	300° 225° 149°	·246 ·431 1·51	7·38 12·93 45·30	·479 ·840 2·940
XVI	282°	·239	7·17	·466

The constancy of the numbers in columns IV and V for any one temperature serves as a test between the two formulae, and for this purpose the first four lots, each of which was taken under identical conditions except for the pressure variation, are of most value. It will be observed that the values from Tables XXII and XXV give the greatest constancy with the square root formula, those from Table XXIV agree best with the direct formula, whilst in the case of Table XXVI the agreement is about the same in both cases. We conclude therefore that the range of pressure differences employed by Schmidt is not great enough to settle this question.

It may perhaps not be out of place to point out that, if we may assume the term involving the square roots of the pressures to be the more important of the two, the values given by Schmidt for the quantity of gas diffusing in unit time at different temperatures are in agreement with what we should expect on the dissociation theory. On that view the temperature variation of the rate of diffusion depends almost entirely on the variation of k_2 the dissociation constant of the hydrogen inside the metal, which in turn is given by the formula $k_2 = C\theta\epsilon^{-q/2\theta}$, where C is a constant and q is the heat necessary to dissociate one molecule of hydrogen inside the metal. Following the method of calculation in *Phil. Mag.*, Ser. 6, Vol. VIII. p. 23 we find

$$\log_{10} L + \tfrac{1}{2}\log_{10}\theta = B + \frac{q}{9.2}\,1/\theta,$$

where L is one of the numbers in column V of the preceding table and B is a constant.

The following values were obtained by taking the means for neighbouring temperatures of the numbers previously quoted from Tables XIV—XVI of Schmidt's paper.

I	II	III
Mean Temperature Centigrade	**$1/\theta$ absolute**	**$\log_{10} L + \frac{1}{2}\log_{10}\theta$**
144°	$2{\cdot}40 \times 10^{-3}$	1·796
168°	$2{\cdot}27 \times 10^{-3}$	1·570
225°	$2{\cdot}01 \times 10^{-3}$	1·273
291°	$1{\cdot}77 \times 10^{-3}$	1·0505

It will be noticed that the numbers in columns II and III come very near to satisfying the linear relation given above.

If we calculate the value of q for the above figures we get $10{\cdot}88 \times 10^3$ calories. The corresponding value for hydrogen dissolved in platinum was found to be $9{\cdot}125 \times 10^3$ calories. These quantities are of the same order of magnitude as the heats of solution of one gram molecule of hydrogen in the metals, which were found by Mond, Ramsay, and Shields* to be $8{\cdot}74 \times 10^3$ and $13{\cdot}22 \times 10^3$ calories for palladium and platinum respectively. It will be noticed that if the dissolved hydrogen is practically all dissociated the sum of these two quantities for any one metal would be the heat of solution of H_2 if no dissociation took place.

This note will not altogether fail in its purpose if it succeeds in drawing attention to the fact that fresh experiments on this subject are required. No doubt accurate experiments are more difficult to make with palladium than with platinum, and to obtain unexceptionable results two points must be carefully borne in mind. One is to make certain of the pressure law by using as great a range of pressure differences as possible, and the other is never to let the external pressure become so great as to admit of the formation of two immiscible solid hydrogen solutions inside the metal.

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* *Phil. Trans.* A. Vol. cxci. p. 105.

Some Optically Active Nitrogen Compounds. By Miss M. B. THOMAS and H. O. JONES, M.A., Clare College.

[*Read* 14 November 1904.]

The work was undertaken in order to institute a quantitative comparison of the molecular rotatory powers of different substituted quaternary ammonium bases. The compounds of this type which have hitherto been described give salts with camphor-sulphonic and brom-camphor-sulphonic acids which are stable and easily soluble in water. These salts can, in many cases, be resolved into their *d* and *l* components by fractional crystallisation from suitable solvents. Since the molecular rotation of salts of active acids and bases in dilute aqueous solution is the sum of those of the acidic and basic ions, the rotation of the basic ion can be obtained by subtraction of that of the acidic ion from the total molecular rotation. Since the rotatory power of an ion appears to be unaffected by many of the conditions which alter that of liquids or of non-electrolytes in solution to a considerable extent, it is possible that any regularities which exist may be more evident in the values for ions. The bases investigated are those in which three of the groups substituting ammonium hydrogen atoms (phenyl, benzyl, and methyl) are kept the same, while the fourth is successively replaced by the isopropyl and isoamyl radicals. The brom-camphor-sulphonates or camphor-sulphonates of these bases respectively were prepared by Pope and Peachey's method, and fractionally crystallised until a constant rotation was obtained.

Phenyl-benzyl-isopropyl-methyl ammonium iodide was obtained by mixing methyl-isopropyl-aniline with benzyl iodide in molecular proportions. The iodide thus formed was recrystallised from methylated spirit (M.P. = 133° C.) and heated with the calculated quantity of silver *d* brom-camphor-sulphonate in ethyl acetate. The solution after filtration from the precipitated silver iodide deposited crystals of phenyl-benzyl-isopropyl-methyl ammonium *d* brom-camphor-sulphonate (M.P. = 184° C.). This was recrystallised from (1) ethyl acetate or (2) *water*. The salt of the laevo base is the less soluble in both liquids, and after three crystallisations from either, a constant molecular rotation ($[M]_D = -123°$) in dilute aqueous solution was obtained. Thus for the laevo basic ion $[M]_D = -393°$ ($[M]_D$ for *d* brom-camphor-sulphonic ion $= 270°$).

l phenyl-benzyl-isopropyl-methyl ammonium iodide was precipitated from the brom-camphor-sulphonate by addition of potassium iodide to the water solution. The active iodide melts at a slightly lower temperature (132°) than the inactive iodide and crystallises in quite a different form.

The crystals of both iodides have been examined by Mr Dain, of Clare College. The laevo iodide crystallises in the tetragonal system, whereas the crystals of the inactive iodide belong to the oblique system. The inactive iodide must therefore be a racemic compound: the melting point of either iodide is however depressed only very slightly by admixture with the other.

The active iodide is stable in alcoholic solution and $[M]_D$ in absolute alcohol $= -428°$. It is however racemised in chloroform solution, a solution with a molecular rotatory power $[M]_D = -508°$ becoming completely inactive after two days.

Phenyl-benzyl-isoamyl-methyl ammonium iodide was prepared by treatment of methyl-isoamyl-aniline with the theoretical quantity of benzyl iodide. After crystallising from alcohol it formed small prisms which melted at 156° C. The *d* camphor-sulphonate was prepared from it in the usual manner. The salt of the laevo base is the less soluble in acetone, but many recrystallisations are required before the separation of the *d* and *l* constituents is complete. After twenty recrystallisations the salt melts at 178—8·5° C. and has $[M]_D = -317°$, and the rotation is not yet constant. The value of $[M]_D$ for the *d*-camphor-sulphonic ion is 51·7° so that the basic ion already has a molecular rotatory power of nearly $-370°$. The iodide recovered from the camphor-sulphonate melts at the same temperature as the inactive, gives $[M]_D = -478°$ in alcohol, and racemises in chloroform solution.

A comparison of these results with those obtained for the corresponding ethyl and allyl compounds shows:

(1) That substitution of the isopropyl or isoamyl for the ethyl group causes a change in the molecular rotation of more than 370° and 350° respectively ($[M]_D$ for phenyl-benzyl-ethyl-methyl ammonium ion $= 19°$).

(2) Substitution of isopropyl or isoamyl for allyl increases the molecular rotation to a great extent ($[M]_D$ for phenyl-benzyl-methyl-allyl ammonium ion $= 166°$).

These results are striking, and several other compounds are in process of resolution in the hope that some regularity in the effect of the various radicals may be observed. It is noteworthy that the effect of an unsaturated group in increasing the rotatory power, which is so frequent in other series of compounds, is not shown in this series.

On the Inheritance of Tortoiseshell and Related Colours in Cats. By L. DONCASTER, M.A., King's College.

[*Read* 28 November 1904.]

It is well known that Tortoiseshell cats are almost always females, and it is commonly said that the corresponding colour in males is orange (otherwise described as red or yellow). In the hope of throwing light on certain other cases of varieties confined to one sex I was led to inquire into the matter more fully, and with that object wrote to a number of owners of pedigree cats asking for further information. I received a considerable number of replies, which are so closely in agreement that there can be little doubt about the accuracy of the statements, and it is wholly upon the evidence so obtained that this note is based.

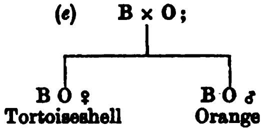
Tortoiseshell kittens may be obtained from the following kinds of matings:

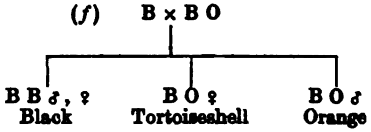
(*a*) Tortoiseshell ♀ × Tortoiseshell ♂.
(*b*) „ × black or blue ♂.
(*c*) „ × orange ♂.
(*d*) Orange ♀ × orange ♂.
(*e*) „ × black or blue ♂.
(*f*) Black or blue ♀ × orange ♂.

In all of these matings in addition to Tortoiseshells, kittens of other colours may appear, viz.:

(*a*) Tortoiseshell ♀ × Tortoiseshell ♂ gives Tort., orange, black.
(*b*) Tortoiseshell ♀ × black or blue ♂ gives Tort. ♀, orange ♂, black ♂, ♀.
(*c*) Tortoiseshell ♀ × orange ♂ gives Tort., orange, black.
(*d*) Orange ♀ × orange ♂ gives *either* Tort., orange (and in one case blue, *see below*); *or*, only orange.
(*e*) Orange ♀ × black ♂ gives Tort. ♀, orange ♂.
(*f*) Black ♀ × orange ♂ gives Tort., black (and probably orange).
(*g*) Black ♀ × black ♂ gives only black (or blue).

From these facts it is clear that tortoiseshell is a heterozygous form; it is produced by mating orange with black, both of which can be bred true, but in the rare cases when a tortoiseshell ♀ has been paired with a tortoiseshell ♂, orange and black kittens have been produced in addition to tortoiseshells. The question then at once arises, why are tortoiseshells almost exclusively females, the number of certainly known males of this colour being very small? The answer appears to be that in the male orange is completely dominant over black, while in the female the dominance is incomplete, and tortoiseshell results. On this hypothesis all the facts given above are explicable: for example, in mating (*e*) orange ♀ × black ♂ gives only tortoiseshell and orange kittens, since both the parents are homozygous, and of the offspring (heterozygous) the females are tortoiseshell, the males orange. In the reverse mating (*f*) black ♀ × orange ♂, the male may be heterozygous, and hence blacks appear in the offspring. If black be represented by *B*, orange by *O*, we get





In the same way in mating (*b*) tortoiseshell ♀ × black ♂, orange appears in the offspring, since the male heterozygotes are orange, the female tortoiseshell.

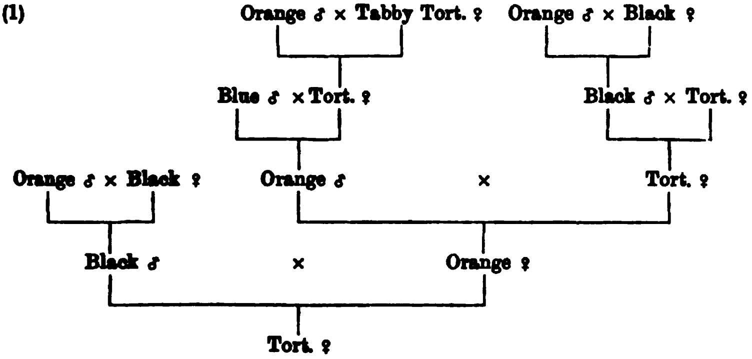
This accounts also for the fact that orange females are very rare, although males are common, for in all matings in which one of the parents is black, orange can appear only in the male offspring. If therefore the great majority of orange males contain recessive black, when they are paired with tortoiseshells, only a quarter of the kittens will be pure orange, and only half of these females.

In addition to the evidence concerning the relations between orange and black, I have obtained a quantity of information respecting cream and blue. These two colours appear to be dilute forms of orange and black, and have exactly similar relations to one another. Two blues, from yellow ancestors, when paired together give only blue offspring, and are therefore homozygous; a cream ♀ × blue ♂ gives blue tortoiseshell (blue and cream) females, cream males, but no blues; a blue ♀ × cream ♂ on the other hand gives blue tortoiseshell females, blues of both sexes (and possibly cream males, but of this I have no certain evidence). Blue tortoiseshell ♀ × blue ♂ gives blue tortoiseshell females, blues of both sexes, and creams (mostly, possibly always males); blue tortoiseshell ♀ × cream ♂ gives blue tortoiseshell females, blues and creams of both sexes. These results are exactly like those obtained with orange and black, and can be accounted for completely on the hypothesis that cream is dominant over blue in the male, but in the female the heterozygote is the blue tortoiseshell.

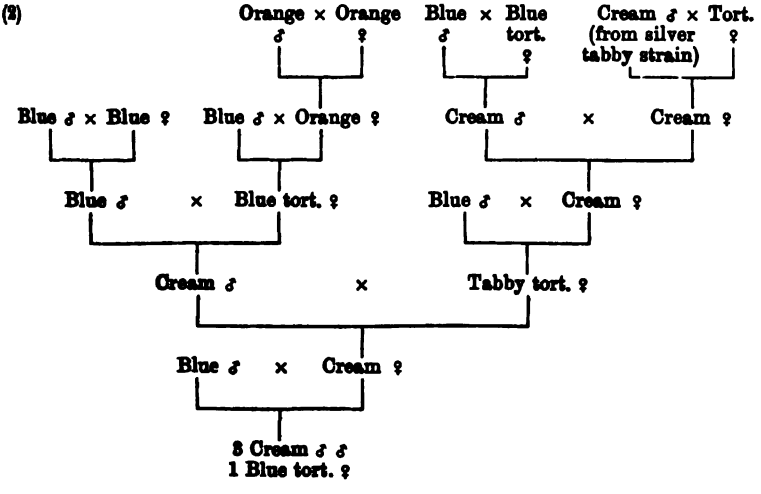
Creams may be obtained by pairing orange with blue, *i.e.* the dilution may be transferred from the blue to the orange, yielding cream; but this does not take place in all the kittens, since orange, blue, and tortoiseshell may also appear. It seems also that although in the female orange is not completely dominant over black, yet it may contain recessive blue, for in one case an orange female with blue in its ancestry, when paired with an orange male, gave three tortoiseshell females and one blue male. Orange may probably contain recessive cream in either sex, and black is certainly a simple dominant over blue.

To illustrate the above I give two genealogical tables *.

(1)



(2)



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* For the second table, with many others, I am indebted to Miss W. Beal; for the first, chiefly to Miss A. M. Roper.

Several smaller points remain which are not quite clear. On the hypothesis that in the male orange is dominant over black and cream over blue, while in the female these combinations give tortoiseshells, yellow males should result from the pairing black ♀ × orange ♂ (or blue ♀ × cream ♂). I have obtained no certain evidence that this occurs; my correspondents mention black (or blue) and tortoiseshell, but say nothing of yellows, but in one mating of this kind a male tortoiseshell was produced.

Male tortoiseshells are occasionally obtained, and it must be supposed that in them the dominance of the orange is incomplete. A tortoiseshell ♂ paired with a female of the same colour gives tortoiseshell, orange, and black kittens. I hear however of one case where a tortoiseshell mated with his own daughters of the same colour produced almost entirely orange or orange-and-white kittens, and a small proportion of blacks; with other tortoiseshell females my correspondent says "he sired tortoiseshells freely."

All the colours mentioned may be associated with a certain amount of white, and this reappears in the offspring without affecting the inheritance of other colours; the presence of white (*i.e.* the piebald character) seems to be a separate allelomorph as in the case of rats and mice.

My investigation of the inheritance of tortoiseshell colour was undertaken in connexion with a general inquiry into the nature of varieties confined to one sex, but does not appear to be analogous to other cases known to me.

Varieties confined to one sex appear to be of several kinds; they may be Mendelian recessives, as in the variety *lacticolor* of *Abraxas grossulariata*, exhibited this year at the British Association, and in this case by suitable mating the variety may be transferred to the male. Human colour-blindness is possibly also of this nature, but is usually found in the male instead of the female. A second type is when a local race resembles the normal form in one sex, but differs in the other, *e.g.* ♀ var. *bryoniae* in *Pieris napi*, or ♂ var. *rustica* in *Spilosoma mendica*. In such cases a cross between the two races yields intermediates, and reciprocal crosses give similar results. Finally in the tortoiseshell cat the female variety seems to be produced by defective dominance in one sex of a colour completely dominant in the other.

In conclusion I wish to acknowledge my indebtedness to my numerous correspondents, especially Miss W. Beal, and Sir C. Alexander, for the large amount of information received from them, only a small part of which I have been able to include in the account given.

On the vibrations of atoms containing 4, 5, 6, 7, *and* 8 *corpuscles and on the effect of a magnetic field on such vibrations.* By J. J. THOMSON, M.A., F.R.S., Cavendish Professor of Experimental Physics, Cambridge.

[*Received* 27 December 1904.]

The paper contains an investigation of the time of vibration of model atoms of some complexity, it was undertaken in the hope that the properties of these models might throw some light on the effect of a magnetic field on the lines in the spectra of luminous bodies.

Case of Four Corpuscles.

We suppose that we have four equal corpuscles with equal charges of negative electricity placed within a sphere of uniform positive electrification; when in equilibrium the corpuscles will be situated at the corners of a regular tetrahedron with its centre at the centre of the sphere. We can easily show that if b is the radius of the sphere of positive electrification, and a the distance of any one of the corpuscles from the centre of the sphere, then when the attraction of the positive sphere on the corpuscle is equal and opposite to the repulsion exerted by the other three corpuscles

$$\frac{a^3}{b^3} = \frac{3\sqrt{3}}{16\sqrt{2}},$$

the positive charge in the sphere being equal in magnitude to the sum of the negative charges on the four corpuscles. This result may be expressed by saying that when in equilibrium the distance between two corpuscles is equal to b.

Let us call the equilibrium positions of the four particles 1, 2, 3, 4 and let us take as our axes of coordinates the lines joining the middle points of opposite sides of the tetrahedron. The line joining the middle points of (12), (34); (13), (24); (14), (23) being taken as the axes of x, y, z respectively; the coordinates of the points 1, 2, 3, 4 are respectively (α, α, α), $(\alpha, -\alpha, -\alpha)$, $(-\alpha, \alpha, -\alpha)$, $(-\alpha, -\alpha, \alpha)$, where $3\alpha^2 = a^2$.

If (ξ_r, η_r, ζ_r) are the displacements of the rth particle from its position of equilibrium, m the mass and e the charge on a corpuscle, we can show that the vibrations of the system are given by the equations:

$$(1) \quad m\frac{d^2p}{dt^2} = -kp, \quad m\frac{d^2q}{dt^2} = -kq, \quad m\frac{d^2r}{dt^2} = -kr,$$

where

$$p = \xi_1 + \xi_2 + \xi_3 + \xi_4, \quad q = \eta_1 + \eta_2 + \eta_3 + \eta_4, \quad r = \zeta_1 + \zeta_2 + \zeta_3 + \zeta_4,$$

and
$$k = \frac{4e^2}{b^3};$$

thus p, q, r which are proportional to the coordinates of the centre of gravity of the four corpuscles are principal coordinates and their vibrations give three periods each equal to $2\pi\sqrt{\frac{m}{k}}$.

If

$$p_1 = \xi_1 + \xi_2 - (\xi_3 + \xi_4), \quad q_1 = \eta_1 + \eta_2 - (\eta_3 + \eta_4), \quad r_1 = \zeta_1 + \zeta_2 - (\zeta_3 + \zeta_4),$$
$$p_2 = \xi_1 + \xi_3 - (\xi_2 + \xi_4), \quad q_2 = \eta_1 + \eta_3 - (\eta_2 + \eta_4), \quad r_2 = \zeta_1 + \zeta_3 - (\zeta_2 + \zeta_4),$$
$$p_3 = \xi_1 + \xi_4 - (\xi_2 + \xi_3), \quad q_3 = \eta_1 + \eta_4 - (\eta_2 + \eta_3), \quad r_3 = \zeta_1 + \zeta_4 - (\zeta_2 + \zeta_3),$$

we find

$$m\frac{d^2}{dt^2}(p_2 - q_1) = 0, \quad m\frac{d^2}{dt^2}(p_3 - r_1) = 0, \quad m\frac{d^2}{dt^2}(q_3 - r_2) = 0,$$

$$m\frac{d^2}{dt^2}(p_2 + q_1) = -k_1(p_2 + q_1), \quad m\frac{d^2}{dt^2}(p_3 + r_1) = -k_1(p_3 + r_1),$$

$$m\frac{d^2}{dt^2}(q_3 + r_2) = -k_1(q_3 + r_2), \quad m\frac{d^2}{dt^2}(p_1 + q_2 + r_3) = -k_2(p_1 + q_2 + r_3),$$

$$m\frac{d^2}{dt^2}(p_1 - q_2) = -k_3(p_1 - q_2), \quad m\frac{d^2}{dt^2}(p_1 - r_3) = -k_3(p_1 - r_3),$$

where
$$k_1 = \frac{6e^2}{b^3} = \tfrac{3}{2}k,$$
$$k_2 = \frac{12e^2}{b^3} = 3k,$$
$$k_3 = \frac{3e^2}{b^3} = \tfrac{3}{4}k.$$

Thus the remaining nine principal coordinates of the system, which has twelve degrees of freedom, are

$p_2 - q_1$, $p_3 - r_1$, $q_3 - r_2$, the three corresponding periods of vibration are infinite;

$p_2 + q_1$, $p_3 + r_1$, $q_3 + r_2$, the three corresponding periods of vibration are $2\pi\sqrt{\frac{m}{k}\frac{2}{3}}$;

$p_1 + q_2 + r_3$ with a period $2\pi\sqrt{\frac{m}{3k}}$;

$p_1 - q_2$, $p_1 - r_3$ with two periods $2\pi\sqrt{\frac{m}{k}\frac{4}{3}}$.

Thus there are four different periods the squares of which are in the proportion of 1, 2, 3, 4.

Effect of a magnetic field on the vibrations of the tetrahedron. If α, β, γ are the components of the external magnetic field, there will be terms in the expression for $m\frac{d^2\xi_r}{dt^2}$, equal to $e\beta\frac{d\zeta_r}{dt} - e\gamma\frac{d\eta_r}{dt}$, in addition to the terms independent of the magnetic field, and corresponding terms in the expressions for $\frac{d^2\eta_r}{dt^2}$, $\frac{d^2\zeta_r}{dt^2}$.

Hence instead of the equations

$$m\frac{d^2p}{dt^2} = -kp, \text{ etc.,}$$

we have

$$m\frac{d^2p}{dt^2} = -kp + e\beta\frac{d}{dt}(\zeta_1 + \zeta_2 + \zeta_3 + \zeta_4) - e\gamma\frac{d}{dt}(\eta_1 + \eta_2 + \eta_3 + \eta_4),$$

or

$$m\frac{d^2p}{dt^2} = -kp + e\beta\frac{dr}{dt} - e\gamma\frac{dq}{dt}.$$

Similarly

$$m\frac{d^2q}{dt^2} = -kq + e\gamma\frac{dp}{dt} - e\alpha\frac{dr}{dt},$$

$$m\frac{d^2r}{dt^2} = -kr + e\alpha\frac{dq}{dt} - e\beta\frac{dp}{dt},$$

or if p, q, r vary as $e^{\iota\mu t}$, we have

$$(m\mu^2 - k)^3 - (m\mu^2 - k)\,\mu^2 e^2(\alpha^2 + \beta^2 + \gamma^2) = 0.$$

The roots of this are

$$\mu_0 = \sqrt{\frac{k}{m}},$$

$$\mu_1 = \mu_0 + \tfrac{1}{2}\frac{He}{m},$$

$$\mu_2 = \mu_0 - \tfrac{1}{2}\frac{He}{m},$$

where $H = \sqrt{\alpha^2 + \beta^2 + \gamma^2}$.

The changes in the periods of this oscillation are the same as those of the single particle investigated by Lorentz.

The effects on the other principal coordinates are more complex.

Let
$$p_2+q_1=\psi_3,\quad p_3+r_1=\phi_2,\quad q_3+r_2=\theta_1,$$
$$p_2-q_1=\psi_3',\quad p_3-r_1=\phi_2',\quad q_3-r_2=\theta_1',$$
$$p_1+q_2+r_3=s,$$
$$p_1-q_2=\lambda_1,$$
$$p_1-r_3=\lambda_2.$$

Then the equation
$$m\frac{d^2\psi_3}{dt^2}=-k_1\psi_3$$

becomes when there is a magnetic field
$$m\frac{d^2\psi_3}{dt^2}=-k_1\psi_3+e\beta\frac{d}{dt}(\zeta_1+\zeta_3-(\zeta_2+\zeta_4))-e\gamma\frac{d}{dt}(\eta_1+\eta_3-(\eta_2+\eta_4))$$
$$+e\gamma\frac{d}{dt}(\xi_1+\xi_2-(\xi_3+\xi_4))-e\alpha\frac{d}{dt}(\zeta_1+\zeta_2-(\zeta_3+\zeta_4)),$$

or $$m\frac{d^2\psi_3}{dt^2}=-k_1\psi_3+e\beta\,\tfrac{1}{2}\frac{d}{dt}(\theta_1-\theta_1')+e\gamma\frac{d\lambda_1}{dt}-e\alpha\,\tfrac{1}{2}\frac{d}{dt}(\phi_2-\phi_2').$$

Now if the vibrations are only slightly disturbed we need only retain on the right-hand side the variables whose free periods are the same as that of ψ_3, i.e. we need only retain the terms in θ_1 and ϕ_2, hence we have
$$m\frac{d^2\psi_3}{dt^2}=-k\psi_3+\tfrac{1}{2}e\beta\frac{d\theta_1}{dt}-\tfrac{1}{2}e\alpha\frac{d\phi_2}{dt}.$$

Similarly $$m\frac{d^2\phi_2}{dt^2}=-k\phi_2+\tfrac{1}{2}e\alpha\frac{d\psi_3}{dt}-\tfrac{1}{2}e\gamma\frac{d\theta_1}{dt},$$
$$m\frac{d^2\theta_1}{dt^2}=-k\theta_1+\tfrac{1}{2}e\gamma\frac{d\phi_2}{dt}-\tfrac{1}{2}e\beta\frac{d\psi_3}{dt}.$$

From the form of these equations we see that they differ from those for a single particle in that the terms involving the magnetic force are multiplied by the factor $\frac{1}{2}$, hence the effect of the magnetic force on this vibration is only $\frac{1}{2}$ the normal effect, while the effect on the vibration corresponding to p, q, r is as we have seen normal. The magnetic field has, to a first approximation, no effect on the vibrations of the coordinates s, λ_1, λ_2.

Case of Five Corpuscles.

One position of equilibrium is when four of the corpuscles are at the corners of a regular tetrahedron while the fifth is at the centre of the tetrahedron.

Let x, y, z be the coordinates of the centre of gravity of the five corpuscles, ξ_1, η_1, ζ_1, ξ_2, η_2, ζ_2, ξ_3, η_3, ζ_3, ξ_4, η_4, ζ_4 the coordinates of the four outer corpuscles relative to the central corpuscle, the direction of the axes being the same as in the last example.

Then if

$$\xi_1+\xi_2+\xi_3+\xi_4=p,\quad \eta_1+\eta_2+\eta_3+\eta_4=q,\quad \zeta_1+\zeta_2+\zeta_3+\zeta_4=r,$$
$$\xi_1+\xi_2-(\xi_3+\xi_4)=p_1,\quad \eta_1+\eta_2-(\eta_3+\eta_4)=q_1,\quad \zeta_1+\zeta_2-(\zeta_3+\zeta_4)=r_1,$$
$$\xi_1+\xi_3-(\xi_2+\xi_4)=p_2,\quad \eta_1+\eta_3-(\eta_2+\eta_4)=q_2,\quad \zeta_1+\zeta_3-(\zeta_2+\zeta_4)=r_2,$$
$$\xi_1+\xi_4-(\xi_2+\xi_3)=p_3,\quad \eta_1+\eta_4-(\eta_2+\eta_3)=q_3,\quad \zeta_1+\zeta_4-(\zeta_2+\zeta_3)=r_3,$$

we have

$$m\frac{d^2x}{dt^2}=-kx,\qquad m\frac{d^2y}{dt^2}=-ky,\qquad m\frac{d^2z}{dt^2}=-kz.$$

Thus, x, y, z are principal coordinates, the time of vibration being $2\pi\sqrt{\frac{m}{k}}$.

We have also

$$m\frac{d^2}{dt^2}(p_1+q_2+r_3)=-k_1(p_1+q_2+r_3),$$
$$m\frac{d^2}{dt^2}(p_1-q_2)=-k_2(p_1-q_2)\text{ and }m\frac{d^2}{dt^2}(p_1-r_3)=-k_2(p_1-r_3),$$

where
$$k_1=3k,\quad k_2=\frac{\dfrac{3}{16\sqrt{2}}}{\dfrac{1}{3\sqrt{3}}+\dfrac{1}{4\sqrt{2}}}k.$$

Thus $p_1+q_2+r_3$, p_1-q_2, p_1-r_3 are principal coordinates.

Again

$$m\frac{d^2}{dt^2}(p_2-q_1)=0,\quad m\frac{d^2}{dt^2}(p_3-r_1)=0,\quad m\frac{d^2}{dt^2}(q_3-r_2)=0.$$

Thus p_2-q_1, p_3-r_1, q_3-r_2 are also principal coordinates.

The remaining six principal coordinates are

$$r+\lambda_1(p_2+q_1),\quad r+\lambda_2(p_2+q_1),$$
$$q+\lambda_1(r_1+p_3),\quad q+\lambda_2(r_1+p_3),$$
$$p+\lambda_1(q_3+r_2),\quad p+\lambda_2(q_3+r_2),$$

where λ_1, λ_2 are the roots of the equation

$$\lambda^2 = \lambda\left(\frac{1}{2} - \frac{3\sqrt{3}}{8\sqrt{8}}\right) + 1,$$

or approximately

$$\lambda_1 = \tfrac{9}{8},$$
$$\lambda_2 = -\tfrac{7}{8}.$$

The corresponding periods are

$$2\pi\sqrt{\frac{m}{k_1}},\quad 2\pi\sqrt{\frac{m}{k_2}},$$

where $\quad k_1 = 7{\cdot}3k, \quad k_2 = {\cdot}038k.$

Effect of a magnetic field on the periods. The effects on the vibrations corresponding to x, y, z are normal, that on

$$p_1 + q_2 + r_3,\quad p_1 - q_2,\quad p_1 - r_3$$

is zero, while the effects on the coordinates

$$r + \lambda_1(p_3 + q_1),\quad r + \lambda_2(p_3 + q_1),$$

are respectively about $\frac{11}{32}$ and $\frac{5}{32}$ of the normal.

Case of Six Corpuscles.

The six particles when in equilibrium will be at the corners of a regular octahedron. Denoting the positions of the corpuscles when in equilibrium by the symbols 1, 2, 3, 4, 5, 6, let us take the line joining 1 and 6 as the axis of x, that joining 2 and 5 as the axis of y and that joining 3 and 4 as the axis of z. If a is the distance of any one of the corpuscles in its undisturbed position from the centre of the sphere of positive electrification then the coordinates of the points 1, 2, 3, 4, 5, 6 are as follows 1 $(a, 0, 0)$, 2 $(0, a, 0)$, 3 $(0, 0, a)$, 4 $(0, 0, -a)$, 5 $(0, -a, 0)$, 6 $(-a, 0, 0)$; let ξ_r, η_r, ζ_r be the displacement of the rth corpuscle from its position of equilibrium, then, m being the mass and e the charge on a corpuscle, the oscillations of the corpuscles will be given by the following equations

$$m\frac{d^2p}{dt^2} = -kp,\quad m\frac{d^2q}{dt^2} = -kq,\quad m\frac{d^2r}{dt^2} = -kr,$$

where

$$p = \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 + \xi_6,$$
$$q = \eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \eta_6,$$
$$r = \zeta_1 + \zeta_2 + \zeta_3 + \zeta_4 + \zeta_5 + \zeta_6,$$

and $k = \frac{6e^2}{b^3}$, where b is the radius of the sphere of positive electricity. Thus p, q, r, which are proportional to the coordinates of the centre of gravity of the six corpuscles, are principal coordinates; the time of vibration of each of these coordinates is

$$2\pi\sqrt{\frac{m}{k}}.$$

If

$$\begin{aligned}
p_1 &= 2(\xi_1 + \xi_6) - (\xi_2 + \xi_5 + \xi_3 + \xi_4),\\
p_2 &= (\xi_2 + \xi_5) - (\xi_3 + \xi_4),\\
q_1 &= 2(\eta_2 + \eta_5) - (\eta_1 + \eta_6 + \eta_3 + \eta_4),\\
q_2 &= \eta_1 + \eta_6 - (\eta_3 + \eta_4),\\
r_1 &= 2(\zeta_3 + \zeta_4) - (\zeta_1 + \zeta_6 + \zeta_2 + \zeta_5),\\
r_2 &= \zeta_1 + \zeta_6 - (\zeta_3 + \zeta_4),
\end{aligned}$$

then we can show that

$$m\frac{d^2p_1}{dt^2} = -k_1p_1, \quad m\frac{d^2p_2}{dt^2} = -k_2p_2,$$

with similar equations for q_1, q_2, r_1, r_2. Hence $p_1, p_2, q_1, q_2, r_1, r_2$ are principal coordinates, the time of vibration corresponding to the coordinates with the suffix one is $2\pi\sqrt{\frac{m}{k_1}}$, that corresponding to those with the suffix two is $2\pi\sqrt{\frac{m}{k_2}}$,

$$k_1 = \frac{\frac{11}{8}\sqrt{2}+1}{4\sqrt{2}+1}k; \quad k_2 = \frac{\frac{5}{8}\sqrt{2}+1}{4\sqrt{2}+1}k.$$

If

$$\begin{aligned}
\xi_1 - \xi_6 &= \theta_1, & \eta_1 - \eta_6 &= \phi_1, & \zeta_1 - \zeta_6 &= \psi_1,\\
\xi_2 - \xi_5 &= \theta_2, & \eta_2 - \eta_5 &= \phi_2, & \zeta_2 - \zeta_5 &= \psi_2,\\
\xi_3 - \xi_4 &= \theta_3, & \eta_3 - \eta_4 &= \phi_3, & \zeta_3 - \zeta_4 &= \psi_3,
\end{aligned}$$

then

$$m\frac{d^2}{dt^2}(\phi_3 - \psi_2) = 0, \quad m\frac{d^2}{dt^2}(\phi_1 - \theta_2) = 0, \quad m\frac{d^2}{dt^2}(\theta_3 - \psi_1) = 0,$$

$$m\frac{d^2}{dt^2}(\phi_3 + \psi_2) = -k_3(\phi_3 + \psi_2), \quad m\frac{d^2}{dt^2}(\phi_1 + \theta_2) = -k_3(\phi_1 + \theta_2),$$

$$m\frac{d^2}{dt^2}(\theta_3 + \psi_1) = k_3(\theta_3 + \psi_1),$$

where

$$k_3 = \frac{6}{4 + \frac{1}{\sqrt{2}}}k.$$

Thus $\phi_3 \pm \psi_2$, $\phi_1 \pm \theta_2$, $\theta_3 \pm \psi_1$ are principal coordinates, three of the times of vibration are infinite, and three equal to

$$2\pi\sqrt{\frac{m}{k_3}}.$$

We have also

$$m\frac{d^2}{dt^2}(\theta_1+\phi_2+\psi_3) = -k_4(\theta_1+\phi_2+\psi_3),$$

$$m\frac{d^2}{dt^2}(\theta_1-\phi_2) = -k_5(\theta_1-\phi_2),\quad m\frac{d^2}{dt^2}(\theta_1-\psi_3) = -k_5(\theta_1-\psi_3).$$

Hence the three remaining principal coordinates are

$$\theta_1+\phi_2+\psi_3,\ \theta_1-\phi_2,\ \theta_1-\psi_3,$$

$$k_4 = \frac{12\sqrt{2}+3}{4\sqrt{2}+1}k \text{ and } k_5 = \frac{3\sqrt{2}+3}{4\sqrt{2}+1}k.$$

Effect of the magnetic field. We can show by the method already used for the tetrahedron, that the effect of the magnetic field on the coordinates p, q, r is normal, that on the coordinates $p_1, p_2, q_1, q_2, r_1, r_2$, $\phi_3+\psi_2$, $\phi_1+\theta_2$, $\theta_3+\psi_1$ half normal, and that on the coordinates $\theta_1+\phi_2+\psi_3$, $\theta_1-\phi_2$, $\theta_1-\psi_3$ zero.

Case of Seven Corpuscles.

An arrangement in which the corpuscles are in equilibrium though not as we shall see in stable equilibrium, is when six are at the corners of an octahedron and one at the centre.

If x, y, z are the coordinates of the centre of gravity of the corpuscles, ξ_r, η_r, ζ_r the displacement of one of the outer corpuscles relative to the central one, then we can show that x, y, z are principal coordinates, and that the equilibrium is stable for displacements which change only the values of x, y, z. Using the notation of the last paragraph, we find $\phi_3 \pm \psi_2$, $\phi_1 \pm \theta_2$, $\theta_3 \pm \psi_1$, $\theta_1+\phi_2+\psi_3$, $\theta_1-\phi_2$, $\theta_1-\psi_3$, p_2, q_2, r_2 are again principal coordinates for whose displacements the equilibrium is stable.

The remaining six principal coordinates are of the form

$$\xi_1+\xi_6+\lambda_1(\xi_2+\xi_3+\xi_4+\xi_5),$$

$$\xi_1+\xi_6+\lambda_2(\xi_2+\xi_3+\xi_4+\xi_5),$$

where λ_1, λ_2 are the roots of a quadratic equation, with corresponding coordinates for the η's and ζs; for these displacements the equilibrium is unstable.

The stable form for seven corpuscles would seem to be five in one plane at the corners of a pentagon, the other two on opposite sides of the plane, the line joining them passing perpendicularly to the plane through the centre of the pentagon, the distance of either of these from the centre is about $\frac{59}{60}$ of the distance of the other five from the same point.

Case of Eight Corpuscles.

These when in equilibrium will be arranged at the corners of a cube. Take as the axes of x, y, z the lines through the centre of this cube parallel to its sides. Denoting the eight corpuscles by the symbols (1), (2), (3), (4), (5), (6), (7), (8), let the coordinates of these when in equilibrium be given by the scheme

$$(1)\ (a, a, a),\quad (2)\ (-a, a, a),\quad (3)\ (a, -a, a),\quad (4)\ (a, a, -a),$$
$$(8)\ (-a, -a, -a),\ (7)\ (a, -a, a),\ (6)\ (-a, a, -a),\ (5)\ (-a, -a, a).$$

Let the displacement of the rth particle from its position of equilibrium be ξ_r, η_r, ζ_r.

For the sake of brevity denote $\xi_r + \xi_{9-r}$ by X_r, $\xi_r - \xi_{9-r}$ by X_r', $\eta_r + \eta_{9-r}$ by Y_r, $\eta_r - \eta_{9-r}$ by Y_r', $\zeta_r + \zeta_{9-r}$ by Z_r, $\zeta_r - \zeta_{9-r}$ by Z_r', then we may show that

$$m\frac{d^2}{dt^2}(X_1+X_2+X_3+X_4) = -k(X_1+X_2+X_3+X_4),$$

$$m\frac{d^2}{dt^2}(Y_1+Y_2+Y_3+Y_4) = -k(Y_1+Y_2+Y_3+Y_4),$$

$$m\frac{d^2}{dt^2}(Z_1+Z_2+Z_3+Z_4) = -k(Z_1+Z_2+Z_3+Z_4),$$

where $k = \frac{8e^2}{b^3}$, and b is the radius of the sphere of positive electrification. Thus ΣX, ΣY, ΣZ are principal coordinates, they are proportional to the coordinates of the centre of gravity; the time of vibration corresponding to these coordinates is $2\pi\sqrt{\frac{m}{k}}$.

If
$$X_1+X_2-(X_3+X_4)+Y_1+Y_2-(Y_3+Y_4) = \psi_1,$$
$$X_1+X_2-(X_3+X_4)-(Y_1+Y_2)+Y_3+Y_4 = \psi_2,$$
$$X_1+X_4-(X_2+X_3)+Z_1+Z_2-(Z_3+Z_4) = \phi_1,$$
$$X_1+X_4-(X_2+X_3)-(Z_1+Z_2)+Z_3+Z_4 = \phi_2,$$
$$Y_1+Y_4-(Y_2+Y_3)+(Z_1+Z_3)-(Z_2+Z_4) = \theta_1,$$
$$Y_1+Y_4-(Y_2+Y_3)-(Z_1+Z_3)+(Z_2+Z_4) = \theta_2,$$

then

$$m\frac{d^2\psi_1}{dt^2} = -k_1\psi_1, \quad m\frac{d^2\phi_1}{dt^2} = -k_1\phi_1, \quad m\frac{d^2\theta_1}{dt^2} = -k_1\theta_1,$$

$$m\frac{d^2\psi_2}{dt^2} = -k_2\psi_2, \quad m\frac{d^2\phi_2}{dt^2} = -k_2\phi_2, \quad m\frac{d^2\theta_2}{dt^2} = -k_2\theta_2,$$

where
$$k_1 = \frac{4 + \frac{4}{\sqrt{2}} + \frac{2}{3\sqrt{3}}}{2 + \sqrt{2} + \frac{2}{3\sqrt{3}}}k,$$

$$k_2 = \frac{4 + \frac{1}{\sqrt{2}} + \frac{2}{3\sqrt{3}}}{2 + \sqrt{2} + \frac{2}{3\sqrt{3}}}k.$$

The arrangement of eight corpuscles at the corners of a cube is however unstable, for, if

$$p = X_1 + X_2 - (X_3 + X_4),$$
$$q = Y_1 + Y_3 - (Y_2 + Y_4),$$
$$r = Z_1 + Z_4 - (Z_2 + Z_3),$$

we find

$$m\frac{d^2}{dt^2}(p-q) = \left(2 - \frac{3}{2\sqrt{2}} - \frac{2}{3\sqrt{3}}\right)\frac{e^2}{a^3}(p-q),$$

and, since $2 - \frac{3}{2\sqrt{2}} - \frac{2}{3\sqrt{3}}$ is positive, the cubical arrangement is unstable for a displacement when $p - q$ has a finite value. We can show also that the cubical arrangement is unstable, even although a corpuscle is placed at the centre.

The stable arrangement for eight particles is when we have six particles at the corners of an octahedron and two others at equal distances from the centre on one of the lines joining opposite corners of the octahedron; the octahedron will no longer be regular, the axis along which the two extra particles are placed being longer than the other two, approximately in the proportion of 4 to 3, the distance of the inner particles from the centre is about $\frac{3}{5}$ of the distance of the furthest corner of the octahedron from the centre.

On the positive electrification of a rays, and the emission of slowly moving cathode rays by radio-active substances. By J. J. THOMSON, M.A., F.R.S.

[*Read* 14 November 1904.]

The experiments described below were undertaken with the object of detecting the positive charge on the α rays. Rutherford has shown that the α rays are deflected by electric and magnetic forces in the same direction as a positively electrified body, but I

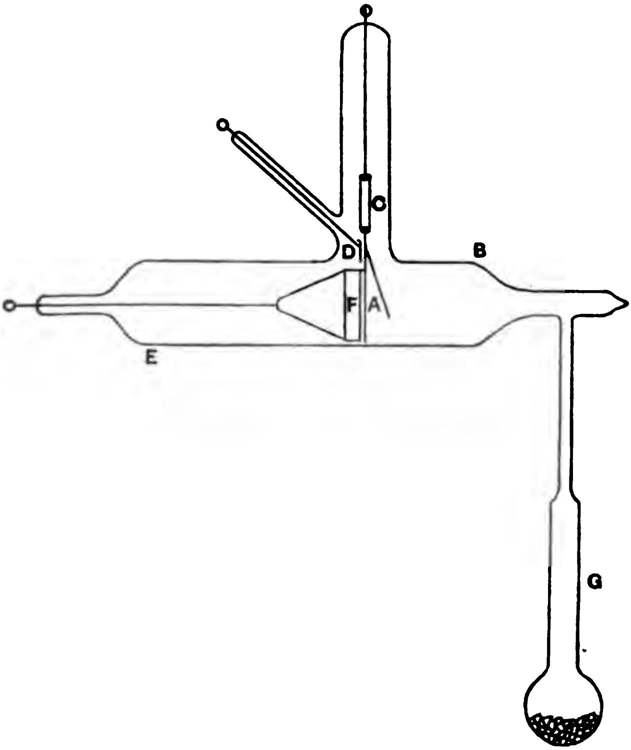

FIG. 1.

am not aware of any experiments in which the positive electricity carried by these rays has been transferred to a conductor and detected by the electrification of the conductor. I only succeeded in doing this after a lengthy investigation in the course of which some properties of polonium and radium, which seem interesting and important, came to light.

The first experiments were made with polonium. This body is supposed only to give out α rays so that it seemed especially suitable for this purpose; it was found however that polonium gives out in addition to α rays large quantities of slowly moving cathode rays.

The polonium (obtained from Sthamer) was deposited on two discs, one bismuth, the other copper. The method used was as follows. A gold-leaf electroscope *A* (Fig. 1) was placed in *B*, a large glass vessel coated with tinfoil, connected with the earth. The leaves of the electroscope were supported by a stick of fused quartz *C*, and were charged by the wire *D*. The polonium discs were placed in the side tube *E*, in the earlier experiments the disc was about 4 cm. from the electroscope. *G* is a tube containing hard charcoal, it was added for the purpose of obtaining high vacua by Dewar's method of immersing charcoal in liquid air; for the experiments we are describing to succeed the vacuum has to be exceedingly good, and I have found Dewar's method of the greatest possible assistance, giving in a few hours better vacua than I previously obtained by several days' pumping.

The vessel containing the electroscope and polonium was set up between the poles of a powerful electro-magnet which when excited gave a field of about 1200 C.G.S. units between the polonium and the electroscope.

In these experiments the electroscope was charged alternately with positive and negative electricity, and the rate of collapse of the leaves measured by a reading microscope provided with a micrometer eye-piece. The potential to which the electroscope was charged was ± 180 volts. If the polonium gave out only α rays we should expect the rate of leak to be much more rapid when the electroscope is charged with negative electricity than when charged with positive. It was found however that exactly the opposite effect occurred, the leak being almost negligible when the charge in the electroscope is negative and very appreciable when the charge is positive. The following observations are samples of a large number all showing the same effect.

Distance of polonium from electroscope 4 cm.

Leak in five minutes in scale divisions of micrometer eye-piece	Charge on electroscope	Magnet
70	+	off
2·5	+	on
5	−	off
5	−	on

Distance of polonium from electroscope 5 cm. (*this was a different tube*).

50	+	off
1	+	on
1	−	off
1	−	on

Thus we see that when the electroscope is charged with positive electricity there is a large leak which is practically stopped by a magnetic field, while when the charge is negative there is very little leak. This shows that the polonium gives out streams of negative electricity, we have however as yet no indication of any emission of positive. I was never able to get an increase in the negative charge of the electroscope, this I attribute to the velocity of the cathode rays given out by the polonium being so low that they are not able to make their way against the repulsion exerted by the negatively electrified electroscope. In order to be sure that the effect was due to the polonium I had the polonium disc mounted on swivels so that by tapping the disc could be made to rotate about a vertical axis and the face on which the polonium is deposited made to face towards or away from the electroscope. When the polonium-coated face of the disc was towards the electroscope, the leak, when the latter was charged positively, was 45 divisions when the magnet was off, six divisions when the magnet was on; when the electroscope was charged negatively the leak was seven divisions when the magnet was off, three when it was on; the disc was then tapped until it turned through 180 degrees, the polonium-coated face now facing down the tube *E*, and away from the electroscope, the leak when the electroscope was positively charged was now only three when the magnet was off, and it was too small to detect when the magnet was on. When the electroscope was negatively charged the leak was one with the magnet off, zero with it on. This shows that the leak observed is due to the polonium. There is however a considerable leak when the disc is so placed that rays proceeding normally from the disc could not strike against the electroscope, thus when the disc was tapped so that it turned through an angle of 90°, from the position in which the polonium faced the electroscope, the leak in the electroscope for a positive charge with the magnet off only fell from 45 to 22, although in the new position no rays at right angles to the disc could reach the electroscope. The negative particles which discharge the disc in this case come from the walls of the vessel and are either diffusely reflected cathode rays originating from the polonium or are due to secondary radiation excited by the rays from the polonium.

The absence of any appreciable leak in the preceding experiments, when the electroscope is charged negatively, even when the negatively charged cathode rays are prevented by the magnetic field from reaching the electroscope, shows that the *a* rays must lose their positive charge, or rather have it neutralized by the negative charge on the cathode rays, as they pass through the swarm of cathode rays. These experiments have given no indication of the positive charge on the *a* rays. I will now pass on to some experiments by which the positive charge on these rays can be detected. In these the polonium disc instead of being at a distance of 4 or 5 cm. from the electroscope is placed as close to it as possible: the distance in the experiments in which the following results were obtained was 1 millimetre.

Leak in scale divisions in five minutes	Charge on electroscope	Magnet
31	+	off
0	+	on
9	–	off
5	–	on

It will be noticed that although with the magnet off the leak is as before much greater when the electroscope is charged positively than when it is charged negatively, owing to the copious emission of slow cathode rays by the polonium, yet now when the magnetic field is on the negatively electrified body shows by far the bigger leak, in fact there is no appreciable leak with the positive charge. We see that now the *a* particles are able to reach the electroscope and give up their charge to it, these particles are but little affected by the magnetic field, while the cathode particles are almost entirely stopped by it. The leak when the electroscope is charged negatively cannot be wholly due to secondary cathode rays emitted by the electroscope when bombarded by the *a* rays from the polonium, for these cathode rays unless moving with a velocity of about 10^{10} cm./sec. would be stopped by the magnetic field, although it is probable that when no magnetic field acts upon the vessel, part of the leak, when the electroscope is negative, may be due to this cause. A series of measurements were made varying the distance between the polonium and the electroscope, and it was found that as the distance increased the leak increased, when the charge in the electroscope was positive, and diminished when it was negative.

Experiments with Radium.

Similar experiments were made using radium instead of polonium, the radium used was one of the earlier samples procured from de Haen and was not very strong. The radium was spread over a horizontal disc placed about 2 millimetres below a similar disc fastened to the electroscope, the results obtained with this were the same as with the polonium. Thus when the electroscope was charged positively the leak was 39 in 10 minutes with the magnet off, 7 with the magnet on, while when the electroscope was charged negatively the leak was 11 with the magnet off, 10 with the magnet on. Thus we have again the positive leak larger than the negative when the magnet is off, and smaller when it is on. The negative leak diminishes as the distance of the radium from the electroscope increases, and when this distance is a few centimetres the negative leak is very small compared with the positive.

The preceding experiments show that polonium and radium emit copious streams of slowly moving negative corpuscles, these streams only differ in the velocity of the particles from the β rays hitherto investigated, and it is to be observed that they would escape detection by the methods hitherto used to investigate β rays, for such methods only detect β rays able to traverse a considerable thickness of aluminium foil, *i.e.* β rays of great velocity, in fact the ability to penetrate this foil has been taken as the definition of a β ray. We see too that if we define an α ray as one absorbed by a thin layer of aluminium foil, that α rays may be of two kinds, one carrying positive, the other negative charges. The existence of these slow negative rays may have a very important bearing on the transformations which radio-active matter undergoes, it may also be the explanation why the radium emanation which does not emit the fast β rays, yet seems to acquire a positive charge and be attracted by negatively electrified bodies; this would be the case if the emanation like polonium gave out the slow β rays. Investigations are in progress at the Cavendish Laboratory to see whether the emission of these slow cathode rays is a general property of radio-active substances.

Another point brought out by these experiments is the way in which these slow β rays tend to neutralize the positive charge on the α rays. This tendency for the α rays to lose their charge has an important bearing on the determination of e/m for the α rays. In these determinations it has been assumed that the α ray retained its positive charge during the whole of its path under the electric and magnetic forces, in the light of the above experiments it seems by no means certain that this is the case. The point could be tested by making experiments with different lengths of paths; if the a ray retains its charge, alteration in the length of path would not affect e/m, if the ray lost its charge it would.

I wish to acknowledge my obligation to my assistant, Mr E. Everett, for the help he has given me with these experiments.

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PROCEEDINGS

OF THE

CAMBRIDGE PHILOSOPHICAL SOCIETY.

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Note on a property of orthogonal covariants of a binary quantic. By ARTHUR BERRY, M.A., King's College.

[*Received* 17 March 1905.]

Let $f \equiv a_0x^n + na_1x^{n-1}y + \dots + a_ny^n$ be a binary quantic, let Ω, O denote the usual operators

$$a_0\frac{d}{da_1} + 2a_1\frac{d}{da_2}\dots, \quad na_1\frac{d}{da_0} + (n-1)a_2\frac{d}{da_1}\dots,$$

and let Δ be written for $\Omega - O$. A function of the coefficients and variables which is unaffected, or changed in sign, when the variables undergo an orthogonal transformation is called an orthogonal covariant. I consider in this note only such "direct" orthogonal covariants as are unchanged by orthogonal transformations of the direct or rotational form

$$x = \cos\theta X - \sin\theta Y, \quad y = -\sin\theta X + \cos\theta Y.$$

It is clearly sufficient to consider only such covariants as are homogeneous both in the variables and in the coefficients. The necessary and sufficient condition that a rational integral function of $a_0, a_1, \dots x, y$ should be a direct orthogonal covariant is known* to be that it should be annihilated by the operator

$$y\frac{d}{dx} - x\frac{d}{dy} - \Delta \text{ ....(1).}$$

Let the covariant be

$$C_0x^m + C_1x^{m-1}y + \dots + C_my^m = 0.$$

---

* See for example Elliott's *Algebra of Quantics*, chap. xv., or Sylvester's original paper quoted below.

Applying the annihilator and equating coefficients of the separate powers of x and y to zero we get the system of equations:

$$\left.\begin{aligned}\Delta C_0 + C_1 &= 0\\ -mC_0 + \Delta C_1 + 2C_2 &= 0\\ -(m-1)\,C_1 + \Delta C_2 + 3C_3 &= 0\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots&\cdots\\ -(m-r)\,C_r + \Delta C_{r+1} + (r+2)\,C_{r+2} &= 0\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots&\cdots\\ -C_{m-1} + \Delta C_m &= 0\end{aligned}\right\}\cdots\cdots\cdots\text{(I)}.$$

Eliminating $C_1, C_2, \ldots C_m$ we see that C_0 satisfies the partial differential equation

$$\phi_m(\Delta)\,C_0 = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\text{(2)},$$

where $\phi_m(\Delta)$ is the symbolic determinant

$$\left|\begin{array}{ccccccc}\Delta, & 1, & 0, & 0, & 0, & \cdots\cdots\cdots\cdots & 0\\ -m, & \Delta, & 2, & 0, & 0, & \cdots\cdots\cdots\cdots & 0\\ 0, & -(m-1), & \Delta, & 3, & 0, & \cdots\cdots\cdots\cdots & \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots\cdots\cdots & \cdots\\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots\cdots\cdots & 0\\ \cdots\cdots & \cdots\cdots & \cdots\cdots & 0, & -2, & \Delta, & m\\ \cdots\cdots & \cdots\cdots & \cdots\cdots & 0, & 0, & -1, & \Delta\end{array}\right|$$

In this determinant the elements in the leading diagonal are all Δ, those in the parallel line immediately above and on the right are $1, 2, \ldots m$, those in the parallel line immediately below and on the left are $-m, -(m-1), \ldots -2, -1$, and all the other elements are zero.

To reduce this determinant let us first add the 3rd column to the 1st, the 4th to the 2nd, the 5th to the 3rd and so on, thus obtaining a modified determinant which it is easy to write but troublesome to print. In this new determinant let us subtract the 1st and 2nd rows respectively from the 3rd and 4th; the 3rd and 4th thus modified respectively from the 5th and 6th, and so on. The last two rows now become

$$\begin{array}{cccccc}0, & 0, & \ldots\, 0, & \Delta, & m,\\ 0, & 0, & \ldots\, 0, & -m, & \Delta,\end{array}$$

so that $\Delta^2 + m^2$ is a factor, and the remaining factor, i.e. the minor obtained by suppressing the mth and $m+1$th rows and columns, is

$$\phi_{m-2}(\Delta).$$

Thus $$\phi_m = (\Delta^2 + m^2)\,\phi_{m-2}(\Delta).$$

Now $$\phi(1) = \Delta^2 + 1, \quad \phi(2) = (\Delta^2 + 2^2)\,\Delta,$$

so that, if m is even

$$\phi(\Delta) \equiv (\Delta^2 + m^2)\,\{\Delta^2 + (m-2)^2\} \ldots (\Delta^2 + 2^2)\,\Delta \ldots\ldots (3),$$

and if m is odd

$$\phi(\Delta) \equiv (\Delta^2 + m^2)\,\{\Delta^2 + (m-2)^2\} \ldots (\Delta^2 + 1^2) \ldots\ldots\ldots (4).$$

The equations (I) shew that, if the source C_0 is found so as to satisfy the differential equation (2) then C_1, C_2, ... and hence the whole covariant are determined uniquely. If the source of a covariant of order m is annihilated by an operator $\phi_{m-2r}(\Delta)$ obtained by omitting r factors at the beginning of $\phi_m(\Delta)$, it is at once evident that the covariant is the product of a covariant of order $m-2r$ by $(x^2+y^2)^r$. It is suggested that, if the source of a covariant of order m is annihilated by an operator made up of some other selection of the factors of $\phi_m(\Delta)$, then the covariant is expressible in terms of covariants of lower order, but I have not succeeded in proving any such result.

The result of this note was given in a question set in the Mathematical Tripos, Part II. 1897. I had not then read Sylvester's original paper "On the Principles of the Calculus of Forms*"; I now find that the equation $\phi(\kappa) = 0$ occurs there (p. 359 of the reprint) as an auxiliary equation in connection with the problem of integrating the differential equation corresponding to (1), but its significance as a differential equation satisfied by the source is not indicated.

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[*] *Cambridge and Dublin Mathematical Journal*, VII. (1852), pp. 179—217, reprinted in *Mathematical Papers*, Vol. I., 48, pp. 328—363.

Note on the Positive Leak from Hot Platinum in Air at Low Pressures. By O. W. RICHARDSON, M.A., Trinity College.

[*Read* 30 January 1905.]

In the *Philosophical Magazine* [6], Vol. VI. p. 80, the author showed that the current from a fresh positively charged hot platinum wire in a good vacuum fell off asymptotically with the time the wire was heated at a constant temperature. Recent experiments have shown that if fresh air be admitted into the apparatus from time to time and the wire reheated in this atmosphere the falling off reaches a limit, or at any rate the rate of decay of the current becomes inappreciable. With the wire in this state the author has recently made experiments to see if the magnitude of the current is affected by the surrounding gas, with the following results.

In commencing an experiment the apparatus was pumped out until the McLeod gauge registered a pressure of about ·0001 mm. and the wire heated until the positive leak did not change by 10 °/₀ of its value in half-an-hour. Air was then admitted to a pressure of about 2 cms. and pumped out again to a pressure of ·2 mms. The temperature of the hot wire, as determined by its resistance, was then brought to its initial value and the positive leak measured. After letting in fresh air the leak was found to fall off at constant temperature in a similar way to that from a new wire. This effect may be due to a number of causes and has not been fully investigated. After a time the rate of decay became small, and when the current did not fall off more than 10 °/₀ in half-an-hour, readings were taken at different pressures. The following table represents a series of results with 3·5 volts mean potential on the filament:—

Pressure	Current
67	29·1
33	20
16	16·8
8	14·5
·5	13·3

The unit of pressure is $\frac{1}{800}$ mm. and that of current is 5·6 × 10^{-4} electrostatic units per second.

The numbers in the above table represent a linear relation between the current and the pressure. Thus the leak consists of two parts, one of which is proportional to, and the other independent of, the pressure. Similar results were obtained with a potential of + 80 volts on the filament. The part of the leak which is independent of the pressure was found to be nearly independent of the potential over a range from 40 to 200 volts.

Some Methods of Increasing the Spark-Length of the Wimshurst Machine. By B. J. PALMER, B.SC. (Communicated by Professor J. J. THOMSON.)

[*Read* January 30, 1905.]

Some interesting experiments have recently been published by Humphreys (*Physical Review*, Vol. x., No. 5) showing how the normal spark-length of a Wimshurst can be greatly increased by taking a spark from the negative terminal. Professor J. J. Thomson suggested I should endeavour to measure this increment and so find out its cause. My experiments in this direction were entirely unsatisfactory, but incidentally I noticed several other ways in which discharge could be forced, and the following paper deals with these.

Many of the phenomena about to be described must have been noticed before by different experimenters, but I do not know of any research other than Humphreys' dealing with the subject, so that they will be new, in part at least, to many.

The magnitude of the lengthening depends on several things, the size, shape, and degree of polish of the terminals between which the discharge takes place, the speed with which the machine is run, the atmospheric conditions, and the size of the Leyden-jars on each terminal.

Unless otherwise stated the following experiments were performed with a two-plate Wimshurst, with plates of glass 40 cms. diameter, brass sectors and terminals of equal size, 2·5 cms. in diameter, and with jars on the machine.

The following were the methods:—

(1) By taking a Spark from the Negative Terminal.

The machine was kept running continuously with uniform speed and the terminals gradually separated until sparks ceased to pass; this took place when the gap was about 10 cms. across. On placing the finger near any part of the negative terminal so that a small spark passed to the knuckle, then a bright discharge took place between the balls; the distance between them was further increased and the discharge could in this way be forced through a distance of 15 cms. If the gap was still greater, a bright discharge no longer passed but a brush discharge took place across it. At a distance of 16 cms. and upwards a brush appeared on that point of the positive ball nearest the negative terminal, and this brush either lost itself in the air between the poles or passed to the nearest portion of the framework of the machine.

When driven with uniform speed there was a limit to the potential difference which could be obtained between the terminals of the Wimshurst. Before the finger was presented to the negative terminal the machine was run in each case until the jars were charged to this maximum potential, and if this was not obtained then the small spark taken by the finger did not produce a spark discharge but only a brush discharge.

It does not appear to make any difference how the spark is taken, the finger, an uninsulated ball or knob of a small Leyden-jar held in the hand, all giving similar results. I was unable to produce any increase, however, by fastening a wire *W* (Fig. 1) with ball *F* attached to the outer coating of the jar and taking the spark with *F* from *B*.

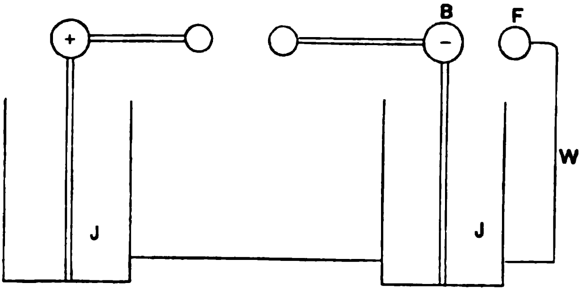


Fig. 1.

The conditions necessary for success are that the jars must be on the machine, and their outer coatings in good metallic connection.

The effect can be produced with a single jar, large or small, by standing it on paraffin wax or any other insulator and connecting a coating to each terminal.

(2) By taking a Spark from the Positive Side.

In endeavouring to measure the increase noted above it seemed that the effect produced with a gap of about 5 cms. or less was the same whether the spark was taken from the positive or negative side. This suggested the idea that the increment could be produced by the former as well as by the latter, and therefore I tried to obtain the effect on the Wimshurst. For a long time I was unsuccessful with a spark-gap of 10 cms. or more; there seemed to be little effect by placing the finger near the positive pole and taking a spark, and it was impossible to produce a great increase in length like that obtained by the method of Expt. 1.

If the potential is very nearly great enough to produce discharge, then a spark taken from the positive terminal will start the discharge, and under these conditions a spark could at times be forced across a gap of 7 cms., but such an occurrence was exceedingly rare, it being very difficult to take a spark with the finger at just the right moment.

I was more successful with a shorter spark-gap. The terminals were pulled apart a distance of 15 cms. so that no discharge took place when the machine was slowly driven by hand. When the jars were charged the machine was stopped and the knobs brought quickly together so that the distance between them was about 4 cms., then on taking a spark from the positive side a bright discharge generally took place between the terminals. The distance between the knobs could be more or less than this, depending upon the time elapsing between charging and discharging; if this was too small an interval, the spark passed as the knobs were brought together, if too great, no effect was produced on touching the positive side, but after one or two trials the discharge was easily obtained.

(3) By means of another Wimshurst Machine placed near.

Hertz (*Wied. Ann.* XXXI., p. 983) has shown that discharge between two terminals takes place at a lower potential when the gap is simultaneously illuminated with ultra-violet light, and it at first seemed that the effects of Expts. 1 and 2 might be caused by the ultra-violet light produced by the small spark taken from the negative terminal with the finger. The following experiment shows that such is not the case. A large amount of ultra-violet light was made by discharging a Ruhmkorff coil between zinc terminals, and this light was focussed on various parts of the spark-gap of the Wimshurst machine; at the same time Expts. 1 and 2 were repeated and no alteration in the effects already obtained could be noticed—a small spark taken from the negative terminal forcing discharge as before.

Since there was a great deal more ultra-violet light produced by the discharge between the zinc terminals than that between the finger and the negative pole of the Wimshurst, it seems that this form of radiation is not the cause of the increase in spark-length.

An interesting example of the effect produced by ultra-violet light is the synchronising in the discharge of two Wimshursts. Two machines were placed with terminals facing each other about 40 cms. apart, and driven simultaneously; the speeds were then altered so that the intervals of time between successive sparks were about the same for both; it was then an easy matter to make the two machines discharge at the same instant and to continue doing this for a number of sparks.

There seemed to be no necessity for the terminals to be of the same size, nor for the spark-gaps to be equal, if one was 6 cms. or more and the other only 2 cms. they synchronised just as easily.

When a board or plate of metal was placed between so that the rays of one spark were shielded from the other, then the effect was practically stopped, but even when the spark-gaps were not so shielded from each other then one machine had to be charged to very near sparking potential before the other discharged. In the latter case the effect could be increased by bringing the machines nearer together.

(4) By holding a Rod of Insulating Substance next the Positive Terminal.

Humphreys has already dealt with this method (*Phys. Review*, Vol. XI., No. 2). The machine was run continuously and the spark-gap gradually increased until sparks ceased to pass, then on bringing a glass rod near the positive pole a discharge took place. The gap could in this way be increased 50 per cent. No effect was produced by holding the rod at rest near the negative terminal beyond a brush discharge between the glass and the pole. On rare occasions if the rod was quickly moved past the negative ball a discharge could be forced provided the machine was near sparking potential at the time.

Many different substances besides glass were tried and the following were the results.

Sealing-wax, ebonite, and many insulators all gave a result similar to the glass rod.

A stick of sealing-wax tipped with tinfoil, an insulated wire, a glass rod with end white-hot, a candle-flame and tip of a blowpipe flame produced discharge when passed near either terminal; with the latter a slight shock was felt in the hand holding the blowpipe.

An air-blast either across the gap or along it produced no effect whatever at either side. The wind was forced both from positive to negative and in the opposite direction by having a terminal as in Fig. 2. *A* was a glass tube holding the hollow

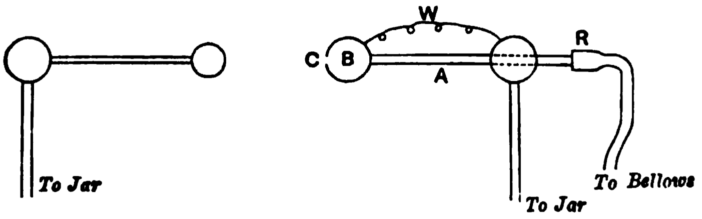


Fig. 2.

terminal *B*. In the latter at *C* there was a small hole. *B* was connected to the nearest metal of the terminal by means of a wire *W*. The air-blast was formed by a pair of foot bellows which were connected by a rubber tubing to the glass tube at *R*.

The jars were next removed from the machine. When the machine was running and the knobs about 6 cms. apart no continuous brush discharge took place across the gap. On bringing the end of a glass rod near the positive terminal and in the gap, a large positive brush was formed extending about three-quarters the distance across; the negative brush was also apparent and there was a narrow dark space between them. Beyond a slight crackling no discharge took place when the rod was brought near the negative terminal.

(5) Suddenly altering the Shape of the Terminal.

All the preceding methods of increasing the gap seemed to depend in some way on a brush discharge being produced at the same instant or just previously to the spark; and since a brush is easily produced on a point, it would follow that if one were suddenly made on a round conductor the brush thereby caused should force discharge. It was possible to lengthen the spark by this means in the following way. A hollow ball of indiarubber *A* was fastened in a cup of brass by means of sealing-wax (Fig. 3); a hollow brass tube *C* was soldered to this cup and a hole was made through the sealing-wax and rubber by passing a hot wire along *C*. If a glass rod *B* with end first pointed and then slightly rounded in the Bunsen flame was pushed along *C* into the ball, then at the point where it touched the rubber a slight protuberance was made. A difficulty was experienced in covering the ball with some conducting material which would yield sufficiently when the ball was stretched. Tinfoil and Dutch-metal were first tried but I was unable to place them on the ball so that they altered with the shape of the rubber; next a paste of plumbago and gum was used and although imperfect the phenomena were easy enough to obtain with it. The ball (diameter 3·5 cms.) and tube formed one terminal of the Wimshurst, and the other pole was of brass and 2·9 cms. diameter.

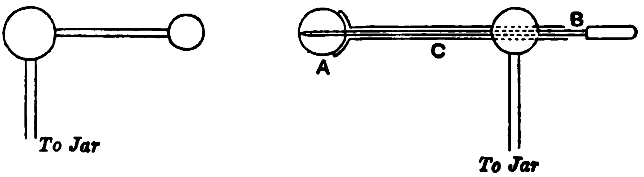


FIG. 3.

The following were the results obtained:

(*a*) With an indiarubber ball as the positive terminal so that a positive brush was formed, and with a spark-gap of 6 cms., the slightest push of the glass rod produced a brush and very often a spark discharge; with one of 8 cms. a big brush discharge was caused quite across the gap; at 9 cms. and upwards the brush divided into two parts, the positive being very large and extending three-quarters of the way across. The spark obtained with the shorter distance was very interesting, it would often pass from the brass cup round to the point on the rubber and then straight across the space between the terminals.

(*b*) With the rubber ball as negative terminal and with a spark-gap of 4 cms., a slight push to the glass rod produced a brush and very often a spark discharge; at 6 cms. a small negative brush was produced and a point of light on the positive ball, while at 10 cms. and above there was a small negative brush as before and no noticeable effect on the positive ball.

The same experiments were next tried without the jars on the machine and the results were much the same as before, except of course that there was a diminution in the brightness of the spark.

Modifications in the Phenomena of Expts. 1 and 2 by Varying the Size of the Terminals.

If the balls were of unequal sizes, the effects of methods 1 and 2 were remarkably modified. The small ball in each case had a diameter of 2 cms. and the larger 4 cms.

The small ball was first made positive, as is often the case with many machines. All the effects of Expt. 1 were increased but none of those of Expt. 2 could be obtained.

The most interesting changes were produced when the smaller ball was the negative terminal, when the effects varied with the length of the gap; at a distance 5·2 cms. or less none of the effects of Expt. 1 could be obtained, but if the spark was taken from the positive side then at times a spark discharge took place; when this did not happen the negative brush appeared on the portion of the negative terminal nearest the positive at the instant of taking the spark. On gradually increasing the distance the negative brush became fainter and fainter but could generally be seen in a darkened room up to a length of 10 cms. At this point a spark taken from the negative terminal produced discharge. No doubt at this length the relative sizes of the two balls have not so great an effect in determining the distribution of potential in various parts of the field, so that the lines of force would be similar in position to those existing under the conditions of Expt. 1, but the increase was not so great, owing perhaps to the difficulty of preventing the negative terminal discharging in the form of a brush when it is smaller than the other ball.

An end of a rod was substituted for the positive terminal and a sphere for the negative as before. As soon as the machine was started the positive brush appeared at the end of the rod and a spark from the negative made the brush increase in size. No effect was observed on taking a spark from the positive side except the momentary annihilation of the positive brush as the potential on that side was lowered. If the terminals were interchanged so that the positive pole was a sphere and the negative the end of a rod, then a spark from the positive produced a slight increase in the negative brush, and often made one or two other small ones shoot out from other points as well. Beyond the sudden extinguishing of the negative brush no effect was produced by taking a spark from the negative side.

With two large balls as terminals the effect was the same as with the two smaller, but with the latter the effects of Expts. 1 and 2 were easier to obtain with a shorter spark-gap, owing undoubtedly to the greater want of uniformity in the field.

There were no indications of any of the above effects if the jars were removed from the machine.

The Sound produced by the Discharge.

It is fairly easy to tell when a disruptive discharge is assisted by the brush produced and when it is not, because the sound of discharge is quite different in each case. In the former case the spark may be long but it is not nearly so loud and sharp as it is when no brush is present. In each of the previous cases when the discharge is forced the sound clearly indicates that a brush is present in the gap at the same time.

When the terminals of the Wimshurst are large balls and the spark-gap great the spark under ordinary conditions is much more abrupt and decisive than when the terminals are small, and this difference is probably due to the fact that there is no simultaneous brush discharge when the poles are large.

All the preceding effects point to the conclusion that a brush discharge is first formed in each case, and this weakens the resistance to disruptive discharge. Professor J. J. Thomson suggested I should try to obtain the effect in gases in which the brush is more or less difficult to obtain than in air. The terminals were first surrounded with a glass tube 5 cms. in diameter and held in position by rubber stoppers, but the proximity of the tube to the metal of the terminals caused a brush discharge between them, so I used a large spherical bulb. At high potentials a brush always formed on the rubber stoppers so that finally the electrodes were held in position with sealing-wax. The gases were in each case dried by passing them over pumice moistened with sulphuric acid. Faraday ("Experimental Researches in Electricity," Series 12, 1425—1454) gives the phenomena of the brush discharge in various gases and some of these were tried with the following results:

(1) Coal-gas. In this gas the brush is difficult to obtain. With terminals 7 cms. apart, a point of light on the positive ball was seen on taking a spark from the negative pole and sometimes a very faint brush. There was no spark discharge produced, but when the gas was flushed out and the air admitted the spark discharge could be easily obtained by Expt. 1.

With a gap of 4 cms. I was rarely able to make a discharge between the terminals and that passing to the finger synchronise, but on the admission of air it was easy enough.

(2) Carbon-dioxide. The brush is also difficult to obtain in this gas. It was easier to make the sparks synchronise at 4 cms. or thereabouts than with coal-gas but it does not require so great a potential difference between the terminals to produce spark discharge and this might partly compensate for the poorness of the brush.

(3) Nitrogen. The poles were 4·5 cms. apart; on driving the machine so slowly that no spark passed under ordinary conditions, then a spark taken from the negative side produced a very bright spark discharge; the same effect could also at times be obtained by taking a spark from the other terminal. A brush is formed in this gas very easily.

CONCLUSION.

It seems that a spark taken from a terminal as in Expts. 1 and 2 produces a surge or wave which is carried over to the other terminal by means of the jars, a sudden change in the potential of the negative pole producing a corresponding change in the positive, because if the jars are absent or even insulated from each other none of these phenomena is produced; changing the size of the terminal moreover shows that the effect is always greatest when the spark is taken from the larger ball and the disturbance carried over to the lesser; around the smaller the electromotive intensity is greatest and this is greatly increased by the sudden change produced by the wave. For a ball of a given size there is a given electromotive intensity at which a brush is formed, and therefore a brush discharge is easiest formed on the smaller terminal; this by producing dissociated ions puts the gas in a condition in which it is electrically weaker, and if the distance is not too great a spark passes, otherwise the brush only is formed.

The experiments made in various gases point to the same conclusion. The phenomena of Expt. 1 were the most difficult to be obtained in gases such as coal-gas and carbon-dioxide, which only produce a brush with difficulty, while with such a gas as nitrogen, where a brush was easily obtained, the effects of the same experiment were easily produced.

Faraday noticed that the spark-length of a machine was always increased by having the smaller ball positive, and many makers of machines arrange the terminals in this way. The lengthening may be due to a positive brush being formed on that terminal just previous to the discharge by spark, and the smaller the ball the lower the potential at which the brush is produced. If the smaller ball is negative a brush is formed there at a lower potential, and this simply discharges the machine, aiding but little in the passage of the spark.

The results of Experiment 5 can also be explained by the brushes produced. Whenever the glass rod is brought near either terminal a slight crackling is heard due to the brush discharge between the metal ball and the glass; this, as before, makes the passage of the spark easier, especially when the positive brush is produced.

With metals, whether insulated or not, flames, etc., the positive brush produces discharge when they are held near the positive terminal; when near the negative a small spark passes and we really have a repetition of Experiment 1.

It is impossible to say why the effects are not so great when the brush is formed on the negative ball until we explain the difference between the positive and negative brushes; their appearance has long been noted to be different and the above experiments show that the presence of the positive in a gap makes a gas much weaker electrically than if the negative were present. In this action we have yet another difference between the two electricities.

All the above experiments were undertaken at the Cavendish Laboratory with the exception of (5) and those with different gases which were done at the Technical Schools, Southend.

I take this opportunity of expressing my gratitude to Professor J. J. Thomson for his kindness, and for the assistance he has given me during the progress of these experiments.

Reduction of Generating Functions by means of Complex Integration. By G. B. MATHEWS, M.A., F.R.S., St John's College, Cambridge.

[*Read* 13 February 1905.]

In solving various Diophantine problems we have to perform an operation Ω, which may be defined as follows. A rational function of a and other variables is formally expanded in positive and negative powers of a; the negative powers of a are then discarded, and a is replaced by unity in the remaining terms. If ϕ is the function in question, the final result is indicated by $\Omega_a\phi$, or simply by $\Omega\phi$, when there is no risk of ambiguity.

Thus, to take a very simple case, let

$$\begin{aligned}\phi &= \frac{1}{(1-ax)\left(1-\dfrac{y}{a}\right)}\\ &= (1+ax+a^2x^2+\ldots)\left(1+\frac{y}{a}+\frac{y^2}{a^2}+\ldots\right)\\ &= \Sigma a^{p-q}x^py^q:\end{aligned}$$

then it is clear that

$$\Omega\phi = \Sigma x^py^q$$

with the condition that $p \geqslant q$. By putting $p = q + r$, we obtain

$$\begin{aligned}\Omega\phi &= \Sigma\,(xy)^qx^r \qquad (q \geqslant 0,\ r \geqslant 0)\\ &= \frac{1}{(1-xy)(1-x)};\end{aligned}$$

but in more complicated cases it is not so easy to find the reduced value of $\Omega\phi$. The object of this note is to show how the principles of complex integration may be applied to the problem.

Suppose that, $|a|$ being greater than 1, we have a convergent expansion

$$\phi(a) = \ldots + b_2a^{-2} + b_1a^{-1} + c_0 + c_1a + c_2a^2 + \ldots;$$

then since

$$\frac{1}{a-1}=a^{-1}+a^{-2}+a^{-3}+\ldots$$

the coefficient of a^{-1} in the expansion of $\phi(a) \mid (a-1)$ is

$$c_0+c_1+c_2+\ldots,$$

that is, $\Omega\phi$. Hence

$$2\pi i\Omega\phi=\int\frac{\phi(a)\,da}{a-1},$$

the integral being taken over a closed simple contour at every point of which the above expansion of $\phi(a) \mid (a-1)$ is legitimate.

But the integral is also equal to

$$2\pi i\Sigma R$$

when ΣR is the sum of the residues of $\phi(a) \mid (a-1)$ within the contour. Calculating this sum by the method of partial fractions, the value of $\Omega\phi$ is determined.

For example, if

$$\phi(a)=\frac{1}{(1-ax)\left(1-\frac{y}{a}\right)}$$

we must suppose $|ax|<1$, $|y|<|a|$, as well as $|a|>1$. We can take as the path of integration the circle which has its radius equal to $|a|$ and centre at the origin; the poles within it are 1 and y, so that

$$\begin{aligned}\Sigma R\left(\frac{1}{(a-1)(1-ax)\left(1-\frac{y}{a}\right)}\right)&=\frac{1}{(1-x)(1-y)}+\frac{y}{(y-1)(1-xy)}\\&=\frac{1}{(1-x)(1-xy)}\end{aligned}$$

on reduction.

The same result may be obtained in this case by taking the residue with respect to the one external pole, for which $a=x^{-1}$, and changing sign; the value of $\Omega\phi$ then comes out immediately.

As another example, let it be required to find

$$\Omega\frac{\frac{w}{a}}{(1-ax)(1-ay)\left(1-\frac{z}{a}\right)\left(1-\frac{w}{a}\right)}.$$

Here

$$\frac{\phi}{(a-1)}=\frac{aw}{(a-1)(1-ax)(1-ay)(a-z)(a-w)};$$

and integrating over the circle $r = |a|$ as before, the external poles are x^{-1}, y^{-1}, so that

$$\Omega\phi = \frac{\dfrac{w}{x}}{x\left(\dfrac{1}{x}-1\right)\left(1-\dfrac{y}{x}\right)\left(\dfrac{1}{x}-z\right)\left(\dfrac{1}{x}-w\right)} + \frac{\dfrac{w}{y}}{y\left(\dfrac{1}{y}-1\right)\left(1-\dfrac{x}{y}\right)\left(\dfrac{1}{y}-z\right)\left(\dfrac{1}{y}-w\right)}$$

$$= \frac{wx^2}{(1-x)(x-y)(1-xz)(1-xw)} - \frac{wy^2}{(1-y)(x-y)(1-yz)(1-yw)}$$

$$= \frac{w(x+y-xy-xyz-xyw+x^2y^2zw)}{(1-x)(1-y)(1-xz)(1-xw)(1-yz)(1-yw)}$$

after an easy reduction.

To test the practicability of the process I applied it to the function

$$\frac{1-\dfrac{1}{a^2}}{(1-ax)(1-a^3x)(1-a^5x)\left(1-\dfrac{y}{a}\right)\left(1-\dfrac{y}{a^3}\right)\left(1-\dfrac{y}{a^5}\right)}$$

from the Ω of which, on putting $y = x$, we obtain the reduced generating function which enumerates the concomitants of a binary quintic. The result was to confirm the calculation of Cayley; the actual labour involved was, of course, considerable, chiefly on account of the occurrence of poles with irrational affixes.

There is no theoretical difficulty in extending the process above explained to cases where the operation Ω is applied to an expansion containing positive and negative powers of two or more quantities a, b, etc. Thus let

$$f = \frac{1}{(1-ax)\left(1-\dfrac{x}{a}\right)(1-abx^2)\left(1-\dfrac{x^2}{ab}\right)},$$

then

$$\Omega_b f = \frac{1}{(1-ax)\left(1-\dfrac{x}{a}\right)(1-ax^2)(1-x^4)} = \frac{\phi}{(4)}$$

where $(4) = 1 - x^4$, and

$$\phi = \frac{1}{(1-ax)\left(1-\dfrac{x}{a}\right)(1-ax^2)}$$

Now

$$\frac{\phi}{(a-1)} = \frac{a}{(a-1)(a-x)(1-ax)(1-ax^2)}$$

and, calculating the residues for the poles 1 and x, we obtain

$$\Omega_a\phi = \Sigma R\left(\frac{\phi}{a-1}\right) = \frac{1}{(1)^2(2)} - \frac{x}{(1)(2)(3)} = \frac{1+x^2}{(1)(2)(3)},$$

and hence

$$\Omega_{a,b}f = \frac{1}{(4)}\Omega_a\phi = \frac{1}{(1)(2)^2(3)}$$

$$= \frac{1}{(1-x)(1-x^2)^2(1-x^3)}.$$

There is one very curious point about this theory which deserves to be noticed. In the actual applications the letters used are mere umbræ, and the question of their numerical values does not come in; but when, as here, the calculus of residues is employed, it is necessary to assume relations of numerical inequality for the purpose of deciding which poles are to be considered as being inside, and which outside, the contour of integration.

It cannot, I fear, be claimed for this method of residues that it supersedes others, such as those employed by MacMahon in his papers on partitions: but it is, at any rate, a useful auxiliary, and the fact that it is theoretically complete entitles it to some consideration.

Soluble forms of metallic dihydroxytartrates. By H. J. H. Fenton, M.A., F.R.S., Christ's College.

[*Read* 27 February 1905.]

Sodium dihydroxytartrate was first obtained by Grüber in 1879 and was afterwards further examined by Kekulé and by Barth. The barium salt was also prepared by Barth and in 1889 Miller succeeded in isolating the free acid. The author in 1898 discovered a new and simple method for preparing the free acid in a pure state and made a further study of its relationships; the lithium, potassium, rubidium, cæsium, and ammonium salts were also prepared and analysed.

From the fact that the sodium salt decomposes on heating into sodium tartronate and carbon dioxide, Grüber concluded that the acid should be regarded as tribasic carboxytartronic acid $C(OH)(COOH)_3$ and this view of its constitution appeared to be borne out by Barth's analysis of the barium salt; in this case the sodium salt would have to be considered as an acid salt. Kekulé's experiments however shewed conclusively that the acid is really dibasic and this result was further established by the author's experiments on the titration of the free acid and analyses of the salts above mentioned. The basicity of the acid and composition of its salts are therefore quite in harmony with Kekulé's formula which represents the acid as

$$COOH.C(OH)_2.C(OH)_2.COOH$$

or tetrahydroxy-succinic acid.

Although this formula is now very generally accepted it cannot be considered as entirely established; the comparative stability of the free acid on heating and the decomposition into tartronic acid and carbon dioxide when its aqueous solution is boiled are properties which at first sight appear to be hardly consistent with such a constitution.

Compare Grüber, *Ber.* 1879 (12) 514; Barth, *Sitz. Acad. Wien*, 82, ii. 1024.

Fenton, *Trans. Chem. Soc.* 1898 (73) 71, 167 and 472.

Debus, *Trans. Chem. Soc.* 1904 (85) 1388.

Investigations are now being undertaken with the object of throwing further light upon this question and in the present communication a short preliminary account is given of some experiments which have been made with regard to the conditions under which the salts can be formed.

All the metallic dihydroxytartrates which have so far been examined contain water of crystallization which is not expelled when the salt is kept in a vacuum desiccator, and any attempt at removal of the water by heat is attended with decomposition of the salt. It appeared therefore to be a matter of interest to ascertain the nature of the salts produced from solvents other than water.

When solutions of the free acid and sodium ethylate in absolute alcohol are mixed together a semi-transparent gelatinous precipitate is obtained which, when collected and washed with alcohol, is very easily soluble in cold water. This result is remarkable when it is remembered that the sodium salt prepared from aqueous solutions is a heavy white crystalline powder almost insoluble in water; its solubility is in fact so slight that it serves for the qualitative and quantitative estimation of sodium (Fenton, *loc. cit.* 167).

The aqueous solution of this soluble sodium salt after standing for a few minutes begins to deposit the ordinary sparingly soluble form of sodium dihydroxytartrate. By titration of the free acid with sodium ethylate in absolute alcoholic solution it is found that the ratio of acid to metal is the same in the soluble salt as in the ordinary form, but it has not yet been found possible to determine the composition of the soluble salt in the isolated condition. When kept in a vacuum desiccator over sulphuric acid it dries up to a white amorphous powder but at the same time suffers a certain amount of decomposition; it is still extremely easily soluble in water but the aqueous solution effervesces with dilute acids and gives a smaller yield of the insoluble salt on standing.

It was at first thought probable that the soluble form might be a salt of the diketo-acid $COOH.CO.CO.COOH$, but analysis shews that the dried product still contains hydrogen, the results being

C 25·18, H 2·47, Na 24·2 per cent.

The normal sodium salt of the diketo-acid would require

C 25·26, Na 24·21 per cent.

It is probable therefore that the dried product is a mixture of this salt with some tartronate and carbonate.

When dihydroxytartaric acid, or one of its soluble salts, is mixed with calcium chloride in aqueous solution the mixture almost immediately sets to a transparent colourless jelly, so that the vessel may be inverted without loss; in this behaviour dihydroxytartaric acid resembles glyoxylic acid, the formula of which is also generally represented as containing two hydroxyl groups associated to the same carbon atom (compare Debus, *loc. cit.* 1391).

But if the dihydroxytartaric acid and calcium chloride are both dissolved in anhydrous methyl or ethyl alcohol the solutions remain perfectly clear on mixing and no separation occurs even after standing for a day or two. A drop of water however will immediately cause the formation of a gelatinous precipitate; this precipitate is mostly dissolved if more water is added but the solution soon afterwards gelatinises. The action of small quantities of water in bringing about the separation of the calcium salt is so sensitive that it is quite easy by this reaction to detect the presence of one or two per cent. of water in alcohol. 'Combined' water, as in chloral hydrate or hydrated calcium chloride, does not produce a similar effect.

Studies on Unsaturated Ketonic Compounds. By S. RUHEMANN, M.A., Gonville and Caius College.

[*Read* 27 February 1905.]

In a paper which I lately published in the *Journal of the Chemical Society* (1905, LXXXVII. 17) it has been shown that, under the influence of bases, olefinic ketonic compounds react with mercaptans to form additive substances only. On using as catalytic agent hydrogen chloride alone or together with zinc chloride, Posner (*Ber.* XXXIV. 1395; XXXV. 799) has found that frequently mixtures are produced owing to the fact that mercaptans, besides reacting additively at the ethylene linking, partly condense with the ketonic group of the olefinic ketones. In the paper referred to, I have described various additive products of olefinic ketonic compounds containing the grouping

$$\overset{|}{\underset{|}{\mathrm{C}}}\mathrm{H : CH—CO—}.$$

I have since applied this reaction to other substances of the same type of ketones and have obtained from benzylideneacetophenone, benzylidenedesoxybenzoïn, benzylidenebenzoylacetic ester the following compounds.

Benzylideneacetophenone yields with *iso*-amylmercaptan, under the influence of piperidine, iso-amylthiolbenzylacetophenone,

$$C_6H_5 . CH(S . C_5H_{11}) . CH_2 . CO . C_6H_5,$$

which melts at 49—50°; with phenylmercaptan the ketone forms

$$C_6H_5 . CH(S . C_6H_5) . CH_2 . CO . C_6H_5 \text{ (M.P. 120—121°)}.$$

The latter substance is identical with the substance which Posner has prepared by the aid of hydrogen chloride as catalytic agent; but from *iso*-amylmercaptan and the olefinic ketone he has obtained an oil which from the analytical result he concludes to be a mixture.

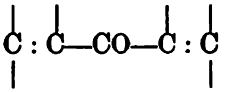
Benzylidenedesoxybenzoïn unites with phenylmercaptan to yield phenylthiolbenzyldesoxybenzoïne,

$$C_6H_5 . CH(S . C_6H_5) . CH(C_6H_5) . CO . C_6H_5 \text{ (M.P. 204—205°)}.$$

Ethyl benzylidenebenzoylacetate forms with phenylmercaptan the additive compound

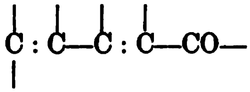
$C_6H_5.CH(S.C_6H_5).CH(CO.C_6H_5).CO_2.C_2H_5$ (M.P. 100—101°).

In my previous communication (*loc. cit.*) I have pointed out that di-olefinic ketones of the type:



$$\overset{|}{\underset{|}{C}} : \overset{|}{C}\text{—CO—}\overset{|}{C} : \overset{|}{\underset{|}{C}}$$

combine with 1 or 2 mols. of mercaptans, whilst, by the aid of hydrogen chloride as catalytic agent, Posner has been able to obtain only additive products with 2 mols. of mercaptans. I have, further, shown that cinnamylideneacetophenone, a di-olefinic ketone of the type:



$$\overset{|}{\underset{|}{C}} : \overset{|}{C}\text{—}\overset{|}{C} : \overset{|}{C}\text{—CO—}$$

unites with 1 mol. only of either *iso*-amylmercaptan or phenylmercaptan. This result differs from Posner's statement (*Ber.* XXXVII. 502) according to which the substance resulting from the reaction between the ketone and phenylmercaptan is an additive compound with 2 mols. of the mercaptan. Prof. Posner has since by letter corrected his error.

Cinnamylideneacetone, $C_6H_5.CH:CH—CH:CH.CO.CH_3$, also unites with 1 mol. of phenylmercaptan only to form a compound which melts at 53—54°. But Posner's pupil, A. Werner (see Posner, *loc. cit.*), has found that the product of the action of hydrogen chloride on a mixture of the ketone and the mercaptan, is an oil which, by potassium permanganate, is transformed into a di-sulphone. From this result it would follow that the oil is an additive product of the di-olefinic ketone with 2 mols. of the mercaptan; yet my experiments, recorded in this paper, and the facts at which Kuvevenagel (*Ber.* XXXVII. 4038) arrived in the course of his study on the reaction of potassium hydrogen sulphite with di-olefinic ketones of the above type, induce me to doubt the correctness of Werner's result.

The behaviour of mercaptans towards the tri-olefinic ketone, *Cinnamylidenebenzylideneacetone*,

$$C_6H_5.CH:CH.CH:CH.CO.CH:CH.C_6H_5,$$

is similar to the action of mercaptans on cinnamylideneacetone or cinnamylideneacetophenone. With 1 mol. of either phenylmercaptan or *iso*-amylmercaptan it yields additive compounds which melt at 69—70° and 133—134° respectively. The ketone, also, adds 2 mols. of mercaptans, but I have been unable to isolate these substances from the former compounds which are produced at the same time.

Mercaptans combine with acetylenic ketones as readily as with olefinic ketones. As yet I have examined in this direction particularly methoxybenzoylphenylacetylene,

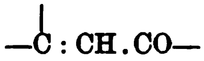
$$C_6H_5.C:C.CO.C_6H_4.O.CH_3,$$

which unites with 1 mol. of phenylmercaptan to yield

$$C_6H_5.C(S.C_6H_5):CH.CO.C_6H_4.O.CH_3 \text{ (M.P. 121—122°)}.$$

This compound differs from the additive products of mercaptans with olefinic ketones inasmuch as it is yellow whilst the latter substances are colourless.

At present I am engaged in the study of the thiol-compounds; it is especially their behaviour towards heat which appears to be worthy of an inquiry for the following reason. The saturated β-hydroxyacids, on heating, split off water and yield unsaturated acids; the additive products of the olefinic ketones:



$$\overset{|}{-C}:CH.CO-$$

with primary organic bases suffer a similar decomposition (see Ruhemann and Watson, *Trans. Chem. Soc.* 1904, LXXXV. 1170) which takes place either in the direction

$$\text{I.}\quad R.CH(NH.R').CH_2.CO- \longrightarrow R.CH:CH.CO- + NH_2.R',$$

or

$$\text{II.}\quad R.CH(NH.R').CH_2.CO- \longrightarrow R.CH:N.R' + CH_3-CO-$$

(R and R′ denoting hydrocarbon radicles).

On the other hand, the additive compounds of acetylenic ketones with organic bases seem to distil without decomposition. As yet I have examined in this direction the compound

$$C_6H_5.C(NH.C_6H_5):CH.CO.C_6H_5,$$

which E. R. Watson (*Trans. Chem. Soc.* 1904, LXXXV. 1324) has obtained from methoxybenzoylacetylene and aniline, and have ascertained that, in a vacuum, it boils unchanged. If the additive products of the unsaturated ketones with mercaptans behave in a similar manner, then it may be expected that the substances which are formed from olefinic ketones will suffer an analogous decomposition, whilst the compounds which result from the union of mercaptans with acetylenic ketones will boil without decomposition.

Some Compounds of Guanidine with Sugars. By R. S. MORRELL, M.A., Gonville and Caius College, and A. E. BELLARS, B.A., Magdalene College.

[*Read* 27 February 1905.]

The action of alkalies and especially of ammonia on sugars has been the subject of a number of papers by Lobry de Brüyn and his colleagues van Leent and van Ekenstein[1]. They have described crystalline compounds of ammonia with glucose, galactose, rhamnose, arabinose, and xylose, of the nature of osamines, but they consider that maltose and lactose form additive products with ammonia. Several of these osamines crystallise with molecules of alcohol of crystallisation. The compound of ammonia with fructose, fructosamine, $C_6H_9O_4N$, is of interest in its connection with chitosamine, $CHOCHNH_2(CHOH)_3CH_2OH$, from which it may be prepared. Fructosamine is by far the most stable of the osamines which generally evolve ammonia at their melting points, and even there is a loss of ammonia when aqueous solutions of osamines are heated to boiling. The study of the influence of dilute alkalies on the optical activity of solutions of glucose led Lobry de Brüyn and van Ekenstein to the conclusion that glucose, fructose, and mannose are reciprocally transformable into one another in aqueous solution.

Several years ago it was noticed by one of us that an alcoholic solution of guanidine gave white minutely crystalline precipitates with alcoholic solutions of glucose and rhamnose. The compounds obtained were exceedingly hygroscopic and could not be recrystallised from any solvent; moreover they were never free from inorganic material, since the guanidine was prepared from its hydrochloride by the action of sodium ethylate. If guanidine were prepared from the carbonate by decomposing the salt with less than the calculated quantity of baryta, and the concentrated aqueous solution were freed from excess of guanidine carbonate by addition of absolute alcohol, it was found that compounds of the base with sugars was obtained free from ash. From ethylic alcohol solutions glucose, fructose, galactose, rhamnose, and maltose were precipitated combined with guanidine as microscopically crystalline substances which are very hygroscopic. The glucose, fructose, maltose compounds are composed of three molecules of the sugar and two of guanidine, but those obtained from galactose and rhamnose seem to contain an equal number of molecules of sugar and base. In several cases there is combined alcohol, which is not removed in a vacuum; Lobry de Brüyn found that the rhamnosamine crystallised out with a molecule of methyl alcohol. The compounds react strongly alkaline in aqueous solutions, and their optical activity is generally much lower than that of the parent sugar; moreover it diminishes slowly with time until a constant value is obtained. The rotation angle of the glucose compound in water becomes in time negative, and that of fructose slowly falls to a constant value, which is of the same sign as that of the parent sugar. The change of sign in the rotation angle of the glucose compound may point to a slow transformation of glucose into fructose as observed by Lobry de Brüyn. We have not yet examined the aqueous solution of the glucose compound for mannose and fructose. Dilute acids decompose the compounds at the ordinary temperature, but the concentration of the acid must be considered, as will be seen from the following experiments:

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[1] *Rec. des Trav. Chim. des Pay Bas*, 12, 286; 13, 218; 14, 98, and 134; 15, 183; 18. 81.

1·72 grs. galactose compounds on treatment with 10 c.c. N.HCl required 50·71 c.c. $\frac{N}{10}$ c.c. NaOH for neutralisation. Percentage of guanidine = 16·86.

1·2501 grs. dissolved in 100 c.c. water and treated with 12·5 c.c. N.HCl required 101·6 c.c. N/10 NaOH. °/$_0$ guanidine = 11·04.

2·089 grs. rhamnose compound on treatment with 12·5 c.c. N.HCl required 73·55 c.c. $\frac{N}{100}$ NaOH. Percentage of guanidine = 14·57.

0·9498 grs. rhamnose compound, dissolved in 25 c.c. water, on treatment with 12·5 c.c. N.HCl required 110·5 c.c. N/10 NaOH. Percentage of guanidine = 8·82.

The compounds of glucose and maltose with guanidine are decomposed completely by the dilutest hydrochloric acid.

Picric acid precipitates guanidine picrate from aqueous solutions of compounds of guanidine with glucose, fructose, galactose, and rhamnose, but the yield of picrate depends on the concentration of the solution in the case of the fructose and rhamnose compounds. The influence of concentration of acid and time on the decomposition of aqueous solutions of these sugar-compounds is still under investigation. Their optical activity is generally much lower than that of the parent sugar, and diminishes slowly until a constant value is obtained. The specific rotation of the glucose compound is $[\alpha_D] = 29·0°$, which sinks in 6 days to $[\alpha_D] = -6·2°$; for the galactose compound $[\alpha_D] = 52·9°$, which becomes in 8 days

$[\alpha_D] = 0{\cdot}98°$; the fructose compound has an initial specific rotation of $[\alpha_D] = -58{\cdot}3°$, falling to $[\alpha_D] = -18{\cdot}7°$ in 8 days. Maltose and rhamnose in their compounds with guanidine show a similar muta-rotation. An aqueous solution of the glucose guanidine compound has its optical activity changed to that of glucose on addition of hydrochloric acid. The extreme solubility in water, and insolubility in neutral organic solvents render the recrystallisation of these substances a matter of great difficulty, but as they were precipitated in a minutely crystalline form from ethylic alcohol analyses were made of the precipitate, which had been washed with absolute alcohol and ether. Only in the case of rhamnose have we been able to obtain a derivative which can be recrystallised with difficulty from ethyl alcohol. Their melting points are low, between 94—98° with effervescence. On drying at 40° the substances soften and become yellow in colour. The yields of these guanidine compounds are excellent; from 18 grams of glucose dissolved in 1500 c.c. of absolute alcohol nearly 23 grams of the additive product were precipitated; the yield is quantitative; moreover the alcoholic filtrate was inactive and changed the colour of a Fehling's solution only slightly on boiling. Heating to 100° modified the properties of these substances in a striking manner; they melt and alcohol is evolved, but no ammonia; their reducing power towards Fehling's solution is diminished, and they give no alkaline reaction in aqueous solution. Their behaviour towards phenylhydrazine acetate is unchanged, except in the case of the galactose derivative, which gives no longer an osazone, and does not react with picric acid to give guanidine picrate. In all the cases examined the percentages of carbon and nitrogen increase on melting the substances. We have not been able to obtain the fused products in a crystalline form, and have analysed them directly. The maltose compound loses four molecules of water besides alcohol, those of rhamnose and galactose one molecule of water.

We are unable at present to assign with certainty constitutional formulae to these substances, but the investigation is being continued, and we hope that a careful examination of the products obtained by heating the substances to 100° C. will throw light on their constitution. In this preliminary notice we desire to express our thanks to the Government Grant Committee of the Royal Society for a grant covering the cost of the material of this research.

The influence of very strong electro-magnetic fields on the spark spectra of gold, antimony, bismuth, lead, and tin. By J. E. PURVIS, M.A., St John's College, Cambridge.

[*Read* 27 February 1905.]

The aim of this preliminary paper is to describe the effects on the appearance of the spectra of the above-mentioned elements, when their spark discharge takes place in very strong electro-magnetic fields.

Description of the apparatus.

The spectroscope used in the experiments is Professor Liveing's large 21-foot concave grating. The spectrum of the second order was used for wave-lengths from about λ 3600 and extending into the ultra-violet; and that of the first order for lines whose wave-lengths are below λ 3600. The width of the slit was varied from time to time, when it was required to examine the effect on the stronger or weaker lines. In the case of very strong wide lines, the slit must be considerably narrower than for finer lines, if the 'Zeemann' effect is to be seen.

Self-induction coils were introduced occasionally: and although, as has been before noticed, the lines appear to become sharper, there is the drawback that they appear to be weakened as well, and also the weaker lines become so much weakened as to have no effect on the photographic plate.

The electro-magnet is of the du Bois form. The poles are conical, and the noses of the pole pieces are 7 mm. diameter. In these experiments the distance between the poles was 4 mm. In this space, with a current of 25 amperes, a field strength of about 41,000 C.G.S. units can be obtained. A current of 21 amperes was employed in these experiments and this gives a field of 40,000 units. The coils wound round the magnet become warm after 40 minutes running, and when larger exposures were necessary to photograph the weaker lines, it was necessary to cool down, and repeat the experiment after the magnet had cooled. This did not seem to interfere with the exactness of the definition of the lines, unless there was an excessively windy day.

The current supplying the magnet was obtained directly from a dynamo. It was easily controlled when allowing for the increased resistance as the coils became warmer, and there was no difficulty in keeping the current constant as shown by the ammeter. The magnet was arranged broadside on, that is, the lines of force between the poles were in a direction perpendicular to the line joining the slit and the grating.

The ends of the electrodes were held firmly in the middle of the gap between the poles by means of a specially constructed mechanical arrangement. The stand of this holder is firmly clamped to the stage of the magnet, and it has three movements. The holder can be moved upwards or downwards; also in a direction perpendicular to this; and thirdly there is a slow adjustment to keep the ends of the electrodes at a constant distance from each other. This slow adjustment is very necessary, because, as the ends of the electrodes wear away during the sparking, and the distance between them becomes greater, the spark begins to discharge to the poles of the magnet. The distance between the ends of the electrodes was usually from 1 to 2 mm.

The electrodes of the metals gold, tin, and lead were thin strips of about 2 mm. wide and 1 mm. thick. Those of antimony and bismuth were made by melting the metals, and sucking the molten metal into narrow glass tubes of about 2 mm. internal diameter. The glass and its solidified contents were then thrown into water, and it was easy to pick off the shattered glass from the surface of the pencil of metal. The pieces were then carefully filed flatter.

The spark was focussed on the slit by a quartz lens of a focal length of about 5 inches and of 1 inch diameter. It was firmly fixed in a holder, and it could be moved upwards or downwards by a fine screw through a distance of 2 or 3 mm. By this means two photographs could be taken on the same plate, one parallel to the other—one of the normal spark and the other when the spark was discharging in the magnetic field.

In order to analyse the nature of the vibrations in the magnetic field, a calcite crystal was introduced between the quartz lens and the slit of the spectroscope. The crystal was specially polished at the rhombic ends, and it was selected of a sufficient size to disperse the two images of the ordinary and extraordinary rays at a convenient distance from each other. The two images were about 2·5 mm. from and above each other when they fell on the slit; and the optic axis of the crystal was perpendicular to the lines of force between the poles of the magnet. By this arrangement the "ordinary" and the "extraordinary" vibrations of the same line could be photographed on the same plate.

As a rule, any division of the lines into triplets is well marked when the lines are observed in the second order. As regards the lines observed in the first order the division is not so well marked, unless they are sharp and narrow. When any line is normally strong and broad, a widening may be noticed, and it is then necessary to analyse it by the calcite to determine if it is merely a widening or if it is a real division. Further, there is obviously a great loss of light when the normal image is divided by the calcite; and, although some lines are distinctly widened when in the field, they are so much weakened by the calcite dispersion that the images do not appear on the plate even after 1½ hours' exposure. Consequently only the well-marked and the strongest lines were observed.

To obtain the discharge, an App's coil was used with a large Leyden jar, which could give a spark of 6 inches long. In the case of antimony, bismuth, lead, and tin, it was found necessary not to have such a violent discharge, because the temperature was often raised so high as to melt the ends of the electrodes.

Maxim's special rapid photographic plates were used and developed with a mixture of metol-hydroquinone. The time of exposure varied from 30 to 90 minutes as required by the nature and intensity of the various lines.

Results.

The following notes describe the effect on the various lines whose wave-lengths are mentioned on the left-hand side.

GOLD.

Wave-length	
λ 4792·8.	A triplet; the two outside members of which seem to vibrate perpendicular to the lines of force between the poles of the magnet, and the inner one parallel thereto.
4607·8.	There seems to be a widening of this line; but the air lines in the region interfere with a more exact analysis.
4488·4.	This line is distinctly widened; and, on analysing it with the calcite crystal, it seems to be a triplet with the two outer members vibrating perpendicular to the lines of force, and the inner one parallel thereto.
4437·3.	A distinct widening of the line, but it is too weak to affect the sensitive plate when the calcite is interposed to analyse it.
4315·3.	A distinct widening of the line; but it is also too weak as in λ 4437·3.
4065·2.	This line is distinctly widened; and, on analysing it with the calcite, it seems to be divided into a triplet, with the two outside members vibrating perpendicular to the lines of force, and the middle one parallel thereto.
4053·0.	It shows a distinct widening, and when analysed by the calcite, it seems to be a triplet, the two outer members vibrating perpendicular, and the inner one parallel, to the lines of force.

Wave-length	
λ 4041·1. 4016·2.	These two lines show a distinct widening—but are too weak to analyse.
3898·0.	This line is a triplet, the two outer members of which vibrate perpendicular to the lines of force, and the inner one parallel thereto.
3804·2.	This line is widened, and, on analysing it, although it is rather weak it seems to be a triplet, with the three components vibrating like λ 3898.
3633·4.	This line is widened, but it is too weak to analyse.
3586·6.	This line appears as a widened reversal or doublet. The reversal is not sharp nor well defined.
3553·7.	This line is widened, but it is also too weak to analyse.
3122·8.	This is a triplet; the two outside members of which vibrate parallel to the lines of force, and the middle one perpendicular thereto.
3122·6.	A weak line which appears to be widened, although this is not easy to make out on account of its weakness, and also particularly its nearness to λ 3122·8.
3029·3.	This line is a doublet or reversal. Tho two members are quite clear, sharp and well defined.
2995·1.	A somewhat broad line, which seems to show signs of widening.
2990·3.	The line is weak, and seems to become a doublet or reversal —but the division is not very clear or well defined.
2913·6.	This line becomes a triplet, the two outside members of which vibrate perpendicular to the lines of force, and the inner one is parallel thereto.
2825·5.	It seems to be slightly widened, but it is too weak to analyse.
2820·1.	A rather broad line, which seems to be slightly widened, but it is too weak to analyse.
2802·3.	This line shows signs of widening.
2748·3.	This line becomes a triplet, the two outside members of which vibrate perpendicular, and the inner one parallel, to the lines of force.
2676·5.	This line becomes a widened doublet or reversal. The two divisions are sharp, clear and well defined.
2428·1.	This line also becomes a widened doublet or reversal. The two members do not appear to be so sharp nor so clearly defined as those of 3029·3 and 2676·5.

With regard to the lines which are divided into triplets, of those arising from 4792·8, 4488·4, 4065·2, 4053, 3898, 3804·2, 2913·6, 2748·3, the two outer members vibrate perpendicular to the lines of force, and the inner one parallel thereto. Whilst of the triplet arising from 3122·8 the two outer members vibrate parallel, and the inner one perpendicular, to the lines of force.

It may be mentioned here that the air lines at 4600 and

3994 show well-marked signs of being widened, as has been noted before.

ANTIMONY.

Wave-length	
λ 4622·0.	This line becomes a widened reversal or doublet. The separation is well marked and clearly defined.
4351·5. 4264·4.	Two broad lines which seem to show signs of widening.
4132·8. 4033·7.	Two lines, which seem to show distinct widening, but are too weak to analyse.
3722·4.	This line becomes a widened reversal or doublet. The separation is quite sharp and clearly defined.
3637·5.	There is a distinct widening of this line. On analysing it with the calcite, it seems to be a triplet, with the two outer components vibrating perpendicular, and the inner one parallel, to the lines of force.
3597·8.	A nebulous line, and seems to show signs of being widened.
3520·3.	A weak line, which appears to be widened, but it is too weak to analyse.
3504·6. 3498·3.	Two nebulous lines, which appear to be widened.
3267·6.	There is well marked widening. When analysed with the calcite, it appears to be a triplet, the two outside members of which vibrate parallel, and the inner one perpendicular, to the lines of force.
3240·5.	A broad nebulous line, which appears to be slightly widened.
3231·6.	This becomes a triplet, the two outer members vibrate parallel, and the inner one perpendicular, to the lines of force.
3039·8.	A broad diffuse line, which is perhaps a little widened.
3029.	This becomes a triplet. On analysing with the calcite, it seems to be divided like 3231·6, but the members of the triplet do not seem to be as sharply defined as those of 3231·6.
2877·1.	There is a distinct widening. On analysing with the calcite, it appears to be a triplet, with the two outside members vibrating parallel, and the inner one perpendicular, to the lines of force.
2768·1.	This line is widened, but it does not seem to be separated by the calcite.
2717·9. 2691·3. 2681·7.	These lines show a widening, but they are too weak to analyse by the calcite.
2668·9.	A widened reversal or doublet. The separation is well marked and clearly defined.

Wave-length	
λ 2651·7. 2611·3.	These lines show distinct widening—but they are too weak to analyse.
2597·2.	It shows a distinct widening; but the calcite did not separate it further.
2572·7.	This line is rather weak, but it seems to show signs of widening.
2527·6.	A distinct widening; and, on analysis, it appears to be a triplet, with the two outer members vibrating parallel, and the inner one perpendicular, to the lines of force.
2477·3. 2444·8.	These lines show a distinct widening—but they are too weak to analyse.

With regard to the lines divided into triplets, it is seen that of those arising from 3267·6, 3231·6, 3029, 2877·1, 2527·6, the two outer members vibrate parallel, and the inner one perpendicular, to the lines of force. On the other hand, of the triplet arising from 3637·5, the two outer members vibrate perpendicular, and the inner one parallel, to the lines of force.

There are three lines on the plates and marked by Hartley and Adeney as belonging to tellurium, which may be mentioned here, namely:

3382·0.	A sharp, well-marked and weak line when the spark is normal; but when it is in the magnetic field there appears to be no impression on the plate, even after 70 minutes' exposure. It may be due to a widening or a division of the line which makes it too weak to affect the sensitive plate.
3273·0.	This is a widened doublet or reversal. The separation is well marked and sharply defined.
3246·6.	This gives a distinct widening. When analysed with the calcite, it seems to be a triplet, with the two outside members vibrating parallel, and the inner one perpendicular, to the lines of force.

BISMUTH.

4724·5	This line becomes a widened doublet or reversal.
4301·5. 4259·2.	Two broad strong lines and they appear to be slightly widened.
4121·2.	The line becomes a triplet, with the two outer components vibrating perpendicular, and the inner one parallel, to the lines of force.
4079·0. 3863·7. 3792·7.	These are strong diffuse lines, and seem to show signs of being widened.

Wave-length

λ 3757·0. This strong diffuse line seems to be widened, and one or two plates also show a weak ill-defined doublet or reversal.

3595·7. This line shows a widening, but no separation could be made out when the calcite was interposed.

3510·5. The line showed distinct widening; and on analysing with the calcite, it seems to be a triplet, with the two outer members vibrating parallel, and the inner one perpendicular, to the lines of force.

3396·7. There is a widening, but no separation could be made out on interposing the calcite crystal.

3075·7. There seems to be a widening, but when the calcite is interposed, the light is too weak to affect the photographic plate.

3067·1. A strong line with diffuse edges and well marked reversal, even when the spark is normal. In the magnetic field each side of the reversal seems to be divided into two, and the appearance on the plate is of four lines, with the two outside members rather diffuse on the outer edges, and the two inner members much sharper.

3023·8. The line gives a widened doublet or reversal.

2992·2. This weak line shows a distinct widening. On analysing with the calcite, the two outer components vibrate perpendicular, and the inner one parallel, to the lines of force.

2988·1. A widened doublet or reversal.

2937·5. The line is distinctly widened. On analysing with the calcite, it seems to be a triplet, with the two outer components vibrating perpendicular, and the inner one parallel, to the lines of force.

2897·2. There is a distinct widening. On analysis, it appears to be a triplet, with the two outer components vibrating perpendicular, and the inner one parallel, to the lines of force.

2779·3.
2627·0. } Both these lines seem to be slightly widened, but they are too weak to analyse with the calcite.

Of the lines divided into triplets, those arising from 4121·2, 2992·2, 2937·5, 2897·2, show that the two outer members vibrate perpendicular, and the inner one parallel, to the lines of force. Whilst of the triplet arising from 3510·5, the two outer members vibrate parallel, and the inner one perpendicular, to the lines of force.

LEAD.

4386·4.
4245·3. } Both these lines are strong and nebulous on the less refrangible side; and both show distinct widening.

Wave-length	
λ 4168·2.	This line shows distinct widening—but it is too weak to affect the sensitive plate when the calcite is introduced.
4061·5.	This becomes a triplet, and the two outside components seem to vibrate perpendicular, and the inner one parallel, to the lines of force.
4057·5.	This becomes a triplet. The two outside components vibrate perpendicular, and the inner one parallel, to the lines of force.
4020·5.	The line shows marked widening, but it is too weak to affect the plate when the calcite is introduced.
3853·2.	A broad line which seems to be widened.
3738·9.	This line becomes a triplet, the two outside members vibrating perpendicular, and the inner one parallel, to the lines of force.
3682·9.	This becomes a triplet, and the components vibrating like those of 3738·9.
3671·0.	It shows distinct widening—but the line is too weak to affect the plate when the calcite is interposed, even after more than 60 minutes' exposure.
3639·2.	This line becomes a triplet, the two outside components vibrating perpendicular, and the inner one parallel, to the lines of force.
3572·6.	A weak line, which seems to be a triplet, the members of which vibrate like those of 3639·2.
2872·2.	The line becomes a widened reversal or doublet.
2832·2.	The line is distinctly widened; and, on analysing it with the calcite, seems to be a triplet, with the two outer members vibrating perpendicular, and the inner one parallel, to the lines of force.
2822·1.	The line is distinctly widened, but it is too weak to analyse.
2801·4.	This line is widened, and the calcite separates it into a triplet, with the two outer components vibrating perpendicular, and the inner one parallel, to the lines of force.
2662·5.	The line is widened. On analysing with the calcite the components are weak, but it seems to be divided into a triplet, with the two outside members vibrating parallel, and the inner one perpendicular, to the lines of force.
2613·4.	There is a distinct widening. With the calcite it is divided into three, and the two outer members vibrate parallel, and the inner one perpendicular, to the lines of force.
2576·4. 2475·7.	These lines are widened, but are too weak to analyse.
2204·3,	A weak nebulous line, which seems to show signs of being widened.

With regard to the lines divided into triplets, of the lines 4061·5, 4057·5, 3738·9, 3682·9, 3639·2, 3572·6, 2832·2, 2801·4, the two outside members vibrate perpendicular, and the inner one parallel, to the lines of force. On the other hand, of the triplets arising from 2662·5 and 2613·4, the two outside members vibrate parallel, and the inner one perpendicular, to the lines of force.

TIN.

Wave-length	
λ 4524·0.	The line becomes a triplet, the two outer members of which vibrate perpendicular to the line of force, and the inner one parallel thereto.
3800·3.	There is a distinct widening, and, when analysed by the calcite, it seems to be a triplet, with the two outside components vibrating perpendicular, and the inner one parallel, to the lines of force.
3655·5.	The line is widened, but the image is too weak to analyse.
3351·8.	A nebulous band, which seems to be very slightly widened. No separation can be made out by the calcite.
3330·0.	This line becomes a doublet or reversal. The separation is clear, sharp and well defined.
3282·9.	A nebulous band, which seems to be very slightly widened. No separation can be made out by the calcite.
3261·6.	The line becomes a triplet, and the two outside members vibrate parallel, and the inner one perpendicular, to the lines of force.
3174·3.	This line becomes a triplet, with the two outer components vibrating parallel, and the inner one perpendicular, to the lines of force.
3033·1.	This becomes a triplet, with the two outer members vibrating perpendicular, and the inner one parallel, to the lines of force. The three components, when analysed by the calcite, do not appear so well defined nor so sharp as in other instances.
3007·9.	This is a triplet, with the members vibrating as in 3033·1. And also here the components separated by the calcite are not so well defined, although the definition is sharper than that of 3033·1.
2862·1.	This line becomes a triplet, the two outer components vibrating perpendicular, and the inner one parallel, to the lines of force.
2849·3.	A somewhat weak line, which is widened. The separation is not very well defined, but it seems to be a triplet, with the two outer members vibrating perpendicular, and the inner one parallel, to the lines of force.
2838·9.	This becomes a triplet, with the components vibrating as the last.
2705·8.	There is a well-marked widening, but the line is too weak to analyse with the calcite.

With regard to the lines divided into triplets, of those which arise from the lines 4524, 3800·3, 3033·1, 3007·9, 2862·1, 2849·3, 2838·9, the two outside components vibrate perpendicular to the lines of force and the inner one parallel thereto. Whilst of the triplets which arise from 3261·6 and 3174·3, the two outer members vibrate parallel, and the inner one perpendicular, to the lines of force.

There is a weak line at 3032·2 which appears to belong to arsenic, and it seems to be widened in the magnetic field.

The general results, therefore, of the effect of very strong electro-magnetic fields on the spark spectra of the above-mentioned elements are:—

(1) That there is a division of some lines into triplets, the components of which vary in the directions of their vibrations.

(2) That other lines are widened, even when they do not seem to be divided.

(3) That there is an increase in the number of lines which are reversed, if these, or some of them, are not real doublets. It may be mentioned that some of these reversals or doublets seem to be as persistent as those which have been noticed when the reversed line has its origin in the arc.

With regard to the doublets or reversals, it does not seem easy to distinguish them from each other when they are observed with the magnet broadside on. But there may be a difference in that when a line is really doubled, the two components seem to be much wider apart than is observed usually in reversals; and they are also sharp, well defined and persistent. However, this is a question which will be investigated later.

The investigation is still in progress, together with other elements and alloys, and some of the photographs will be published later.

Finally, I desire to express my best thanks to Professor Liveing for allowing me the use of his spectroscope; to Mr G. F. C. Searle for measuring the magnet after it was constructed; and to Mr C. T. Heycock for the gift of the electrodes of pure gold.

On the relation in size between the megalosphere and the microspheric and megalospheric tests in the Nummulites. By J. J. Lister, M.A., F.R.S., St John's College.

[*Read* 13 March 1905.]

At the meeting of the Society on 31 October, 1904, the author drew attention to the fact that in the three English species of *Nummulites*, viz. *N. laevigatus*, *variolarius* and '*elegans*,' both megalospheric and microspheric forms were represented and associated in the Bracklesham and Barton beds of the Hampshire basin.

A comparison of the sizes of the megalospheres in these species suggested that a definite relation might exist between them and the sizes of the whole microspheric tests.

To examine this question the following species have been examined: *Nummulites complanatus* (*tchihatcheffi*), *N. perforatus* (*Lucasanus*), *N. gizehensis* (*curvispirus*), *N. perforatus* var. *obesus* (*Lucasanus* var. *obsoletus*), *N. laevigatus* (*Lamarcki*), *Assilina exponens* (*mamillata*), *Nummulites biarritzensis* (*guettardi*), *N. discorbinus* (*sub-discorbinus*), *N. Orbignyi* (*wemmelensis* var. *elegans*) and *N. variolarius* (*Heberti*). (The name included in brackets is that of the alternate form of the species. In all cases except the last it happens that the name of the microspheric form has priority over that of the megalospheric.)

The species are here arranged in order of the sizes of the megalospheres, and this is found to coincide with the order of the volumes of the microspheric tests (with the exception of the variety *obesus* of *N. perforatus*, the microspheric test of which falls one place out in the series).

Polystomella crispa is the only member of the *Nummulitidae* of which the details of the life-history are approximately known, and in this the only mode of origin of the megalospheric form which has been observed is, by an asexual process of reproduction, from a microspheric parent. Hence it would appear that in this mode of reproduction, in the *Nummulitidae*, the size of the offspring is approximately proportional to the volume of the protoplasm of the parent.

The microsphere probably arises as a zygote formed by the conjugation of the zoospores produced by the megalospheric individuals; and the size of the microsphere is found not to vary outside 15–20 μ in the four species (*N. gizehensis*, *laevigatus*, *Orbignyi* and *variolarius*) in which it has been measured.

The two modes of reproduction are thus contrasted in the size of the offspring.

The great reduction in the size of the megalospheric test in the earlier species on the list (*N. complanatus*, *perforatus*, &c.) was compared with the corresponding phenomenon offered by Ferns, in which the prothallus is dwarfed by the large size of the sporophyte.

The old Moraines of South Victoria Land. By H. T. FERRAR, M.A., Sidney Sussex College. (Communicated by Mr A. E. SHIPLEY.)

[*Read* 13 March 1905.]

The paper first dealt with the Topography of South Victoria Land, a land consisting of a range of mountains some 800 miles long in a North and South direction, with a steep Eastward face on an average 10,000 feet high, facing the sea and buttressing a vast interior ice-field. Details were given of the stranded moraines on Cape Adare, on the Possession Islands and on Franklin Island as well as those high on the slopes of Mount Erebus and Terror. The latter could only have been landed there by the Ross Ice-sheet being thicker than it is at present. Reversed glaciers, glaciers not reaching the sea and beheaded glaciers were mentioned; all pointing to the same conclusion, a retreat of the ice.

This retreat is now going on, so that increase of cold could not produce a greater glaciation. If this former greater extension was due to a warmer climate, why have the New Zealand glaciers decreased of late, and what is the connection of the 'Ice-age' of Europe with the 'Great Glacier Epoch' of New Zealand and Patagonia?

Notes on a Collection of Parasites belonging to the Museum of University College, Dundee. By A. E. SHIPLEY, M.A., F.R.S., Fellow and Tutor of Christ's College, Cambridge, and University Lecturer in the Morphology of the Invertebrata.

[*Received* 7 March 1905.]

This collection of parasites was handed to me some considerable time ago for determination; in working them out I have in many cases consulted Dr von Linstow to whom I owe my best thanks for his ready help. The collection contained fifteen species of Nematoda of which eleven belonged to the genus *Ascaris* and ten species of Cestoda of which five belonged to the genus *Bothriocephalus*.

As befits an important centre of the whaling industry the parasites collected in the Museum at Dundee are largely taken from marine animals. The seal, the walrus, the narwhal, the dolphin, several fishes and the cormorant figure amongst the hosts. A complete list of these is given at the end of the Paper.

NEMATODA.

1. *ASCARIS ADUNCA* Rud.

A. rigida Rud.
A. biuncinata Molin.
Diesing, System. Helm. II. p. 171.
Molin, SB. Ak. Wien. XXXIII. p. 297.
Schneider, Monog. d. Nematoden, p. 48.
Ramsay Wright, P. Canad. Inst. I. 1879, p. 72.
von Linstow, Arch. Naturg. 1878, p. 236.
" " " 1884, p. 130.
" Ergebn. Hamburger Magalhaenische Sammelreise, I.

Numerous specimens from *Lophius* sp. The organ affected was indecipherable. The nematode is described from the stomach and intestine of the *Alosa vulgaris* Cuv. and Val.

2. *ASCARIS ATTENUATA* Molin.

Molin, SB. Ak. Wien, XXX. p. 147.

A large number of specimens with no host recorded, they were presumably taken from a Python as von Linstow records this species in the intestine of *Python tigris* Daud. Another bottle contained specimens from *Python molurus* Gray.

3. *ASCARIS CLAVATA* Rud.

Diesing, Syst. Helm. II. p. 176 and Revision d. Nematoden, p. 664.
von Linstow, Arch. Naturg. 1880, p. 43.
„ Arch. mikr. Anat. LX. 1902, p. 2.

Several specimens taken from the duodenum of the Cod *Gadus morrhua*, it is also recorded from the Haddock *Gadus aeglefinus*.

4. *ASCARIS DECIPIENS* Krabbe.

Krabbe, Vid. Medd. pp. 45—47.
von Linstow, Fauna artica, Römer and Schaudinn, I. p. 119, Jena, 1900.

No host is recorded for these specimens but it is common enough in the intestine of *Phoca barbata* Müller and *Phoca groenlandica* Müller, *Phoca hispida* Schreb and *Phoca vitullina* Lin.

Another collection taken on the S.S. *Eclipse* were found in a bearded seal *Phoca barbata* Müller at Lat. 68° 30′ N. and Longit. 55° 10′ W.

Another collection was made from the stomach of the Walrus *Trichechus rosmarus* Lin. taken off Greenland.

5. *ASCARIS MYSTAX* Rud.

Diesing, Syst. Helm. II. p. 179.
„ SB. Ak. Wien, XLII.
Schneider, Monog. d. Nematoden, p. 38.
Bischoff, Zeitschr. wiss. Zool. VI. p. 377.
Baird, Rep. New York Zool. Soc. VIII. 1903, p. 129.

A few specimens from a dog. This Nematode is also a human parasite, being occasionally found in the intestines.

6. *ASCARIS OSCULATA* Rud.

Schneider's Monogr. d. Nematoden, p. 44.
Krabbe, Vid. Medd. p. 45.
Nehring, SB. Ges. naturf. Berlin, 1884, p. 59.

von Linstow, Fauna artica, Römer and Schaudinn, I. p. 119, Jena, 1900.
von Linstow, Arch. mikr. Anat. XLIV. p. 509.
Joyeux-Laffuie, Bull. Soc. Normand. VIII. 1895, p. 144.
Stiles and Haswell, Internal Parasites of the Pribilof Fur Seal, Washington, 1899.

Numerous specimens from *Phoca barbata* but from what part of the body is not recorded. It has been described from the intestine of this species of seal as well as from that of *Halichoerus grypus* Nilss., *Monachus albiventer* Gray, *Phoca cristata* Fahr., *Trichechus rosmarus* Lin., *Leptonyx monachus* Fahr., *Phoca groenlandica* Müller, *Phoca annellata* Nilss., *Phoca pantherina*, and *Phoca vitulina*. A second set of specimens came from the Harp Seal *Phoca groenländica*.

7. *ASCARIS RUBICUNDA* Schneider.

Schneider's Monogr. d. Nematoden, p. 42.
Stossich, Boll. Soc. Adriat. XVI. 1895.
Müller, Arch. Naturg. LXIII. 1897, p. 9.

A few specimens from an undetermined Python. This species has been described from the oesophagus and stomach of *Python molurus* Gray.

8. *ASCARIS SIMPLEX* Rud.

Rudolphi, Entoz. Hist. II. p. 170, and Synops. p. 49.
Krabbe, Vid. Medd. p. 47.
von Linstow, Voyage of the Challenger, XXIII. Pt LXXI. p. 2.
Stiles and Haswell, Internal Parasites of the Pribilof Fur Seal, Washington, 1899.

Many specimens from the Dolphin *Delphinus delphis* Lin. The species has also been described from *Phocaena communis* Cuv., *Platanista gangetica* Cuv. It is found in the stomach and oesophagus.

9. *ASCARIS SPICULIFERA* Rud.

Diesing, Syst. Helm. II. p. 157.
Leidy, P. Ac. Philad. 1858, p. 102.
Schneider, Monog. d. Nemat. p. 45.
Parona, Elmintol. Sard. p. 70.
von Linstow, Voyage of the Challenger, XXIII.

A few specimens from the stomach of the Cormorant, *Phalocrocorax* sp. It is a common species in sea-birds.

10. *ASCARIS TYPICA* (Diesing).

Conocephalus typicus Dies.
SB. Ak. Wien, XLII. (1860), 1861, p. 669.
C. W. Stiles and A. Haswell, The Internal Parasites of the Pribilof Fur Seal, Pt III. p. 127, Washington, 1899.

A few examples from the stomach of *Globicephalus globiceps*, others from that of *Delphinus delphis* Lin.

11. *ASCARIS* sp.

This Ascaris from an unknown host was unidentifiable.

12. *PHYSALOPTERA DILATATA* Rud.

Diesing, Syst. Helm. II. pp. 233—234.
Molin, Monogr. del. Gen. *Physaloptera*, pp. 648—649.

A few specimens of what seem to be *Physaloptera dilatata* were taken from the Hussar Monkey, *Cercopithecus pyrrhonotus* (probably from the stomach). Von Linstow records it from *Logathrix humboldtii* Geoffr. and from *Cebus fatuellus* Erxl.

13. *PHYSALOPTERA TERDENTATA* Molin.

Molin, Verh. Ges. Wien, 1882, p. 127, and Monog. d. genus *Physaloptera*, SB. Ak. Wien, 1860, p. 651.

Four specimens from a Leopard. This species has hitherto been described from the Puma, *Felis concolar* and it is quite possible that the word leopard has been loosely applied in this instance.

14. *PSEUDALIUS ALATUS* Leuck.

Wiegmann's Arch. Naturg. 1848, pp. 26—28.
von Linstow, P. R. Soc. Edinb. XVI. 1888.

A large collection from a Narwhal, *Monodon monoceros* taken off Greenland. The part of the body from which the Nematodes were taken is not mentioned. Von Linstow's specimens came from the vessels of the inside of the cranium, the Eustachian tubes and the pharynx.

15. *SPIROPTERINA CORONATA* van Ben.

van Beneden; Mem. sur l. vers intestinaux, Paris, 1861.

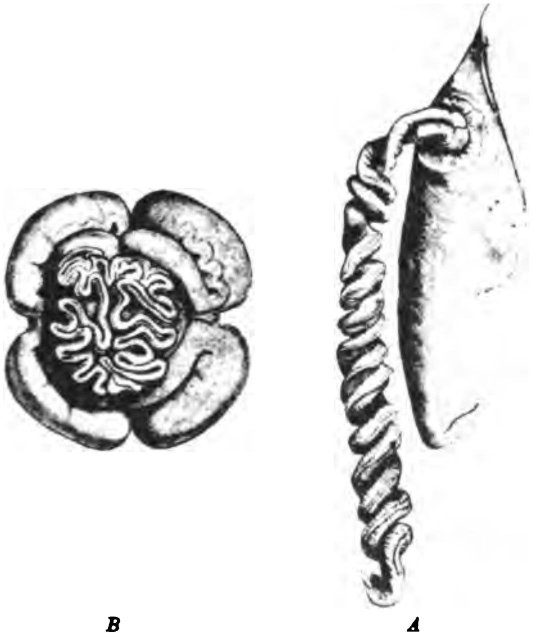
Several specimens from "a Dogfish." It usually occurs in the liver.

CESTODA.

16. *ANTHOBOTHRIUM TORTUM* v. Lins.

von Linstow, Arch. Bakter. XXXVII. 1904, p. 682.

This curious tapeworm was found with its head embedded in the tissues of the stomach wall of a *Phoca barbata*. The length is some 23 mm., the breadth 3 mm., the edges of the animal are thickened and the animal is twisted or coiled round its longitudinal axis. The head bears firm cushions and in the centre a maze of convoluted ridges as is shown in the figure. The proglottides are anteriorly 0·11 mm. long, posteriorly 0·24 mm. There are no calcareous bodies and the reproductive organs had not developed. The central area was well marked off from the peripheral by transverse muscles.



Anthobothrium tortum von Lins. From *Phoca barbata*.
A. Entire animal × 4. *B*. Anterior view of head × 8.

17. *ANTHOCEPHALUS REPTANS* Wag.

Diesing, Syst. Helm. I. p. 563.
„ SB. Ak. Wien, XLVIII. p. 313.
Olsson, Lund. Univ. Ars-skr. IV. p. 9.

A few specimens from the muscles of *Brama rayi* Bloch: it is also recorded from the muscles, peritoneum, liver of *Pogonias chromis* Cuv. and Val.

18. *BOTHRIOCEPHALUS ANGUSTATUS* Rud.

Diesing, Syst. Helm. I. p. 594.

Numerous fragments taken from the intestine of the Salmon, *Salmo salar*. Diesing records it from *Scorpaena scrofa* Lin.

19. *BOTHRIOCEPHALUS ANTHOCEPHALUS* Rud.

Krabbe, Helminthologist. Undersögels. i Denmark og pa Island, Copenhagen, 1868.
von Linstow, Bull. Ac. St Petersb. XV. p. 271.

Several examples from an unknown host probably *Phoca barbata*. This seems to be the parasite for which Monticelli has proposed the generic name of *Pyramicocephalus*, Boll. Soc. Napoli, Ser. I. IV. 1890, p. 189.

20. *BOTHRIOCEPHALUS LANCEOLATUS* Krabbe.

Krabbe, Helminthologist. Undersögel. i Denmark og pa Island, Copenhagen, 1868.
von Linstow, Arch. Naturg. 1878, p. 218.
Buysson, H. du., Rev. Sci. Bourbonnais, XVI. p. 133.

Several specimens from the intestine of *Phoca barbata*, this species has also been described from *Gadus morrhua* Cuv.

21. *BOTHRIOCEPHALUS MICROCEPHALUS* Rud.

Diesing, Syst. Helm. I. p. 592.
„ SB. Ak. Wien, XLVIII.
Monticelli, Osservazioni sul Bothriocephalus microcephalus, Rud., Napoli, 1888.
Lühe, Zeitschr. wiss. Zool. LXVIII. 1900, p. 43.

Very numerous specimens taken from *Orthagoriscus mola* Schneider, the Sunfish. It is also recorded from *O. nasus* Raf. It is found in the stomach, intestine and gills.

22. *BOTHRIOCEPHALUS RUGOSUS* Goeze.

Diesing, Syst. Helminth. I. p. 591.
„ Rev. d. Cephal. Param. pp. 239—240.
Wagner, Zeitschr. wiss. Zool. IX. pp. 73—77.
Olsson, Acta Univ. Lund. IV. p. 10.
von Linstow, Arch. Naturg. 1888, pp. 242—245.
Schaninsland, Jen. Zeitschr. XIX. 1883.

A few specimens from the Haddock, *Gadus aeglefinus* Lin. It is usually found in the intestine and the pyloric appendages. This species also occurs in the Cod and in *Merlangus carbonarius* Cuv., *Lota vulgaris*, *Motella mustela* Cuv., and probably in many other fishes.

23. *CYSTICERCUS DELPHINI* Rud.

Box, Histoire des Vers, I. p. 324.
van Beneden, C. R. Ac. Sci. LXVII. p. 1051.

A fair number of specimens taken from the blubber of *Delphinus delphis* L. These cysticerci are said to be the larval forms of some species of *Phyllobothrium*.

24. *SOLENOPHORUS MEGALOCEPHALUS* Creplin.

Bothridium pythonis Blanchard.
Diesing, Syst. Helminth. I. p. 595.
Blanchard, Ann. Sci. nat. 3rd Ser. XI. p. 118.
Griessbach, Arch. mikr. Anat. XXII. 1883, p. 365.
Moniez, Bull. du Dép. du Nord, 2nd Ser. 2 ann. 1879, p. 113.
Stossich, Boll. Soc. Adriat. XVI. 1895.

Two specimens from a *Python sebae*. This species has also been described from the intestine of *Python molurus* and *P. tigris*.

25. *TAENIA RACEMOSA* Rud.

Diesing, Syst. Helm. I. p. 511.

Many specimens from *Eunectes murinus* Wagl., it has also been described from the intestine of *Bothrops jararacca* Wagl.

LIST OF HOSTS WITH THE PARASITES DESCRIBED IN THE FOREGOING PAPER.

NEMATODA.

HOST	*PARASITE*	*POSITION IN HOST*
Canis familiaris	*Ascaris mystax* Rud.	Undetermined
Cercopithecus pyrrhonotus	*Physaloptera dilatata* Rud.	Stomach ?
Delphinus delphis	*Ascaris simplex* Rud.	Stomach and oesophagus
Delphinus delphis	*Ascaris typica* (Diesing)	Stomach
"Dogfish"	*Spiropterina coronata* van Ben.	? Liver
Felis pardus	*Physaloptera terdentata* Molin.	Undetermined
Gadus morrhua	*Ascaris clavata* Rud.	Duodenum
Globiocephalus globiceps	*Ascaris typica* (Diesing)	Stomach
Lophius sp.	*Ascaris adunca* Rud.	Undetermined
Monodon monoceros	*Pseudalius alatus* Leuck.	Undetermined
Phalocrocorax sp.	*Ascaris spiculifera* Rud.	Stomach
Phoca barbata	*Ascaris decipiens* Krabbe	Undetermined
Phoca barbata	*Ascaris osculata* Rud.	Undetermined
? *Python* sp.	*Ascaris attenuata* Molin.	Undetermined
? *Python* sp.	*Ascaris rubicunda* Schneider	Undetermined
Trichechus rosmarus	*Ascaris decipiens* Krabbe	Stomach

CESTODA.

Brama rayi	*Anthocephalus reptans* Wag.	Undetermined
Delphinus delphis	*Cysticercus delphini* Rud.	Blubber
Eunectes murinus	*Taenia racemosa* Rud.	Undetermined
Gadus aeglefinus	*Bothriocephalus rugosus* Goeze	Undetermined
Orthagoriscus mola	*Bothriocephalus microcephalus* Rud.	Stomach, intestine, gills
Phoca barbata	*Anthobothrium tortum* v. Lins.	Stomach
? *Phoca barbata*	*Bothriocephalus anthocephalus* Rud.	Undetermined
Phoca barbata	*Bothriocephalus lanceolatus* Krabbe	Intestine
Python sebae	*Solenophorus megalocephalus* Creplin	Intestine
Salmo salar	*Bothriocephalus angustatus* Rud.	Intestine

On the Maturation and Early Development of the Unfertilized Egg in certain Sawflies (Tenthredinidae). By L. DONCASTER, M.A., King's College*.

[*Received* 24 February 1905.]

In a former note (*Proc. Camb. Phil. Soc.* Vol. XII. 1904, p. 474) I gave an account of the maturation and fate of the polar bodies in *Nematus ribesii*, a sawfly in which unfertilized eggs yield males. The present note summarizes my investigations on other species of the family.

The species observed, in addition to *N. ribesii*, are *Nematus pavidus* and *N. lacteus*, *Poecilosoma luteolum*, *Croesus varus* and *Hemichroa rufa*. Of these species, *N. pavidus* is known to produce males from virgin eggs; *N. lacteus* lays eggs readily when not impregnated, and these develope to young larvae, but I know of no case in which they have been reared to adult flies. The species resembles *N. pavidus* so closely, however, that it probably belongs to the male-producing group. In *N. lacteus* the maturation and fate of the polar bodies was very similar to that described for *N. ribesii*; the second polar mitoses gave rise to four similar nuclei lying near the dorsal edge of the egg, of which the innermost is the egg-nucleus, the three outer the polar nuclei. The egg-nucleus sinks into the yolk and begins to divide to form the embryo, the outermost polar nucleus degenerates, and the two inner nuclei move together until they come into close contact. It appears however that the fusion is less complete than in *N. ribesii*, and the two nuclei break up into chromosomes, giving two groups lying side by side, each containing 8 chromosomes. One of these groups usually rapidly disappears, the other persists for some time longer. The series of *N. pavidus* was unfortunately not complete, but the stages obtained make it almost certain that the course of events is similar to that described in *N. lacteus*; it appears therefore that both these species are essentially similar to *N. ribesii* in their early development.

Poecilosoma luteolum is known to produce only ♀s from unfertilized eggs, and can be bred for several generations without any males appearing. The maturation mitoses take place as before, giving rise to four nuclei which differ from those of the other species in being of different sizes. The outermost is smallest, the second larger, and the two inner ones larger still. There is a considerably greater space between the two inner polar nuclei than in the male-producing species, and these two nuclei show no tendency to conjugate. The egg-nucleus sinks in and begins to divide, and the polar nuclei move slowly to the edge of the egg, begin to shrivel, and finally degenerate, usually without showing any signs of breaking up into chromosomes. In some eggs, however, at a stage at which in normal eggs the polar nuclei would have nearly or quite disappeared, a compact mass of chromosomes is found in the "polar protoplasm," and it seems probable that in these rare cases a fusion of all three polar nuclei has taken place, as was found by Henking in certain insects.

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* A full paper with figures and discussion will be published elsewhere.

In *Hemichroa rufa*, which produces mostly females with a few males from unfertilized eggs, the course of events does not differ greatly from that in *P. luteolum*. The polar nuclei are all about the same size, and the two outer degenerate rapidly; the innermost on the other hand becomes resolved into chromosomes which persist for some time without undergoing further change. As in *P. luteolum* the two inner polar nuclei lie far apart, and show no tendency to conjugate.

Croesus varus produces only females from unfertilized eggs, and the male is not certainly known. The earliest stages obtained of this species were after the second polar mitosis, and show the two inner polar nuclei in the polar protoplasm, and the degenerating remains of the outermost. The two inner polar nuclei are very close together, as if preparing to conjugate, but the next stage shows that one of them has become resolved into chromosomes, while the other has disappeared. The chromosomes derived from the innermost polar nucleus (nucleus of the second polar body) are at first eight in number, but before they degenerate they divide so that at a later stage a greater number is found, as was frequently the case in *N. ribesii* and *N. pavidus*.

It appears therefore that in the arrhenotokous species the inner daughter nucleus of the first polar body conjugates with the second polar nucleus, the conjugation being more or less complete in different cases; in the thelytokous species, on the other hand, no tendency to conjugation between these nuclei is observable, unless their close proximity in *C. varus* indicates an abortive attempt at conjugation in that species. It is of interest in this connexion that in a very small proportion of eggs of *N. ribesii*, as was mentioned in my previous note, the development takes place after the thelytokous type, and it is known that a small percentage of females may be bred from virgin eggs of that species.

In my note on *N. ribesii* it was mentioned that the chromosome number in the mitoses of the yolk-nuclei derived from the egg-nucleus remained the same as in the maturation divisions, this has been confirmed by other sections of this species, and is seen even more clearly in *P. luteolum*, where the chromosome number is clearly eight, both in the polar mitoses and in those found in the blastoderm. This raises an important question, for *P. luteolum* can continue to reproduce parthenogenetically for several generations, and if no doubling of the number takes place it seems impossible that reduction can occur. The question cannot be settled definitely until the oogenesis or spermatogenesis is worked out, but my provisional conclusion is that both maturation divisions are equational, and that no reduction takes place.

Another point of interest concerns the centrosome. In none of the species studied have I been able to find centrosomes in the polar mitoses, but in the divisions of the yolk-nuclei and in the blastoderm they are conspicuous, consisting of a central deeply stained granule surrounded by a clear field. Since the eggs are unfertilized they cannot be derived from the spermatozoon, and it seems certain therefore that they arise *de novo*, as was found by Wilson in artificially parthenogenetic eggs of Echinoderms.

Densities of the Earth's Crust beneath continents and oceans compared. By Rev. O. FISHER, M.A., F.G.S., Hon. Fellow of Jesus College.

[*Received* 10 March 1905.]

The following paper refers to the subject treated of with Mr Brill's assistance in the second edition of my *Physics of the Earth's Crust*[*]. Our conclusions were criticised by Mr Blake[†], while Mr Jukes-Browne desired further conditions to be taken into account than we had considered[‡]. Since I referred to our results in a paper at the British Association 1904, during the discussion of the causes of compression of the earth's crust[§], I have been led to go through the problem afresh, making, as I think, considerable improvements in the demonstration, and obtaining some additional results which are of interest.

Neglecting the centrifugal effect, let us regard the earth as a sphere, whose surface consists of flat land and water. If it is spherical, gravity must be constant all over its surface. Observation upon the force of gravity upon islands, as well as Hecker's direct observations in crossing the Atlantic[‖], confirm this view.

Let us therefore assume the earth to consist centrically of a nucleus of concentric layers each of uniform density, covered with a concentric shell varying from place to place in the number, density, and thickness, of the layers of which it is composed; but whose normal attraction over its outer spherical surface is uniform. This attraction, combined with that of the nucleus, will cause gravity over the surface to be uniform.

Since the density of the ocean is less than that of the land, and its depth variable, there must be some arrangement of the densities and thicknesses of the solid crust to compensate for these variations, so as to render the normal attraction constant; and our enquiry consists in seeking some law which will effect this object.

The form being considered spherical, and the normal attraction of the solid crust and the ocean as constant over the surface, it is equivalent to that of some hypothetical uniform shell. If we then substitute a different frustrum for a frustrum of the supposed shell, such that the normal attraction still remains uniform, it is clear that the substituted frustrum must contribute the same amount to the whole normal attraction at any given distance (θ) from it that the frustrum of the original shell did. So if we substitute another differently constituted but similarly shaped frustrum elsewhere, and the normal attraction continues uniform, it must contribute the same amount of attraction at any given distance (θ) that the frustrum of the original uniform shell did.

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[*] *Physics of the Earth's Crust*, 2nd ed., ch. XVII. and Appendix, ch. XXVI.

[†] *Annals of British Geology*, 1892.

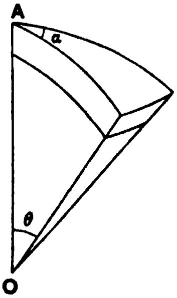
[‡] *Nature*, Nov. 21, 1889, Vol. XLI. p. 54.

[§] *Geological Magazine*, Decade V. Vol. I. p. 495, 1904.

[‖] O. Hecker. *Bestimmung der Schwerkraft auf dem Atlantischen Ozean sowie in Rio de Janeiro, Lissabon und Madrid.* Veröffentlichungen des K. Preussischen Geodätischen Institutes. Neue Folge. No. 11.

Hence these two frustra, though differing from one another, have the same effect on the normal attraction as frustra of the supposed uniform shell, and therefore the same as one another.

Consider a sector of a shell of sectorial angle α whose apex is A, and let it subtend at the centre an angle θ. Let its thickness be t_1 and its density τ_1, and a the radius of the sphere. Let it be assumed, as we shall hereafter prove, that under certain conditions the vertical attraction of the sector at A can be expanded in a converging series of the form



$$a\alpha\tau_1\left\{\frac{t_1}{a}f_1(\theta)+\left(\frac{t_1}{a}\right)^2 f_2(\theta)+\&c.\right\}.$$

Let this be overlapped by another sector of thickness t_2 and density τ_2. Then the vertical attraction at A of the composite sector will be

$$a\alpha\left\{\left(\tau_1\frac{t_1}{a}+\tau_2\frac{t_2}{a}\right)f_1(\theta)+\left(\tau_1\left(\frac{t_1}{a}\right)^2+\tau_2\left(\frac{t_2}{a}\right)^2\right)f_2(\theta)+\&c.\right\},$$

and in like manner if there is any number m of overlapping sectors, their vertical attraction at A will be expressed by

$$a\alpha\left\{\Sigma_m\tau\left(\frac{t}{a}\right)f_1(\theta)+\Sigma_m\tau\left(\frac{t}{a}\right)^2 f_2(\theta)+\&c.\right\}.$$

Now suppose another composite sector with p layers t_1', t_2', &c. of densities τ_1', τ_2', &c., θ and α remaining the same. Then its vertical attraction at A will be

$$a\alpha\left\{\Sigma_p\tau'\left(\frac{t'}{a}\right)f_1(\theta)+\Sigma_p\tau'\left(\frac{t'}{a}\right)^2 f_2(\theta)+\&c.\right\}.$$

If now we suppose that the vertical attractions at A of these two composite sectors are equal we must have

$$a\alpha\left\{\Sigma_m\tau\left(\frac{t}{a}\right)f_1(\theta)+\&c.+\Sigma_m\tau\left(\frac{t}{a}\right)^r f_r(\theta)+\&c.\right\}$$

$$=a\alpha\left\{\Sigma_p\tau'\left(\frac{t'}{a}\right)f_1(\theta)+\&c.+\Sigma_p\tau'\left(\frac{t'}{a}\right)^r f_r(\theta)+\&c.\right\},$$

and therefore,

$$\left(\Sigma_m \tau \frac{t}{a} - \Sigma_p \tau' \left(\frac{t'}{a}\right)\right) f_1(\theta) + \&c. + \left(\Sigma_m \tau \left(\frac{t}{a}\right)^r - \Sigma_p \tau' \left(\frac{t'}{a}\right)^r\right) f_r(\theta)$$
$$+ \&c. = 0.$$

This will be satisfied to the order of approximation of

$$\left(\frac{t}{a}\right)^r \text{ and } \left(\frac{t'}{a}\right)^r,$$

independently of the value of θ, if

$$\Sigma_m \tau \left(\frac{t}{a}\right) = \Sigma_p \tau' \left(\frac{t'}{a}\right), \&c., \quad \Sigma_m \tau \left(\frac{t}{a}\right)^r = \Sigma_p \tau' \left(\frac{t'}{a}\right)^r.$$

It follows that two equal sectors, whose densities and thicknesses satisfy these conditions, will to the degree of approximation named produce the same vertical attraction upon particles at their apices, independently of the value of θ, *i.e.* independently of the length of the sector. If a shorter length of them has the same property, so will their difference. Hence the remaining frustra will have the like property. Now since α and θ may have any values, neither of them being involved in the above equations, it appears that, subject to the above proviso, we may make the pair of frustra of any shape we please and of any size, and place them at the same distance θ from A. Consequently two frustra similar and equal to one another in form at the surface, the layers of which fulfil the above conditions, in regard to densities and thicknesses, will produce equal vertical attraction the one to the other at any point on the surface of the sphere equidistant from each of them, and will therefore be suitable to constitute parts of a spherical shell whose attraction is uniform over the whole sphere.

We will now shew that the vertical attraction can be expressed in a converging series of the above form, provided that the thickness t of the shell is less than the chord, $2a \sin \frac{\theta}{2}$, of the sector.

The vertical attraction of the sector at A will be

$$\tau \int_{a-t}^{t} \int_0^{\theta} \int_0^{\alpha} \frac{r^2 \sin \theta \, (a - r \cos \theta)}{(r^2 - 2ar \cos \theta + a^2)^{\frac{3}{2}}} \, d\alpha \, d\theta \, dr,$$

$$= \tau\alpha \int_{a-t}^{t} \int_0^{\theta} \frac{r^2}{a^2} \frac{\left(1 - \frac{r}{a} \cos \theta\right) \sin \theta}{\left(1 - 2\frac{r}{a} \cos \theta + \frac{r^2}{a^2}\right)^{\frac{3}{2}}} \, d\theta \, dr.$$

Let $\frac{r}{a} = u$, and let $\cos \theta = \mu$.

Then $dr = a\,du$, and $d\mu = - \sin \theta \, d\theta$.

The integral now becomes

$$\tau a a \int_{1-\frac{t}{a}}^{1} \int_{\cos\theta}^{1} \frac{u^2(1-\mu u)}{(1-2\mu u+u^2)^{\frac{3}{2}}}\, d\mu\, du$$

$$= \tau a a \int_{1-\frac{t}{a}}^{1} u^2 du \left[\frac{\mu - u}{\sqrt{1-2\mu u+u^2}}\right]_{\cos\theta}^{1}$$

$$= \tau a a \int_{1-\frac{t}{a}}^{1} u^2 du \left(1 - \frac{\cos\theta - u}{\sqrt{1-2u\cos\theta+u^2}}\right).$$

Put $u = 1 - x$. $\therefore du = - dx$.

The integral becomes

$$\tau a a \int_0^{\frac{t}{a}} (1-x)^2 dx \left(1 - \frac{\cos\theta - 1 + x}{\sqrt{1 - 2\cos\theta + (1-x)^2}}\right)$$

$$= \tau a a \int_0^{\frac{t}{a}} (1-x)^2 dx \left(1 - \frac{x - 2\sin^2\frac{\theta}{2}}{\sqrt{4\sin^2\frac{\theta}{2}(1-x) + x^2}}\right).$$

But

$$\frac{x - 2\sin^2\frac{\theta}{2}}{\sqrt{4\sin^2\frac{\theta}{2}(1-x)+x^2}}$$

$$= \frac{x - 2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\sqrt{1 - 2\sin\frac{\theta}{2}\frac{x}{2\sin\frac{\theta}{2}} + \frac{x^2}{4\sin^2\frac{\theta}{2}}}}.$$

And this can be expanded in a series of ascending powers of

$$\frac{x}{2\sin\frac{\theta}{2}},$$

which when taken between the limits will give ascending powers of

$$\frac{t}{2a\sin\frac{\theta}{2}},$$

and the series will be convergent when t is less than $2a\sin\frac{\theta}{2}$.

Hence expanding, integrating, and taking the limits, the vertical attraction at A can be expressed in a series of the form

$$\tau a a \left\{\frac{t}{a} f_1(\theta) + \left(\frac{t^2}{a^2}\right) f_2(\theta) + \&c.\right\}^*,$$

which will be convergent when t is less than $2a \sin\frac{\theta}{2}$, *i.e.* when it is less than the chord of the sector.

The limitation as regards convergency requires that the distances from the frustra of the places at which we are comparing the vertical attraction of two similarly-shaped frustra should be greater than the thickness of either of them. But since the dimensions of continents and oceans are large when compared with the depths which we propose to consider, this limitation will not be important.

Carrying the approximation to the $(n-1)$th powers of

$$\left(\frac{t}{a}\right) \text{ and } \left(\frac{t'}{a}\right),$$

and eliminating a and transposing, we have the $n-1$ equations

$$\begin{aligned}
\Sigma_m(\tau t) \quad &- \Sigma_p(\tau' t') \quad = 0,\\
\Sigma_m(\tau t^2) \quad &- \Sigma_p(\tau' t'^2) \quad = 0,\\
\vdots \qquad & \qquad \vdots\\
\Sigma_m(\tau t^{n-1}) &- \Sigma_p(\tau' t'^{n-1}) = 0.
\end{aligned}$$

If we assume the sum of the densities to be equal to some unknown number b, with that addition the equations may be written in the general form[†]

$$\begin{array}{llllll}
\tau_1 & +\tau_2 & +\tau_3 & +\ldots+\tau_n & = b, & \times c_{n-1},\\
\tau_1 t_1 & +\tau_2 t_2 & +\tau_3 t_3 & +\ldots+\tau_n t_n & = 0, & \times c_{n-2},\\
\tau_1 t_1^2 & +\tau_2 t_2^2 & +\tau_3 t_3^2 & +\ldots+\tau_n t_n^2 & = 0, & \times c_{n-3},\\
\vdots & \vdots & \vdots & \qquad\quad\vdots & & \\
\tau_1 t_1^{n-1} & +\tau_2 t_2^{n-1} & +\tau_3 t_3^{n-1} & +\ldots+\tau_n t_n^{n-1} & = 0, & \times 1.
\end{array}$$

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[*] In fact,

$$f_1(\theta) = 1 + \sin\frac{\theta}{2},$$

$$f_2(\theta) = -\left(1 + \frac{3}{4}\sin\frac{\theta}{2} + \frac{1}{4\sin\frac{\theta}{2}}\right),$$

$$f_3(\theta) = \frac{1}{8} + \frac{1}{8}\sin\frac{\theta}{2} + \frac{5}{24\sin\frac{\theta}{2}},$$

$$f_4(\theta) = \frac{1}{64}\sin\frac{\theta}{2} - \frac{1}{32}\sin\frac{\theta}{2} + \frac{1}{64\sin^3\frac{\theta}{2}},$$

$$\&c. = \&c.$$

Phil. Mag. Apr. 1894, p. 377.

[†] See Todhunter's *Theory of Equations*, Art. 290.

Multiply these equations by $c_{n-1}, c_{n-2}, \ldots, c_1, 1$, where $c_{n-1}, c_{n-2}, \ldots, c_1$, are arbitrary constants to be determined by the condition that when the equations so multiplied are added together as they stand, the $n-1$ columns after the first shall vanish, but not so the first.

Then we shall have

$$\begin{aligned}
\tau_1(t_1^{n-1} &+ c_1t_1^{n-2} + \ldots + c_{n-2}t_1 + c_{n-1}) = c_{n-1}b,\\
\tau_2(t_2^{n-1} &+ c_1t_2^{n-2} + \ldots + c_{n-2}t_2 + c_{n-1}) = 0,\\
\tau_3(t_3^{n-1} &+ c_1t_3^{n-2} + \ldots + c_{n-2}t_3 + c_{n-1}) = 0,\\
\vdots \quad &\quad \vdots \qquad\qquad\quad \vdots \qquad\quad \vdots\\
\tau_n(t_n^{n-1} &+ c_1t_n^{n-2} + \ldots + c_{n-2}t_n + c_{n-1}) = 0.
\end{aligned}$$

It appears therefore that the substitution of t_2, t_3, &c., t_n for t_1 causes

$$\tau_1(t_1^{n-1} + c_1t_1^{n-2} + \ldots + c_{n-2}t_1 + c_{n-1})$$

to vanish. Hence this latter expression is by the theory of equations equal to the product

$$\tau_1(t_1-t_2)(t_1-t_3)\ldots(t_1-t_n);$$

whilst it is also equal to $c_{n-1}b$. Also

$$c_{n-1} = (-1)^{n-1}t_2t_3\ldots t_n.$$

Hence

$$\tau_1t_1(t_1-t_2)(t_1-t_3)\ldots(t_1-t_n) = (-1)^{n-1}t_1t_2\ldots t_nb.$$

Similarly, by assuming another series of arbitrary factors $d_1, d_2, \ldots d_{n-1}$, such that the sum of every column except that containing t_2 vanishes, we can prove that

$$\tau_2t_2(t_2-t_1)(t_2-t_3)\ldots(t_2-t_n) = (-1)^{n-1}t_1t_2\ldots t_nb,$$

and so on.

Consequently we have the $n-1$ equations,

$$\begin{aligned}
&\tau_1t_1(t_1-t_2)(t_1-t_3)\ldots(t_1-t_n)\\
&= \tau_2t_2(t_2-t_1)(t_2-t_3)\ldots(t_2-t_n)\\
&= \tau_3t_3(t_3-t_1)(t_3-t_2)\ldots(t_3-t_n)\\
&\qquad \text{\&c.} \qquad\qquad \text{\&c.}\\
&= \tau_nt_n(t_n-t_1)(t_n-t_2)\ldots(t_n-t_{n-1}),
\end{aligned}$$

each of these products being equal to $(-1)^{n-1}t_1t_2\ldots t_nb$.

Consider the equal products

$$\begin{aligned}
&\tau_1t_1(t_1-t_2)(t_1-t_3)(t_1-t_4)\ldots \text{\&c.} \ldots\ldots\ldots\ldots\ldots(1),\\
&= \tau_2t_2(t_2-t_1)(t_2-t_3)(t_2-t_4)\ldots \text{\&c.} \ldots\ldots\ldots\ldots\ldots(2),\\
&= \tau_3t_3(t_3-t_1)(t_3-t_2)(t_3-t_4)\ldots \text{\&c.} \ldots\ldots\ldots\ldots\ldots(3),\\
&= \tau_4t_4(t_4-t_1)(t_4-t_2)(t_4-t_3)\ldots \text{\&c.} \ldots\ldots\ldots\ldots\ldots(4).
\end{aligned}$$

These being equal, if one of them vanishes, they must all vanish. Hence if $t_1 = t_2$, they must all vanish, because (1) does so. Now $t_1 = t_2$ causes (2) to vanish as well as (1), but none of the rest. Hence $t_1 = t_2$ must necessitate some further relation among the t's which will cause the remaining products to vanish.

If $$t_1 = t_2 = t_3 = t_4, \text{ \&c.},$$

each product will contain a vanishing factor, and they will all vanish. Hence if two layers are equal, the conditions of the problem will be satisfied by their being all equal, which would in effect reduce the shell to a single layer.

But the conditions might also be satisfied if the t's with odd suffixes are equal to those with even. Thus

$$t_1 = t_2 \text{ causes (1) and (2) to vanish,}$$

$$t_3 = t_4 \text{ causes (3) and (4) to vanish,}$$

and so on.

And if the t's with odd suffixes are in one of the frustra, and those with the even in the other (for making τ_2 and τ_4 negative will cause no difference in the demonstration), this implies that the layers in the two frustra are of equal thickness. Hence if two layers t_1 and t_2 are equal, the conditions will be satisfied if all the corresponding layers in the two frustra are of equal thickness. In this case it is clear that the number of layers in the two frustra must be the same, and in no other way than these two can the conditions be fulfilled, when a layer in one frustrum is equal to one in the other.

Next consider the products (3) and (4), and divide out the common factor $(t_3 - t_4)$. Now make $t_3 = t_4$ and we have the two sides of the resulting equation identical except the density factors. These must therefore be equal, or $\tau_3 = \tau_4$. This shows that two layers in two frustra being of equal thickness, they must also be of equal density.

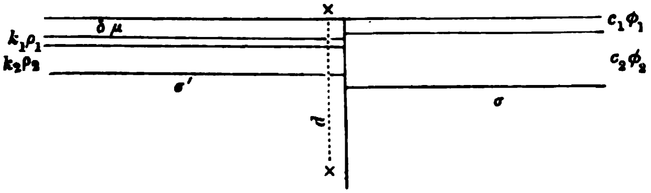
Thus we arrive at the remarkable result that if under the conditions of the problem the layers in two frustra are equal in thickness, each to each, they will also be equal in density; and all the other corresponding layers in like manner will be equal each to each in thickness and in density.

Since in the case of the earth, the upper layer in the land being of rock, differs in density from the upper layer on the side of the ocean, which is water, it follows that all the other layers on the side of the land must differ from those on the side of the ocean in thickness and density.

We see then that if at an early stage of the world's history it was covered by an ocean of uniform depth, the globe which the ocean covered must have consisted of concentric layers each of uniform thickness and density. This would lead us to think that the stable condition must at some time have been *forcibly* disturbed, as it would have been according to my hypothesis that the oceanic areas were produced by the disruption of a portion of the earth's crust, which flew off to make the moon*. But this is a digression.

Let us now suppose that the crust of the earth on the land side consists of two layers of thicknesses c_1 and c_2, and densities ϕ_1 and ϕ_2, and that beneath them there is a substratum of unknown depth and of density σ.

On the side of the ocean let δ be the depth of the water at any place, μ its density, and suppose the solid crust at that place to consist of two layers of thicknesses k_1 and k_2, and densities ρ_1 and ρ_2, and suppose the density of the substratum there to be σ', and let us take into consideration the state of affairs down to a depth d measured from the surface.



This is a generalization; for when considered throughout its whole thickness at any place, the crust must be either of uniform mean density, or else it must be on the average more dense in the upper than in the lower portion, or *vice versâ*, and the upper or lower portion may be the thicker or thinner of the two. It will appear that our formulae enable us to discover what the relations as to density and thickness are.

Then if $\Sigma_m(\tau t)$ refers to the ocean side, and $\Sigma_p(\tau' t')$ to the land side, and if this arrangement of thicknesses and densities approximately represents the case of nature, they must satisfy the equations on p. 110, viz.

$$\Sigma_m(\tau t) = \Sigma_p(\tau' t'),$$
$$\text{\&c.} = \text{\&c.}$$

Taking account of the overlapping of the t's, it will be seen that on the side of the ocean

$$\sigma' = \tau_1,\ \rho_2 = \tau_1 + \tau_2,\ \rho_1 = \tau_1 + \tau_2 + \tau_3,\ \mu = \tau_1 + \tau_2 + \tau_3 + \tau_4.$$

Also $\quad t_1 = d,\ t_2 = \delta + k_1 + k_2,\ t_3 = \delta + k_1,\ t_4 = \delta.$

Hence $\quad \tau_4 = \mu - \rho_1,\ \tau_3 = \rho_1 - \rho_2,\ \tau_2 = \rho_2 - \sigma',\ \tau_1 = \sigma',$

and similarly on the side of the land.

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* *Nature*, Vol. xxv. p. 243. 1882.

Accordingly the equation

$$\Sigma_m(\tau t) = \Sigma_p(\tau' t')$$

becomes

$$(\mu-\rho_1)\delta + (\rho_1-\rho_2)(\delta+k_1) + (\rho_2-\sigma')(\delta+k_1+k_2) + \sigma' d$$
$$= (\phi_1-\phi_2)c_1 + (\phi_2-\sigma)(c_1+c_2) + \sigma d.$$

And transposing,

$$(\mu-\rho_1)\delta + (\rho_1-\rho_2)(\delta+k_1) + (\rho_2-\sigma')(\delta+k_1+k_2)$$
$$+ (\sigma'-\sigma)d + (\phi_2-\phi_1)c_1 + (\sigma-\phi_2)(c_1+c_2) = 0.$$

The equations being now carried to the fifth power of t, terms of the order of $(t/a)^4$ are retained, which will give a close approximation.

Comparing the above equation with the general formulae

$$\tau_1 t_1 + \tau_2 t_2 + \tau_3 t_3 + \&c. = 0,$$
$$\&c., \qquad \&c.,$$

the equations

$$\tau_1 t_1 (t_1-t_2)(t_1-t_3)(t_1-t_4) = \tau_2 t_2 (t_2-t_1)(t_2-t_3)(t_2-t_4),$$
$$\&c. = \&c.,$$

by substituting the proper values for the t's and τ's, give the five equations

$$(\mu-\rho_1)\delta(-k_1)(-k_1-k_2)(\delta-d)(\delta-c_1)(\delta-c_1-c_2) \ldots\ldots(1)$$
$$= (\rho_1-\rho_2)(\delta+k_1)k_1(-k_2)(\delta+k_1-d)$$
$$(\delta+k_1-c_1)(\delta+k_1-c_1-c_2) \ldots(2)$$
$$= (\rho_2-\sigma')(\delta+k_1+k_2)(k_1+k_2)k_2(\delta+k_1+k_2-d)$$
$$(\delta+k_1+k_2-c_1)(\delta+k_1+k_2-c_1-c_2) \ldots(3)$$
$$= (\sigma'-\sigma)d(d-\delta)(d-\delta-k_1)(d-\delta-k_1-k_2)$$
$$(d-c_1)(d-c_1-c_2) \ldots(4)$$
$$= (\phi_2-\phi_1)c_1(c_1-\delta)(c_1-\delta-k_1)(c_1-\delta-k_1-k_2)(c_1-d)(-c_2) \ldots(5)$$
$$= (\sigma-\phi_2)(c_1+c_2)(c_1+c_2-\delta)(c_1+c_2-\delta-k_1)$$
$$(c_1+c_2-\delta-k_1-k_2)(c_1+c_2-d)c_2 \ldots(6).$$

Of the thicknesses here involved, d exceeds all the others on either side. Of the rest it is easy to see which factors are positive and which negative when the thicknesses are on the same side; but when they are on different sides, as for instance δ and c_1, the signs of the factors are not self-evident. The density factor $(\mu-\rho_1)$ is negative, because the density of water is less than that of any kind of rock; and we may safely assume that the density of the upper layer of the land consisting chiefly of sedimentary rocks, is less than that of the lower, consisting of plutonic rocks. Hence $(\phi_2 - \phi_1)$ will be positive. So also the density of the substratum being greater than that of the crust, $(\sigma - \phi_2)$ will be positive.

Hence, changing the signs of all factors which are evidently negative into positive, and in (5) changing provisionally $(c_1 - \delta)$ into $(\delta - c_1)$, to make it agree with (1), we shall have changed the signs of an odd number of factors in each except in (4), viz. in (1) five, in (2) three, in (3) three, in (5) five, in (6) one. All the factors in (4) are obviously positive except the density factor $(\sigma' - \sigma)$. We are therefore obliged to change it provisionally into $(\sigma - \sigma')$. Then since the several products, being equal, had the same sign originally, they will all have the same sign now as one another; and what that sign will be will depend upon the sign of $(\delta - c_1)$ which we must seek to discover.

Our five equations will now be

$$(\rho_1 - \mu)\,\delta k_1(k_1 + k_2)(d - \delta)(\delta - c_1)(c_1 + c_2 - \delta) \ldots\ldots\ldots(1)$$

$$= (\rho_1 - \rho_2)(\delta + k_1)k_1k_2(d - \delta - k_1)(\delta + k_1 - c_1)(c_1 + c_2 - \delta - k_1) \ldots(2)$$

$$= (\sigma' - \rho_2)(\delta + k_1 + k_2)(k_1 + k_2)k_2(d - \delta - k_1 - k_2)$$
$$(\delta + k_1 + k_2 - c_1)(c_1 + c_2 - \delta - k_1 - k_2) \ldots(3)$$

$$= (\sigma - \sigma')\,d\,(d - \delta)(d - \delta - k_1)(d - \delta - k_1 - k_2)$$
$$(d - c_1)(d - c_1 - c_2) \ldots(4)$$

$$= (\phi_2 - \phi_1)\,c_1(\delta - c_1)(\delta - c_1 + k_1)(\delta - c_1 + k_1 + k_2)(d - c_1)\,c_2 \quad \ldots(5)$$

$$= (\sigma - \phi_2)(c_1 + c_2)(c_1 + c_2 - \delta)(c_1 + c_2 - \delta - k_1)$$
$$(c_1 + c_2 - \delta - k_1 - k_2)(d - c_1 - c_2)\,c_2 \ldots(6).$$

In order to determine the sign of $(\delta - c_1)$, that is whether the depth of the ocean or the thickness of the upper layer of the continental crust is the greater, consider the products (1) and (5), and we have the equation

$$(\rho_1 - \mu)\,\delta(d - \delta)(\delta - c_1)(c_1 + c_2 - \delta)k_1(k_1 + k_2)$$
$$= (\phi_2 - \phi_1)\,c_1(\delta - c_1)(d - c_1)(c_1 + c_2 - c_1)(k_1 + \delta - c_1)(k_1 + k_2 + \delta - c_1),$$

and each side of this equation must have the same sign. It is obvious that the factor $(\delta - c_1)$ will not affect the question. We therefore divide it out, and it remains to find the relative magnitude of δ and c_1, such that

$$(\rho_1 - \mu)\,\delta(d - \delta)(c_1 + c_2 - \delta)\,k_1(k_1 + k_2)$$

may have the same sign as

$$(\phi_2 - \phi_1)\,c_1(d - c_1)(c_1 + c_2 - c_1)(k_1 + \delta - c_1)(k_1 + k_2 + \delta - c_1).$$

Write α and β for the factors which do not involve k_1 and k_2, and we have

$$\alpha k_1(k_1 + k_2) = \beta(k_1 + \delta - c_1)(k_1 + k_2 + \delta - c_1) \quad \ldots\ldots\ldots(A),$$

where
$$\alpha = (\rho_1 - \mu)\,\delta\,(d - \delta)(c_1 + c_2 - \delta)$$
$$= (\rho_1 - \mu) f(\delta) \text{ suppose,}$$
and
$$\beta = (\phi_2 - \phi_1)\,c_1\,(d - c_1)(c_1 + c_2 - c_1)$$
$$= (\phi_2 - \phi_1) f(c_1).$$

Equation (A) gives a quadratic in $(\delta - c_1)$ with the result

$$\delta - c_1 = -\frac{2k_1 + k_2}{2} \pm \sqrt{\frac{\alpha - \beta}{\beta} k_1 (k_1 + k_2) + \frac{(2k_1 + k_2)^2}{4}}$$
$$= -\frac{2k_1 + k_2}{2} \pm \sqrt{\frac{\alpha}{\beta} k_1 (k_1 + k_2) + \frac{k_2^2}{4}}.$$

It is evident that α is positive unless δ is greater than $c_1 + c_2$, that is than the whole thickness of the continental crust, which is out of the question, and β is clearly positive. Hence if we adopt the negative sign of the root we have

$$c_1 = \delta + k_1 + \frac{k_2}{2} + \sqrt{\frac{\alpha}{\beta} k_1 (k_1 + k_2) + \frac{k_2^2}{4}}.$$
$$\therefore\ c_1 > \delta + k_1 + k_2,$$

which would make the upper layer of the continental crust greater than the depth of the ocean and the whole thickness of the sub-oceanic crust taken together. If there are to be two layers on the side of the land, the upper layer cannot be so thick as this. Hence we need not regard the negative sign of the root, and then $\delta - c_1$ is positive or negative as

$$\frac{\alpha}{\beta} k_1 (k_1 + k_2) + \frac{k_2^2}{4} >< \left(\frac{2k_1 + k_2}{2}\right)^2$$
$$>< k_1^2 + k_1 k_2 + \frac{k_2^2}{4},$$

or as $\frac{\alpha}{\beta} >< 1$; that is, as

$$\frac{\rho_1 - \mu}{\phi_2 - \phi_1} f(\delta) >< f(c_1).$$

Now δ being the depth of the ocean varies from place to place, but it is capable of measurement; while c_1, the thickness of the upper layer of the continental crust, may be considered constant, but cannot be measured. It must therefore be estimated with reference to some assumed value of δ. If then we can show that $\delta - c_1$ is positive, *i.e.* that $c_1 < \delta$, for some given value of δ, it will be less than δ for all larger values of δ, provided they do not nearly approach $c_1 + c_2$, which we have assumed with evident good reason to be greater than δ.

We have proved that $\delta - c_1$ is positive, if

$$\frac{\rho_1 - \mu}{\phi_2 - \phi_1} f(\delta) > f(c_1).$$

To discover under what conditions this is the case, we must assume some probable values as a working hypothesis. ρ_1 being the density of some kind of rock, and μ the density of water, we may put $\rho_1 - \mu = 1{\cdot}6$. If then we assume $\sigma = 3$, for the density of the substratum under the land, and 0·2 as the difference between the densities of the two layers of the continental crust, we have

$$\frac{\rho_1 - \mu}{\phi_2 - \phi_1} = \frac{1{\cdot}6}{0{\cdot}2} = 8.$$

Let us also assume the entire thickness, $c_1 + c_2$, of the continental crust to be 25 miles, and let us consider the state of affairs to the depth of 100 miles, so that $d = 100$.

Then tracing the curve

$$f(x) = x(25 - x)(100 - x),$$

we observe that it cuts the axis of x at the origin, and at $x = 25$, and $x = 100$,

$$f(x) \text{ is a maximum when } x = 11{\cdot}62,$$

and the value of the function then becomes 13741. If we now seek the value of x which makes $8f(x)$ to have the same value as the maximum value of $f(x)$, *i.e.* to solve the equation

$$8f(x) = 13741,$$

or

$$x^3 - 125x^2 + 2500x - 1717 = 0,$$

we find that 0·5 substituted for x gives a negative result, while 1 gives a positive, which shows that a root lies between 0·5 and 1.

Hence $8f(\delta)$ is greater than the greatest value of $f(c_1)$ when δ has a value somewhere between a half and one mile. Therefore δ is certainly greater than c_1 if $\delta = 1$ mile, which is very much less than the mean depth of the ocean. We may therefore safely assume $\delta - c_1$ as positive for all values of δ with which we are concerned, and for which the conditions of the problem are fulfilled.

We say advisedly "for which the conditions of the problem are fulfilled," because our demonstration assumes that the continental and oceanic areas are abruptly separated, whereas in nature they merge into one another, because there is an area of deposition fringing the coasts which is neither oceanic nor continental; over these areas our formulae will not apply, and indeed there is reason to think that there the assumption of the equality of gravity is not warranted, because in crossing the Atlantic Hecker observed that gravity was slightly in excess as the land was approached; and the deflection of the plumb-line seawards in geodetic surveys points to the same conclusion.

Having decided that $(\delta - c_1)$ must be positive, all the five equal products must be positive.

Hence, by examining these successively, we see that

from (1), $\rho_1 > \mu$ and $\delta > c_1$,
from (2), $\rho_1 > \rho_2$ and $c_1 + c_2 > \delta + k_1$,
from (3), $\sigma' > \rho_2$ and $c_1 + c_2 > \delta + k_1 + k_2$,
from (4), $\sigma > \sigma'$,
from (5), $\phi_2 > \phi_1$ and $\delta + k_1 > c_1$,
from (6), $\sigma > \phi_2$.

The original assumptions were, that the density of the ocean is less than that of rock, that the density of the upper layer of the continental crust is less than that of the lower layer, and that the density of the substratum of the continental crust is greater than that of the lower layer. The results we have obtained are, (1) that the thickness of the upper layer of the continental crust is less than the depth of the ocean at any place where gravity has its average value. From (2) that the density of the upper layer of the suboceanic crust is greater than that of the lower layer, and since from (5) we see that k_1 is less than $(\delta - c_1)$ we gather that this dense upper layer must be very thin. From (3) we learn that the total thickness $(c_1 + c_2)$ of the continental crust is greater than the depth of the ocean and the thickness of the suboceanic crust together, viz. than $(\delta + k_1 + k_2)$. From (4) we learn that σ is greater than σ', *i.e.* that the density of the substratum beneath the continent is greater than that beneath the ocean.

The bearing of these results upon questions of geology appears to be, that, assuming a liquid substratum beneath the crust of the earth, the solid crust is thinner beneath the oceans than beneath the land, but is at the same time more dense. The substratum being thus less dense beneath the suboceanic crust will be affected by upward convection currents, which must necessarily flow horizontally towards the continents, and descend there. The continental crust dips more deeply into the substratum, and consequently these horizontal currents, impinging on it, will tend to compress it, and cause corrugations in it. The upper layer of the suboceanic crust, being very thin and very dense, probably consists of basic lava flows from suboceanic volcanoes, which by oxidation would produce the red clay covering the abysmal areas. The upper layer of the continental crust is thinner than the depth of the true ocean, and of small density, which would agree with its sedimentary origin.

The Chlorination of Methyl derivatives of Pyridine. Part I: 2-Methyl Pyridine. By W. J. SELL, M.A., F.R.S., Christ's College.

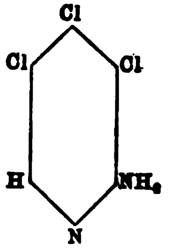
[*Read* 14 November 1904.]

Preliminary experiments having shown that 2-methyl pyridine blackens when chlorine is passed into it and that a similar result attends all attempts at chlorination by means of phosphorus pentachloride, recourse was had to the method found successful in the case of pyridine, viz. the direct chlorination of the 2-methyl pyridine hydrochloride saturated with hydrochloric acid. This product remains liquid at the ordinary temperature of the laboratory. The work of Ramsay (*Phil. Mag.* [5] 2, 1876, II) led him to the conclusion that picoline hydrochloride was not acted upon by chlorine even when heated with it. The author's results confirm this at the ordinary temperature, but show that at about 100—105° C. rapid replacement of hydrogen takes place and a number of chlorinated products are produced. After complete chlorination the still liquid products were cooled by ice and a current of air passed through for some hours when a large crystalline precipitate was produced. This precipitate was drained off from the liquid products and recrystallised from spirit. The crystals are prismatic, freely soluble in alcohol and most organic solvents, but insoluble in water. They constitute rather over 52 per cent. of the 2-methyl pyridine taken, and so far are the only solid product of the change. Analysis shows that the formula C_6HCl_6N should be assigned to this body. The substance melts at 102—103° C. and is isomeric with a compound M.P. 60 obtained by Ost (*J. Pr. Chem.* [2] 27, 257) by the interaction of phosphorus pentachloride on comenamic acid. In addition to the compound mentioned above there is a liquid product consisting of compounds less highly chlorinated, the work on which although advanced is still incomplete.

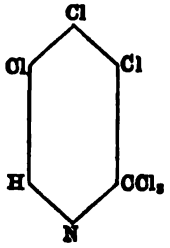
It is worthy of remark that there is a total absence of the dipyridyl derivative which constituted such a large proportion of the products in the chlorination of pyridine hydrochloride (*Trans. Chem. Soc.* 1899, 75, 979; 1901, 79, 899). The solid compound mentioned above, M.P. 102—103°, heated with eighty per cent. sulphuric acid, gives a nearly quantitative yield of trichlorpicolinic acid C_5HCl_3NCOOH, showing that three of the six atoms of chlorine in the original compound belong to the side-chain. This trichlorpicolinic acid when distilled with glycerol breaks down, producing carbon dioxide and a trichlorpyridine, melting at 72—73°, produced not only by the direct chlorination of pyridine, but also by the action of phosphorus pentachloride on that substance (*Trans. Chem. Soc.* 1898, 73, 442).

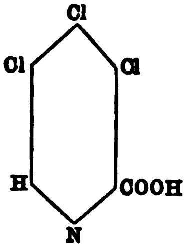
From the highly basic and other properties of this trichlorpyridine it was suspected that the chlorine atoms in the molecule occupied the positions 3, 4, 5, and the experimental verification of this surmise was carried out as follows:

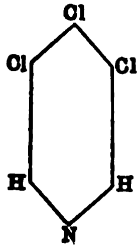
We may take for comparison the substance 2-amino 3, 4, 5 trichlorpyridine whose formula may be thus depicted (*Trans. Chem. Soc.* 1899, 75, 980; 1900, 77, 771):



This compound should also be obtainable from the trichlorpicolinic acid mentioned above by first converting it into its amide, and then by the "Hofmann reaction" into amino-trichlorpyridine. These reactions were carefully carried out and resulted in a substance identical with the compound whose graphic formula is above depicted. We may, therefore, formulate the three substances which are the subject of this paper as follows:







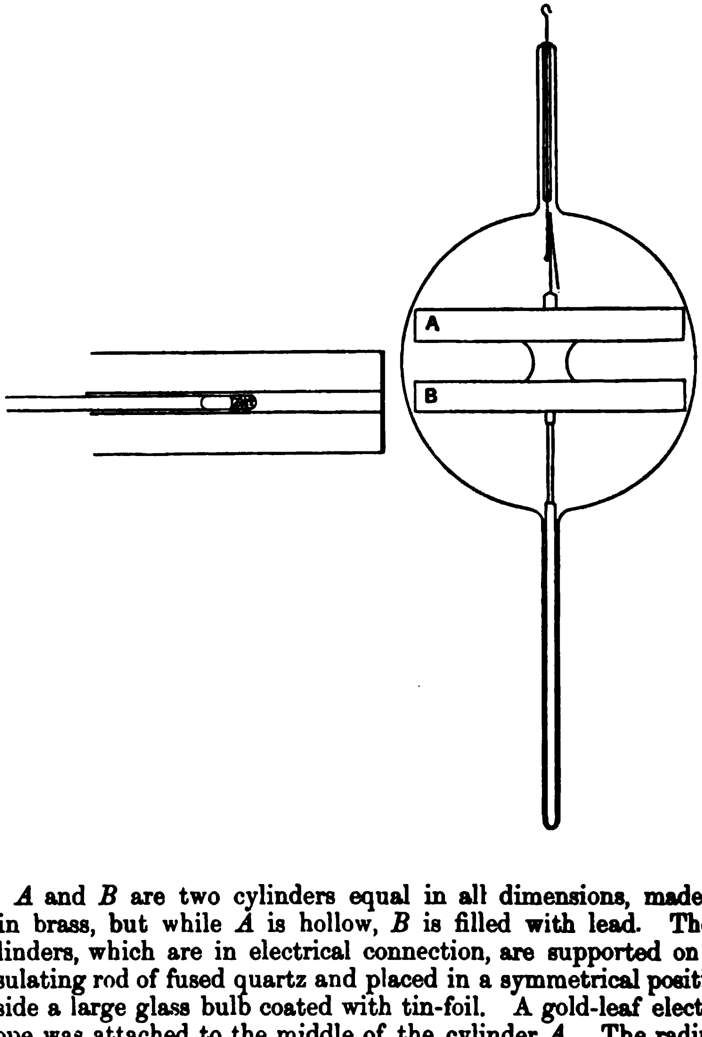
Do the γ rays carry a charge of negative electricity? By J. J. THOMSON, M.A., F.R.S., Cavendish Professor of Experimental Physics.

[*Read* 30 January 1905.]

As the result of a series of interesting experiments Paschen (*Physikalische Zeitschrift*, v. p. 563) came to the conclusion that the γ rays emitted by radium are, like the β rays, charged with negative electricity. Paschen's experiments are of two types; in the first type the radium is placed inside an insulated box made of thick lead, the lead being thick enough to absorb all the β rays, but not so thick as to stop more than a fraction of the γ rays. Under these circumstances the insulated box gets charged up with positive electricity; this Paschen regards as proving that the γ rays which have passed through the lead have carried with them charges of negative electricity, leaving an excess of positive behind. The positive charge on the box may, however, be explained in another way; we know that when γ rays enter or leave a piece of metal secondary rays are produced which like the β rays carry with them a charge of negative electricity, thus the negative electricity which escapes from the box may be carried by the secondary and not by the γ rays. The experiment in this form is inconclusive: to make it conclusive it would be necessary to place the lead box in a magnetic field strong enough to prevent the negatively electrified particles from leaving the box; if the γ rays carry the charge the magnetic field would not diminish the positive charge on the box, as they are not affected by magnetic force; if the secondary rays carry the charge the magnetic field would stop the charging up of the box.

In the second type of Paschen's experiments an insulated piece of metal was bombarded by γ rays, and the metal was found to acquire a charge of negative electricity. Here again the result seems inconclusive, for to prevent the charge leaking from a charged body exposed to radium the body must be placed in a good vacuum: thus the metal in Paschen's experiments had to be placed in a vessel from which the air could be exhausted; now the walls of the vessel are struck by the γ rays and emit secondary radiation which carries a charge of negative electricity, thus the negative electrification may again be due to secondary radiation and not directly to the γ rays.

To test whether the secondary rays are or are not the source of the electrification observed in Paschen's experiments I tried the experiment represented in the figure.



A and *B* are two cylinders equal in all dimensions, made of thin brass, but while *A* is hollow, *B* is filled with lead. These cylinders, which are in electrical connection, are supported on an insulating rod of fused quartz and placed in a symmetrical position inside a large glass bulb coated with tin-foil. A gold-leaf electroscope was attached to the middle of the cylinder *A*. The radium (30 mg. of radium bromide) was outside the glass vessel, it was placed far back in a cylindrical cavity drilled in a lead cylinder, the opening of the cavity was closed by a plate of lead 2 mm. thick to absorb the β rays. This arrangement gave a pencil of γ rays parallel to the axis of the lead cylinder, this cylinder was placed on a levelling table and adjusted so that by altering the level of the table the pencil of γ rays could be directed along the axis of either A or B. The vessel containing the cylinders was exhausted to a very high vacuum by Dewar's method of absorption by carbon cooled with liquid air.

The cylinders were charged (1) positively, (2) negatively, and the rates of leak observed in these cases when the γ rays passed (α) through the hollow cylinder, (β) through the cylinder filled with lead. The leak was small, so that it was necessary to take observations over a considerable time; from 6 to 30 hours was the time taken in the actual experiments. In this experiment there are two sources of leak, (1) the leak along the quartz support, (2) the leak due to the charges carried by the γ and secondary rays. If l_1 is the leak when the cylinders are charged positively, l_2 that when they are charged negatively, then the leak along the support is $\frac{1}{2}(l_1+l_2)$ and that due to the rays $\frac{1}{2}(l_1-l_2)$. Now let us consider how the leak should differ according as the γ rays pass through the hollow cylinder or the one filled with lead. If the charges are carried by the γ rays, then since these are absorbed by the lead cylinder and not by the hollow one, the leak due to the rays $\frac{1}{2}(l_1-l_2)$ should be numerically greater when the rays pass through the lead cylinder than through the hollow one; while if the leak is due to secondary radiation, the leak due to the rays ought to be greater when the rays passed through the hollow cylinder than through the lead one, for the rays give rise to secondary radiation when they come out of the hollow cylinder as well as when they go in, while since the rays are absorbed by the lead there is no secondary radiation at emergence from the solid cylinder. Now it was found in every case that the leak due to the rays was greater when the rays passed through the hollow than when they passed through the solid cylinder, and also that the sign of the leak corresponded to the *escape* of *negative* electricity from the cylinder: each of the conclusions is consistent with the view that the effects are due to secondary radiation, and inconsistent with the view that the γ rays carry a charge of negative electricity.

On the absence of excited radio-activity due to temporary exposure to γ rays. By J. J. THOMSON, M.A., F.R.S., Cavendish Professor of Experimental Physics.

[*Read* 30 January 1905.]

Experiments were made to see if the radiation given out by metals could be temporarily increased by exposure to the radiation from radium. The method used was to measure the saturation current inside a closed metallic vessel, then to place a sealed glass tube containing 30 mg. of radium bromide inside the vessel and leave it there for times varying from one hour to ten days; the radium was then removed and as soon as possible afterwards the saturation leak again measured; experiments were made with vessels made of lead, brass, tin, but no increase in the saturation current attributable to exposure to the radium was ever detected. The measurement of the saturation current took at least five minutes after the removal of the radium, so that a very short-lived increase might escape detection by this method. This point is dealt with in the following paper by Professor Bumstead.

On the absence of excited radio-activity due to temporary exposure to γ rays. By Professor H. A. Bumstead. (Communicated by Professor J. J. Thomson.)

[*Read* 30 January 1905.]

The following experiments form a continuation of the investigation described in the preceding paper by Professor J. J. Thomson, and were undertaken at his suggestion. Their object was to ascertain whether certain metals, after removal from exposure to the β and γ rays of radium, continue to give out secondary rays for a very short time—a time too short for the effect to be detected in Professor Thomson's experiments. For this purpose a horizontal copper disc 18 cm. in diameter was mounted so that it could be rotated about a vertical axis at various speeds by a small motor. Above the disc, at one side, was a cylindrical tin vessel, 6·5 cm. in diameter and 12·5 cm. high, provided with a central rod, insulated by sulphur and protected by an earthed guard tube in the usual manner. The vessel could be charged to various potentials by means of a battery of small storage cells, and the central rod was connected to one of C. T. R. Wilson's tilted electroscopes which served to measure the ionization current in the cylinder. In the bottom of the cylinder was a hole, 5·5 cm. in diameter, which was covered with aluminium leaf 0·00005 cm. thick; this window was 0·7 cm. above the rotating disc. Above a point on the disc whose angular distance from the centre of the testing vessel was about 120° was supported a sealed tube of thin glass containing 30 milligrams of pure radium bromide; the radium was 10·5 cm. above the disc, and was surrounded by a hollow lead cylinder whose walls were 5·5 cm. thick. The bottom of this lead cylinder was 2·5 cm. above the rotating disc; its internal diameter was 5 cm., so that the part of the disc exposed to the radium rays had a greater area than the aluminium window in the testing vessel. The total height of the lead cylinder was 19 cm.

Notwithstanding the 5·5 cm. of lead between the radium and the testing vessel the γ rays which got through the lead and the secondary rays due to them were sufficiently intense to make the current in the testing vessel (with the disc at rest) about 60 times that due to the "spontaneous" ionization of the air. In order therefore to make the test one of any delicacy it became necessary to compensate for this large leak. To accomplish this, a second testing cylinder of the same dimensions as the first, but without the window in the bottom, was mounted on a moveable arm in such a way that it was (like the first) exposed to the rays which came through the lead cylinder, but was screened from the disc. Its central rod was connected to the central rod of the first testing cylinder (and thus to the gold leaf) by means of a wire dipping into a very small cup of mercury; this cup was immediately beneath the pivot about which the moveable arm supporting the second testing cylinder rotated, and thus the second cylinder could be moved closer to or further from the lead screen surrounding the radium. When the two cylinders were charged to equal and opposite potentials (usually 40 volts), one could balance the leak in one by the opposite leak in the other, the ionization in the second cylinder being adjustable by moving it toward or away from the radium. It took a great deal of time to get exact compensation on account of the very slow motion of the gold leaf when near to a balance; but it was easy to make the resultant leak less than 1 per cent. of the uncompensated leak. The connections between the central rods of the testing cylinders and the electroscope were surrounded by metallic screens joined to earth; and the rotating disc and motor were upon a separate table, so that the apparatus was not disturbed by the rotation of the disc.

The ordinary course of the observations was to get an approximate balance with disc at rest, allowing the cylinder which was exposed to the disc to preponderate slightly. The potential acquired by the central rods and gold leaf in 10 minutes was measured by observing the deflection of the gold leaf with a micrometer microscope, and immediately thereafter producing the same deflection by means of a potentiometer. In this way variations in the sensitiveness and zero point of the electroscope were eliminated. A series of such observations was taken with the disc alternately at rest and in motion, and the results compared.

A considerable number of measurements were taken in this way, with the copper disc bare, and also when it was covered with tin-foil, lead-foil, and with blotting-paper which had been soaked in a solution of uranium nitrate and dried. The disc was rotated at two different speeds; with the slower speed, the disc made approximately 150 revolutions per minute, so that the interval between the exposure to the rays and the passage under the testing cylinder was about $\frac{1}{8}$ of a second. It was difficult to determine the higher speed, since the application of the counter appreciably diminished the velocity of rotation. It was however greater than 2200 revolutions per minute, so that the time interval in this case was less than 0·009 seconds. With none of the materials tried, and at neither speed, was there any certain indication that the ionization in the first cylinder was greater when the disc was in motion than when it was at rest. A false positive result was sometimes obtained owing to the aluminium leaf becoming slightly torn near the edge of the window, and allowing ionized air to enter from outside, a greater quantity when the air was in motion than when it was at rest. This difficulty, however, was easily overcome by care in making the window. A second difficulty was that the rotation of the disc, notwithstanding its independent support, had a tendency to disturb the balance slightly, probably by causing a slight motion of the compensating cylinder. This effect however usually persisted when the next observation was taken with the disc at rest, as will be seen from some readings which are given below as examples. The results quoted are a fair sample of the measurements which were taken under favourable conditions; there were many others which were vitiated by disturbing causes of one sort or another. The numbers give the potential in volts acquired by the gold leaf in 10 minutes.

COPPER. Slow speed.

	Disc at rest	Disc in motion
(1)	0·095	—
(2)	—	0·08
(3)	0·067	—

LEAD. Slow speed.

	Disc at rest	Disc in motion
(1)	0·22	—
(2)	—	0·28
(3)	0·26	—
(4)	—	0·32
(5)	0·28	—

TIN. Slow speed.

	Disc at rest	Disc in motion
(1)	0·08	—
(2)	—	0·21
(3)	0·23	—
(4)	—	0·33
(5)	0·40	—

BLOTTING-PAPER SOAKED IN URANIUM NITRATE. Slow speed.

	Disc at rest	Disc in motion
(1)	0·03	—
(2)	—	0·07
(3)	0·09	—

LEAD. High speed.

	Disc at rest	Disc in motion
(1)	Balance	—
(2)	—	0·20
(3)	0·22	—
(4)	—	0·35
(5)	0·35	—

TIN. High speed.

	Disc at rest	Disc in motion
(1)	0·15	—
(2)	—	0·11
(3)	0·10	—

Some comparisons will serve to give an idea of the delicacy of the test represented by these numbers. When the radium was absent and both vessels charged alike, the leak due to "spontaneous" ionization was 0·87 volts in 10 minutes. With the radium in place and the compensating cylinder disconnected, the leak in the other cylinder due to the rays which had penetrated 5·5 cm. of lead was at the rate of 42· volts in 10 minutes. With the radium in place and the two cylinders balanced, 0·250 grams of potassium uranium sulphate were spread as uniformly as possible over a circle of the same diameter as the aluminium window and placed on the disc immediately below the window. This gave a leak of 1·82 volts in 10 minutes. Thus a distribution of uranium salt, over the disc, whose surface-density was 1 milligram per square centimetre would give a leak under the experimental conditions of 0·17 volts in 10 minutes; and the results quoted above show clearly that the difference between the effects with the disc at rest and in motion is much less than this.

With the four substances tested therefore the result is negative. If they retain the power of giving out any rays capable of penetrating 0·7 cm. of air and 0·00005 cm. of aluminium, for 0·009 seconds after exposure to the β and γ rays from 30 milligrams of radium, these rays must be considerably less intense than those due to a layer of uranium salt whose surface-density is 1 milligram per square centimetre.

In conclusion, I wish to thank Professor Thomson for the privilege of working in the Cavendish Laboratory, and for many valuable suggestions during the course of these experiments.

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PROCEEDINGS

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Note on some Physical Properties of Sodium Vapour. By P. V. BEVAN, M.A., Trinity College.

[*Read* 1 May 1905.]

In the *Philosophical Magazine*, Vol. III. 1902, Professor R. W. Wood described some exceedingly interesting experiments with sodium vapour in which he was able to observe and measure the anomalous dispersion of the vapour of this metal. In his paper he describes the formation of a curious cloud of sodium vapour, and in a later paper, *Phil. Mag.* 1904, Vol. VIII. p. 296, he states "the vapour of sodium appears to possess a cohesion almost as great as that of a liquid. Even in a vacuum tube it appears to have a free surface." This behaviour of a vapour was so extraordinary that it seemed worth while to see under what conditions this phenomenon took place, and the experiments to be described in this note were made with this purpose.

It is a very simple matter to observe this cloud. All that is necessary is to heat a piece of metallic sodium in a tube or bulb that is partially exhausted, or that contains hydrogen, and observe through the tube an ordinary bunsen flame which is coloured yellow by sodium. The sodium vapour given off absorbs the sodium light and the cloud is easily visible.

The cloud possesses in general a very sharply defined boundary across which diffusion of sodium vapour does not appear to take place. The first point investigated was to decide whether this cloud had its definite boundary *in vacuo*. It was found that a considerable pressure of gas was necessary for the cloud to possess this definite boundary. If the sodium be heated in a high vacuum it distils practically instantaneously like any other vapour. The method adopted was to attach a bulb of hardest Jena glass to a mercury pump, having first placed some metallic sodium in the stem of the bulb; and then after exhaustion to melt the sodium and shake a small globule into the bulb. After this introduction of sodium into the bulb it was necessary to again exhaust with the pump, as a good deal of hydrogen was given off from the sodium and also from the action of sodium on sodium hydroxide which is always present with the original sodium. If a good vacuum was produced and then the sodium in the bulb heated no definite cloud could be observed, but the sodium distilled at once and settled on the cooler parts of the bulb. The distillation is in fact so easy that the sodium can in a few minutes be distilled from the stem into the bulb almost completely. It thus appears that *in vacuo* the sodium vapour behaves as regards distillation like any other vapour.

It was found also that in an atmosphere of hydrogen the definite cloud could not be observed if the hydrogen were perfectly dry and no sodium hydroxide were present. The absence of oxygen thus seems to destroy this cloud-forming property. At present I have not determined whether pure dry oxygen facilitates the formation of the cloud, but this is a point which I hope to decide later. If however into the bulb which has been dried thoroughly and exhausted, sodium be distilled, and then hydrogen dried with phosphorus pentoxide be admitted, no definite surface to the sodium vapour can be observed on heating. Near the heated sodium there can be seen with sodium light a cloud of the vapour, but this cloud gradually decreases in density and fades into the region of no absorption without any definite boundary. In fact the sodium vapour behaves in this case just as iodine vapour does when a small piece of iodine is heated gently.

It seems that when an ordinary piece of sodium is heated *in vacuo* a large quantity of hydrogen is given off, some of it occluded originally in the sodium and some of it formed by the action of sodium on sodium hydroxide, and this gives rise to an atmosphere sufficiently dense to enable the sodium vapour cloud to form.

These experiments are very easily performed in a Jena bulb, and such bulbs can be used for a considerable time if they are well heated before the introduction of the sodium. Without this preliminary heating the sodium acts on the surface of the bulb, owing apparently to a film of moisture on the surface, and causes the glass to be coated with a white non-transparent film. At a high temperature the glass appears to dissolve the sodium, forming a brown transparent glass.

Another experiment was made showing that sodium vaporizes in observable quantities when just above its melting-point. In the *Berichte der Deutschen Chemischen Gesellschaft*, No. 1, 1905, Kraft and Bergfeld showed that sodium distilled at 140° C. Apparently these observers did not obtain distillation of sodium vapour at a lower temperature. It seemed worth while to try whether at the temperature of boiling water it would be possible to notice any distillation. A bulb was taken with a little sodium deposited on its upper part, the lower part being quite clear. The lower part was placed in cold water and a jet of steam was played on the upper part. After some hours a very distinct film of sodium was observed on the lower part of the bulb, showing that, at a temperature 2° above the melting-point of sodium, vapour was formed in observable quantities. No distillation could be observed from sodium at the ordinary air temperature when a clear part of the bulb was placed in liquid air. Sodium apparently therefore does not sublime at ordinary temperatures.

A Null Method of Measuring Small Ionisations. By NORMAN R. CAMPBELL, B.A., Trinity College.

[*Read* 1 May 1905.]

For the measurement of spontaneous ionisations a method has been tried and found successful, which consists in balancing the current through the ionised gas against that through a constant volume of air, of which the pressure can be varied, ionised by a constant amount of uranium.

The vessel (*A*) in which the unknown ionisation is to be measured is provided with an electrode surrounded by a guard ring of the usual type, and connected to a Wilson gold-leaf electroscope. To the same electroscope is connected the electrode of the "compensator" (*B*). This is an air-tight cylinder of brass with a central wire electrode insulated from its guard ring by amber; in the cylinder is placed a packet containing a suitable amount of uranium nitrate wrapped in thin aluminium foil. A side tube leads to a three-way tap, the other branches of which lead to a water-pump and inlet tube for regulating the pressure.

The quantity of uranium is first adjusted so that when the exterior of the pot is raised to a potential equal and opposite to that of the walls of the vessel (*A*) and great enough to send a saturation current through the air, the current is slightly greater than the largest current which has to be measured through the vessel (*A*).

A is then disconnected from the electroscope and the compensator standardised by observing the rate of rise of potential of the electrode for different pressures in the vessel. The capacity of *B* is measured by repeating one of the same observations with a condenser of known capacity connected in parallel with the electrode. Thus the relation between the pressure in the vessel and the current flowing through it is obtained.

The unknown ionisation is then measured by observing the pressure in the compensator when the sum of the currents flowing through *A* and *B* is zero as indicated by the absence of movement in the leaf of the electroscope.

The arrangement has the following advantages over the ordinary methods:—

1. After standardisation little care is needed that the insulation is good, for the potential of the electrodes never rises appreciably above zero.

2. Only one capacity, that of the compensator, need be measured in order that all the currents may be expressed in absolute units.

3. Advantage is taken of the great sensitiveness of the Wilson electroscope near its zero. Experience shows that observations may be made more rapidly, as well as more accurately, than by the usual method.

4. All the benefits of null methods in detecting small changes are obtained.

On the Reflection of Sound at a Paraboloid. Part II. By The Rev. H. J. Sharpe, late Fellow of St John's College, Cambridge.

[*Received* April 1905.]

38. The present Paper is a continuation of one with the same title that appeared in 1899 in Vol. x. Part III. of the *Proceedings* of the Society. In it the Articles, Figures, and Equations will be numbered on continuously from the 1st Paper. But before going any further perhaps it will be well to correct some Errata in the previous Paper. In Art. 3, No. 19 should be No. 119. In Art. 11 after "$u=0$" add "v being finite." In Art. 12 for the sentence "Also for a given reflector, &c." read "Also for a given reflector, and a given distance from O, high sounds are more magnified than low ones." In Art. 13 for $A_0 = B_0 = \pi^{\frac{1}{2}}$ read $A_0 = B_0 = \pi^{-\frac{1}{2}}$. In Art. 17, after the words "will be proportional to," instead of what follows, read

$$1 - \frac{p^2u^2}{2^2} + \frac{p^4u^4}{2^2 . 4^2} - \&c.$$

The sentence after this can be readily corrected.

In Art. 19 after (40) in the next line *dele* "or negative." In the next line after the words "large values of A are" insert "positive and." Towards the end of Art. after the words "when $u=0$" insert "and v is finite." In the last two lines of Art. 24 some correction is required. In Art. 21, Equations (47) to (48) require correction. In Art. 20 in 3rd line for "large value" read "large values." Art. 30 requires some correction. In Art. 36 and so in the results (92) and (93) I am doubtful about the results obtained, but I hope to exhibit a better way of evaluating the definite integrals involved. In Art. 37 a correction must be made, for which my thanks are due to Prof. Larmor.

Dele the sentence beginning "Here if v is increased, &c." The true explanation is that as the progressive wave gets further and further along the axis, its velocity which was at first rather $> a$ gradually slows down till it ultimately $= a$.

39. I propose in this Paper to consider more fully the case of $A = 0$, but before doing so I should like to consider one or two simple problems, which are analogous to the whole Problem before us, whatever be the value of A, and which, it seems to me, throw some light upon it. Suppose (Fig. 8) xx' an infinite cylindrical tube filled with air, stopped at O and open towards x' and x.

Suppose a source of sound at S spread equally over the section Ss of the tube. Let $OS = OI = c$ and let $OP = x$, where P may be on the right or left of Ss.

To find the motion.

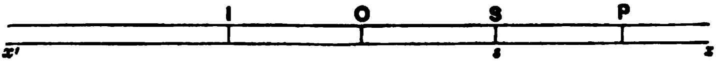

FIG. 8.

Let $\phi_1, \phi_2, \phi_3, \phi_4$ be velocity-potentials, and let

$$\phi_1 \equiv \cos m(at - x + c),$$

$$\phi_2 \equiv \cos m(at + x - c),$$

$$\phi_3 \equiv \cos m(at - x - c),$$

$$\phi_4 \equiv \cos m(at + x + c).$$

In the part OS ϕ_2 and ϕ_3 exist and the vibration is stationary.

In the part Sx ϕ_1 and ϕ_3 exist and the vibration is progressive. The velocity vanishes at O and is discontinuous at S.

Next suppose (Fig. 9) SOx a conical tube with vertex at O and

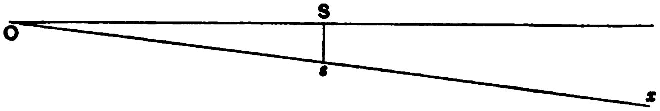

FIG. 9.

open at x. Suppose a source of sound at S spread equally over the spherical section Ss of the tube. Let $OS = c$.

Let ϕ_1, ϕ_2, ϕ_3 be velocity-potentials, and let

$$\phi_1 \equiv \frac{1}{r} \cos m(r - at - c),$$

$$\phi_2 \equiv \frac{1}{r} \cos m(r + at - c),$$

$$\phi_3 \equiv -\frac{1}{r} \cos m(r - at + c).$$

Then in OSs ϕ_2 and ϕ_3 exist and the vibration is stationary;
in Ssx ϕ_1 and ϕ_3 exist and the vibration is progressive.

The velocity vanishes at O and is discontinuous at S.

From both cases we see that, whether the source be near O or far off, there will always be near O a region of stationary vibration.

Now studying equations (46), (50) and (51), we see from (51) that there are *two* solutions of (50), which are approximately

$$V_1 = \frac{\epsilon^{2(Av)^{\frac{1}{2}}}}{2\pi^{\frac{1}{2}}(Av)^{\frac{1}{4}}}, \quad V_2 = \frac{\epsilon^{-2(Av)^{\frac{1}{2}}}}{2\pi^{\frac{1}{2}}(Av)^{\frac{1}{4}}}.$$

The latter corresponds to what Stokes calls (in his Paper on the Discontinuity of Arbitrary Constants, &c.) "the inferior function." The result is, that, whether there be a source in OL or not (see Fig. 2), as long as $v < A$, in the region so defined, the vibration will be *nearly stationary*. This is, I think, what we would expect, and we may compare this with the simple cases above considered.

Again in Fig. 9 suppose there is no source at S. We can have for possible velocity-potentials,

$$F_1 = (\sin mr \sin mat)/r, \quad F_2 = (\cos mr \cos mat)/r.$$

Either represents stationary vibration, and $F_2 + F_1$ gives a progressive wave from source at O.

F_1, F_2 here correspond exactly to V_1, V_2, the two solutions of

$$v\frac{d^2V}{dv^2} + \frac{dV}{dv} + (v - A)\,V = 0,$$

and a linear combination of these two gives the outwards going wave for large values of v.

40. We proceed now to consider in more detail the case of $A = 0$, which was considered in Arts. 10—13 of my former Paper. I was not aware when I wrote that Paper of all the details that are known with regard to the zeros and turning points of the two functions $J_0(x)$ and $J_1(x)$. We proceed to supply the deficiency. It may be observed that (though perhaps finite and large values of A give us the most interesting cases) the case of $A = 0$ has this advantage, that it can be completely worked out to any required degree of accuracy.

41. In what follows, and till further notice, we shall use u and v in the sense of the *first* part of Art. 6, that is (Fig. 2)

$$u = 2OU, \quad v = 2OV.$$

And first we will suppose the paraboloid $u = l$ a sound-*sending* reflector, LO (Fig. 2) a line of sources, giving rise to a progressive wave. By Art. 17 corrected as above Art. 38, or by Art. 34 the law of the strength of source at any point U in LO (Fig. 2) is U_1 where

$$U_1 = 1 - \frac{p^2u^2}{2^2} + \frac{p^4u^4}{2^2 \,.\, 4^2} - \&\text{c.} \equiv J_0(pu).$$

It is interesting to know what value of pu gives the numerically greatest value of $J_0(pu)$. This is at once got from Table I. in Gray and Mathews' *Treatise on Bessel's Functions.* It will be observed that for brevity I have put down only two places of decimals. The turning points of $J_0(pu)$ are given by

$pu = 0$	3·83	7·02	10·17	13·32	&c.,
and then $J_0(pu) = 1$	−·40	+·30	−·24	+·21	&c.

We see at once from this that $u = 0$ gives the point of *numerically* greatest strength, which is the *geometrical focus of the reflector.*

We will next suppose $u = l$ to be a sound-*receiving* reflector, and so the vibration to be stationary. We will endeavour to find the "focus of reflection" (Art. 12) in the axis LOx (Fig. 2).

By Art. 11 the velocity at a point U in LO (Fig. 2) is (putting $v = 0$ and omitting the time-factor)

$$= \frac{2dU_1}{du} = 2p\frac{dJ_0(pu)}{d(pu)} = -2pJ_1(pu).$$

Now the turning points of $J_1(pu)$ are given by

$pu = 1{\cdot}84$	5·33	8·54	11·71	&c.	......(96),
and then $J_1(pu) = {\cdot}58$	−·34	·27	−·23	&c.	

so that $pu = 1{\cdot}84$ gives the point in LO (Fig. 2) of numerically greatest velocity. But since for all points in LO we must have $pu < pl$, we must also have

$$1{\cdot}84 < pl,$$

if there is to be a "focus of reflection" in LO. But p is given by the condition (22),

$$\frac{dU_1}{d(pu)} = 0 = J_1(pu), \text{ when } u = l.$$

But the zeros of the function $J_1(pl)$ are given by

$$pl = 0,\quad 3{\cdot}83,\quad 7{\cdot}01,\quad 10{\cdot}17,\quad 13{\cdot}32, \text{ \&c. } \ldots\ldots(97).$$

Passing over 0, the deepest sound is given by $pl = 3{\cdot}83$, so that even if the sound be as low as to be given by $pl = 3{\cdot}83$ there will be a focus of reflection in LO.

Again by Art. 11 the velocity at a point V in Ox (Fig. 2) is (putting $u = 0$ and omitting the time-factor)

$$= 2\frac{dV_1}{dv} = 2p\frac{dJ_0(pv)}{d(pv)} = -2pJ_1(pv),$$

so that $pv = 1{\cdot}84$ gives another "focus of reflection" in Ox.

The two foci of reflection F_1 and F_2 are indicated in Fig. 10, where $OF_1 = OF_2$ and $OF_1 = \frac{1 \cdot 84}{p}$ and velocity at $F_1 = 2p \times \cdot 58 =$ velocity at F_2, but in an opposite direction.

We see therefore, that (for a given reflector) as p increases (that is for higher and higher notes) the two foci F_1 and F_2 *draw nearer together and the velocity at each one increases.*

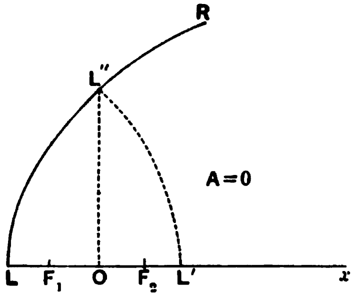

FIG. 10.

It is readily seen from equation (15), putting $A = 0$, that the velocity at O vanishes. Thus when p becomes large, the velocity in the neighbourhood of O tends to become discontinuous, but as p has finite limits (Art. 5) actual discontinuity cannot be set up. If we make $OL' = OL$ and draw the parabola $L''L'$, we can show from the formula given for the velocities in Art. 21 that the motion in the space $L''LL'$ is symmetrical with regard to the lines $L''O$ and LL'. (Of course only half the figure is drawn.)

42. To complete the case of $A = 0$ we have now to evaluate the definite integrals in (33). This was done very imperfectly in Art. 15. The following seems a better way. It is true that the evaluation can be performed in a shorter way, but it will be found convenient for *future* investigations, first to suppose A finite, and then as a particular case to put it $= 0$. Let I_1, I_2 be the two definite integral solutions of the equation

$$x\frac{d^2X}{dx^2} + \frac{dX}{dx} + (x + A)\,X = 0 \quad \text{..............(98)},$$

and let $I_2 \equiv \int_0^{\frac{1}{2}\pi} \cos\left(x\cos\theta + A\log\cot\frac{\theta}{2}\right).\log\sin^2\theta\, d\theta,$

and let X_1, X_2 be the solutions in *series*, the latter involving $\log x$. Then by (18)

$$X_1 = \frac{1}{\pi}\left(\epsilon^{\frac{1}{2}\pi A} + \epsilon^{-\frac{1}{2}\pi A}\right) I_1 \text{ and } X_2 = \frac{1}{\pi}\left(\epsilon^{\frac{1}{2}\pi A} + \epsilon^{-\frac{1}{2}\pi A}\right)\left(I_1 \log x + I_2\right) \quad \text{.........(99)}.$$

We will now suppose x large and A/x a small fraction. In I_1 put $\cos\theta = \dfrac{x-t}{x}$, and we get

$$I_1 = \int_0^x \cos\left\{x - t - \frac{A}{2}\log\frac{t}{2x-t}\right\} \cdot \frac{dt}{t^{\frac{1}{2}}(2x-t)^{\frac{1}{2}}}.$$

Applying to this a method suggested by Art. 405 of Todhunter's *Laplace's Functions*, we have

$$A\log\frac{t}{2x-t} = A\log t - A\log 2x - A\log\left(1 - \frac{t}{2x}\right).$$

Here for $A\log\left(1-\dfrac{t}{2x}\right)$ we may put zero, when t is not large, and when t is large, the corresponding elements of the integral are of no account, because then $t^{-\frac{1}{2}}$ is small.

For the same kind of reason, in the multiplier, for $(2x-t)^{\frac{1}{2}}$ we may put $(2x)^{\frac{1}{2}}$.

Then, when x and A have the kind of values supposed,

$$I_1 = \int_0^\infty \cos\left\{x + \frac{A}{2}\log 2x - t - \tfrac{1}{2}A\log t\right\} \cdot \frac{dt}{(2tx)^{\frac{1}{2}}}.$$

In a similar way we get

$$I_2 = \int_0^x \cos\left\{x - t - \frac{A}{2}\log\frac{t}{2x-t}\right\} \cdot \log\frac{2tx-t^2}{x^2} \cdot \frac{dt}{(2tx-t^2)^{\frac{1}{2}}},$$

and when x is large and A/x small, this becomes

$$I_2 = \int_0^\infty \cos\left\{x + \frac{A}{2}\log 2x - t - \frac{A}{2}\log t\right\} \cdot (\log 2t - \log x) \cdot \frac{dt}{(2tx)^{\frac{1}{2}}}$$

$$\therefore\ I_2 = \int_0^\infty \cos\left\{x + \frac{A}{2}\log 2x - t - \frac{A}{2}\log t\right\} \cdot \log 2t \cdot \frac{dt}{(2tx)^{\frac{1}{2}}}.$$

Put $t = 2\psi^2$ and we get

$$I_1 = \int_0^\infty \cos\left\{x + \frac{A}{2}\log x - 2\psi^2 - A\log\psi\right\} \cdot \frac{2d\psi}{x^{\frac{1}{2}}},\ \text{and}$$

$$I_2 = \int_0^\infty \cos\left\{x + \frac{A}{2}\log x - 2\psi^2 - A\log\psi\right\} \cdot (\log 4 + 2\log\psi) \cdot \frac{2d\psi}{x^{\frac{1}{2}}}.$$

Now put $\dfrac{I_5}{I_6} \equiv \displaystyle\int_0^\infty \begin{matrix}\cos\\ \sin\end{matrix}(2\psi^2 + A\log\psi) \cdot d\psi$, and we get

$$x^{\frac{1}{2}} I_1 = 2\cos\left(x + \frac{A}{2}\log x\right) \cdot I_5 + 2\sin\left(x + \frac{A}{2}\log x\right) \cdot I_6 \quad \ldots(100),$$

$$x^{\frac{1}{2}} I_2 = 2\log 4 \cdot \left[\cos\left(x + \frac{A}{2}\log x\right) \cdot I_5 + \sin\left(x + \frac{A}{2}\log x\right) \cdot I_6\right]$$

$$+ 4\cos\left(x + \frac{A}{2}\log x\right) \cdot \frac{dI_6}{dA} - 4\sin\left(x + \frac{A}{2}\log x\right) \cdot \frac{dI_5}{dA} \quad \ldots(101)$$

We have now to evaluate I_5 and I_6. We shall show that they can be expressed as Gamma functions of a complex, but before doing this we will first establish a relation between them. It is easy to show from (98) that

$$x\left(X_1\frac{dX_2}{dx} - X_2\frac{dX_1}{dx}\right) = C \;\ldots\ldots\ldots\ldots\ldots(102),$$

where C is some quantity independent of x. We can determine its value from (15) and (16) by putting x, X_1, X_2 for u, U_1, U_2, then putting for dX_2/dx and dX_1/dx their values, and making x vanish. We then get $C = 1$. Of course (102) may be regarded as a First Integral of (98). If x is large and A/x small, we can show (Art. 23) that the leading terms in X_1 and X_2 have the following forms

$$\left.\begin{aligned} x^{\frac{1}{2}}X_1 &= C_1\cos\left(x + \frac{A}{2}\log x\right) + C_2\sin\left(x + \frac{A}{2}\log x\right) \\ x^{\frac{1}{2}}X_2 &= C_3\cos\left(x + \frac{A}{2}\log x\right) + C_4\sin\left(x + \frac{A}{2}\log x\right) \end{aligned}\right\} \ldots(103),$$

where C_1, C_2, C_3, C_4 are certain functions of A. From these last equations finding the values of dX_1/dx and dX_2/dx (*retaining only the largest terms*) and putting them in (102), it will be found that we shall get

$$C_1C_4 - C_2C_3 = 1 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(104).$$

Comparing together the equations (99), (100), (101), and (103) we shall get

$$\left.\begin{aligned} &C_1 = \frac{2}{\pi}\left(\epsilon^{\frac{1}{2}\pi A} + \epsilon^{-\frac{1}{2}\pi A}\right) \times I_5 \text{ and } C_2 = \frac{2}{\pi}\left(\epsilon^{\frac{1}{2}\pi A} + \epsilon^{-\frac{1}{2}\pi A}\right) \times I_6 \\ &\qquad C_3 = \frac{1}{\pi}\left(\epsilon^{\frac{1}{2}\pi A} + \epsilon^{-\frac{1}{2}\pi A}\right)\left(2I_5\log 4 + 4\frac{dI_6}{dA}\right) \\ \text{and}\quad &\qquad C_4 = \frac{1}{\pi}\left(\epsilon^{\frac{1}{2}\pi A} + \epsilon^{-\frac{1}{2}\pi A}\right)\left(2I_6\log 4 - 4\frac{dI_5}{dA}\right) \end{aligned}\right\} \ldots(105).$$

Put these values of C_1, C_2, C_3, C_4 in (103) and we shall get

$$\frac{8}{\pi^2}\left(I_5\frac{dI_5}{dA} + I_6\frac{dI_6}{dA}\right) = \frac{-\epsilon^{\pi A}}{(\epsilon^{\pi A}+1)^2}.$$

Integrating we get

$$(I_5)^2 + (I_6)^2 = \frac{\pi}{4} \times \frac{1}{\epsilon^{\pi A}+1} \;\ldots\ldots\ldots\ldots\ldots(106).$$

It can readily be shown that no constant need be added.

It is important to notice that in equations (100), (101), (104), (106) A *may have any value*. For the formula (106) I am indebted to the late Sir G. G. Stokes. It will presently be obtained in another manner. It will be noticed that (106) is here obtained without the use of complex quantities.

From (103) and (104) it follows that when x is large and A/x small,

$$\left.\begin{aligned} C_4 X_1 - C_3 X_2 &= \frac{\cos\left(x + \frac{A}{2}\log x\right)}{x^{\frac{1}{2}}} \\ C_1 X_2 - C_2 X_1 &= \frac{\sin\left(x + \frac{A}{2}\log x\right)}{x^{\frac{1}{2}}} \end{aligned}\right\} \quad \ldots\ldots\ldots(107).$$

If smaller terms are required than those given in (103) they may be got thus. In (98) put $x^{\frac{1}{2}} X \equiv W$. Then

$$x\frac{d^2 W}{dx^2} + \left(x + A + \frac{1}{4x}\right) W = 0.$$

If in this we put

$$\begin{aligned} x^{\frac{1}{2}} X = W = \left(A_0 + \frac{A_1}{x} + \frac{A_2}{x^2} + \&c.\right)\cos\left(x + \frac{A}{2}\log x\right) \\ + \left(B_0 + \frac{B_1}{x} + \frac{B_2}{x^2} + \&c.\right)\sin\left(x + \frac{A}{2}\log x\right) \quad \ldots(108), \end{aligned}$$

we shall find that $A_1 A_2$, &c., $B_1 B_2$, &c., can all be obtained in terms of A_0 and B_0 by the following equations:

$$\left\{n(n-1) - \frac{A^2}{4} + \frac{1}{4}\right\} A_{n-1} - 2nB_n - \frac{A}{2}(2n-1) B_{n-1} = 0,$$

$$\left\{n(n-1) - \frac{A^2}{4} + \frac{1}{4}\right\} A_{n-1} + 2nA_n + \frac{A}{2}(2n-1) A_{n-1} = 0.$$

If in the above we put A_0, $B_0 = C_1$, C_2 respectively we get X_1 and if we put A_0, $B_0 = C_3$, C_4 we get X_2. The above series are semiconvergent if x is large and A/x small.

43. We will now show that I_5 and I_6 depend upon

$$\Gamma\left(\frac{1}{2} + \frac{iA}{2}\right).$$

The following is, if I remember rightly, practically Stokes's investigation.

Put $$I \equiv \int_0^\infty e^{i(2x^2 + A\log x)} \times dx.$$

In it put $x = y^{\frac{1}{2}}$. Then

$$I = \frac{1}{2}\int_0^\infty e^{2iy + \frac{Ai}{2}\log y - \frac{1}{2}\log y} \times dy.$$

Put $y = is$. Then

$$I = \frac{i}{2} e^{-\frac{\pi A}{4} - \frac{\pi i}{4}} \times \int_0^\infty e^{-2s + \left(\frac{Ai}{2} - \frac{1}{2}\right) \log s} \times ds.$$

Now we know that if k and n are real

$$\frac{\Gamma(n)}{k^n} = \int_0^\infty e^{-ks + (n-1) \log s} \times dx.$$

But in this formula n may be imaginary.

Comparing the last two equations we will suppose $k = 2$ and $n = (1 + Ai)/2$ and putting

$$\Gamma\left(\frac{1}{2} + \frac{Ai}{2}\right) \equiv 2p + 2iq \quad \text{..............(109)},$$

and putting c for $\cos\left(\frac{A}{2} \log 2\right)$ and s for $\sin\left(\frac{A}{2} \log 2\right)$ we shall get after some reduction

$$I = \tfrac{1}{2} e^{-\frac{\pi A}{4}} \{-qc + ps + pc + qs + i(qc - ps + pc + qs)\} \quad \text{...(110)},$$

from which $\left(\text{putting } B \equiv A \log 2^{\frac{1}{2}} - \frac{\pi}{4}\right)$ we get

$$\left.\begin{aligned} I_5 &\equiv \int_0^\infty \cos(2x^2 + A \log x)\, dx = 2^{-\frac{1}{2}} e^{-\frac{\pi A}{4}} (q \sin B + p \cos B) \\ I_6 &\equiv \int_0^\infty \sin(2x^2 + A \log x)\, dx = 2^{-\frac{1}{2}} e^{-\frac{\pi A}{4}} (q \cos B - p \sin B) \end{aligned}\right\} \text{(111)}.$$

The above is Stokes's result. From them we get

$$(I_5)^2 + (I_6)^2 = \tfrac{1}{2} e^{-\frac{\pi A}{2}} (p^2 + q^2).$$

But it is known that if m be real and < 1,

$$\Gamma(m)\, \Gamma(1 - m) = \frac{\pi}{\sin m\pi}.$$

But here m may be a mixed imaginary, so put $m = \frac{1}{2} + \frac{Ai}{2}$, and we get

$$\Gamma\left(\frac{1}{2} + \frac{iA}{2}\right) \Gamma\left(\frac{1}{2} - \frac{iA}{2}\right) = \frac{2\pi}{e^{\frac{\pi A}{2}} + e^{-\frac{\pi A}{2}}} = 4(p^2 + q^2) \text{...(112)},$$

so we arrive again at the formula (106). It is interesting to notice the two entirely different ways by which (106) is established. We proceed to find the value of $\Gamma\left(\frac{1}{2} + \frac{iA}{2}\right)$ when $A < 1$.

It is proved in Art. 264 of Todhunter's *Integral Calculus* that if n be real and r an integer,

$$\Gamma(n) = \text{limit of } \frac{1 \,.\, 2 \,.\, 3 \ldots r}{n(n+1)\ldots(n+r-1)} \times r^{n-1},$$

when r is infinite. It can be shown that this formula is true even when n is complex. First then put $n = \frac{1}{2} + \frac{iA}{2}$, next put $n = \frac{1}{2} - \frac{iA}{2}$ and divide the 1st result by the 2nd result. We thus get

$$\Gamma\left(\frac{1}{2} + \frac{iA}{2}\right) \Big/ \Gamma\left(\frac{1}{2} - \frac{iA}{2}\right)$$

$$= \text{limit of } \frac{\left(\frac{1}{2} - \frac{iA}{2}\right)\left(\frac{3}{2} - \frac{iA}{2}\right)\left(\frac{5}{2} - \frac{iA}{2}\right)\ldots\left(\frac{2r-1}{2} - \frac{iA}{2}\right)}{\left(\frac{1}{2} + \frac{iA}{2}\right)\left(\frac{3}{2} + \frac{iA}{2}\right)\left(\frac{5}{2} + \frac{iA}{2}\right)\ldots\left(\frac{2r-1}{2} + \frac{iA}{2}\right)} \times r^{iA},$$

when r is infinite. For shortness put $iA \equiv s$. Then

$$\Gamma\left(\frac{1}{2} + \frac{iA}{2}\right) \Big/ \Gamma\left(\frac{1}{2} - \frac{iA}{2}\right)$$

$$= \text{limit of } \frac{(1-s)\left(1 - \frac{s}{3}\right)\left(1 - \frac{s}{5}\right)\ldots\left(1 - \frac{s}{2r-1}\right)}{(1+s)\left(1 + \frac{s}{3}\right)\left(1 + \frac{s}{5}\right)\ldots\left(1 + \frac{s}{2r-1}\right)} \times r^{s} \ldots (113),$$

when r is infinite.

Now, to make these infinite products convergent, use the device of Weierstrass given in Art. 287 of Hobson's *Trigonometry*. Noting that

$$\left(1 + \frac{s}{n}\right) e^{-\frac{s}{n}} = e^{-\frac{s^2}{2n^2} + \frac{s^3}{3n^3} - \frac{s^4}{4n^4} + \&c.} \ldots\ldots\ldots\ldots (114)$$

give to n the successive values 1, 3, 5, &c., $(2r-1)$ and multiply the numerator and denominator of (113) by

$$e^{-s\left(1 + \frac{1}{3} + \frac{1}{5} + \ldots + \frac{1}{2r-1}\right)} \equiv e^{-sr_1} \text{ suppose.}$$

Further put $$1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots + \frac{1}{(2r-1)^2} \equiv r_2,$$

$$1 + \frac{1}{3^3} + \frac{1}{5^3} + \ldots + \frac{1}{(2r-1)^3} \equiv r_3,$$

$$\&\text{c.}, \qquad \equiv \&\text{c.}$$

We thus get

$$\Gamma\left(\frac{1}{2}+\frac{iA}{2}\right) \Big/ \Gamma\left(\frac{1}{2}-\frac{iA}{2}\right)$$

$$=\frac{(1-z)\left(1-\frac{z}{3}\right)\left(1-\frac{z}{5}\right)\dots\left(1-\frac{z}{2r-1}\right)}{\epsilon^{-\frac{z^2}{2}r_2+\frac{z^3}{3}r_3-\frac{z^4}{4}r_4+\&c.}}\times r^z\times\epsilon^{-zr_1}\ \dots(115).$$

Next in (114) giving to n the successive values

$$-1,\ -3,\ -5,\ \&c.,\ -(2r-1)$$

and multiplying the numerator and denominator of (115) by

$$\epsilon^{z\left(1+\frac{1}{3}+\frac{1}{5}+\dots+\frac{1}{2r-1}\right)}\equiv\epsilon^{zr_1},$$

we get

$$\frac{\Gamma\left(\frac{1}{2}+\frac{iA}{2}\right)}{\Gamma\left(\frac{1}{2}-\frac{iA}{2}\right)}=\frac{\epsilon^{-\frac{z^2}{2}r_2-\frac{z^3}{3}r_3-\frac{z^4}{4}r_4-\&c.}}{\epsilon^{-\frac{z^2}{2}r_2+\frac{z^3}{3}r_3-\frac{z^4}{4}r_4+\&c.}}\times r^z\times\epsilon^{-2zr_1}\ (r=\infty).$$

Or

$$\frac{\Gamma\left(\frac{1}{2}+\frac{iA}{2}\right)}{\Gamma\left(\frac{1}{2}-\frac{iA}{2}\right)}=\epsilon^{-2\left(\frac{z^3}{3}r_3+\frac{z^5}{5}r_5+\&c.\text{ to }\infty\right)}\times\epsilon^{z\log r}\times\epsilon^{-2zr_1}.$$

From (109) the above

$$=\frac{p+iq}{p-iq}=\frac{(p+iq)^2}{p^2+q^2}.$$

Therefore extracting the square root we get

$$\frac{p+iq}{(p^2+q^2)^{\frac{1}{2}}}=\pm\,\epsilon^{-\left(zr_1+\frac{z^3}{3}r_3+\frac{z^5}{5}r_5+\&c.\text{ to }\infty\right)}\times\epsilon^{\frac{1}{2}z\log r}\ (r=\infty)$$

$$=\pm\,\epsilon^{-i\left(Ar_1-\frac{A^3}{3}r_3+\frac{A^5}{5}r_5-\&c.\text{ to }\infty\right)+\frac{1}{2}iA\log r}\ (r=\infty).$$

So that if we put

$$Ar_1-\frac{A^3}{3}r_3+\frac{A^5}{5}-\&c.\text{ to }\infty-\tfrac{1}{2}A\log r\equiv E\ \dots(116),$$

we get

$$\frac{p+iq}{(p^2+q^2)^{\frac{1}{2}}}=\pm(\cos E-i\sin E).$$

If $A=0$ it is known that $q=0$ and that $p=\frac{\pi^{\frac{1}{2}}}{2}$, so that in the above equations the upper sign is to be taken, and we get

$$p=(p^2+q^2)^{\frac{1}{2}}\cos E\ \text{ and }\ q=-(p^2+q^2)^{\frac{1}{2}}\sin E.$$

Putting these values in (111) we get

$$\left.\begin{aligned} I_5 &\equiv \int_0^\infty \cos(2x^2 + A\log x)\,dx = 2^{-\frac{1}{2}}\,\epsilon^{-\frac{\pi A}{4}}(p^2+q^2)^{\frac{1}{2}}\cos(E+B) \\ I_6 &\equiv \int_0^\infty \sin(2x^2 + A\log x)\,dx = -2^{-\frac{1}{2}}\,\epsilon^{-\frac{\pi A}{4}}(p^2+q^2)^{\frac{1}{2}}\sin(E+B) \end{aligned}\right\} \quad \text{.........(117).}$$

By means of (112) we thus get a 3rd corroboration of Stokes's formula (106). It will be observed that the formulae (117) for I_5 and I_6 are of value only when A does not exceed 1, for if A be > 1 we see from (116) that the series for E is divergent.

We will next examine what results we get when in the above investigations $-A$ is put for A, for the Physical Problem requires both of these cases to be considered. Suppose I_5, I_6 then become I_7 and I_8 respectively. It will be found that instead of (111) we shall get

$$I_7 \equiv \int_0^\infty \cos(2x^2 - A\log x)\,dx = 2^{-\frac{1}{2}}\,\epsilon^{\frac{\pi A}{4}}(-q\sin B_1 + p\cos B_1),$$

$$I_8 \equiv \int_0^\infty \sin(2x^2 - A\log x)\,dx = 2^{-\frac{1}{2}}\,\epsilon^{\frac{\pi A}{4}}(-q\cos B_1 - p\sin B_1),$$

where
$$B_1 \equiv -A\log 2^{\frac{1}{2}} - \frac{\pi}{4},$$

and p and q have the same values as before.

Instead of (117) we shall get

$$\left.\begin{aligned} I_7 &\equiv \int_0^\infty \cos(2x^2 - A\log x)\,dx = -2^{-\frac{1}{2}}\,\epsilon^{\frac{\pi A}{4}}(p^2+q^2)^{\frac{1}{2}}\sin(E+B) \\ I_8 &\equiv \int_0^\infty \sin(2x^2 - A\log x)\,dx = 2^{-\frac{1}{2}}\,\epsilon^{\frac{\pi A}{4}}(p^2+q^2)^{\frac{1}{2}}\cos(E+B) \end{aligned}\right\} \quad \text{.........(118),}$$

where B, E and $(p^2+q^2)^{\frac{1}{2}}$ are known from (110), (116), and (112).

It can be easily shown that the limit of $(r_1 - \frac{1}{2}\log r)$ when r is ∞ is $\frac{1}{2}\gamma + \log_e 2$ (γ being Euler's constant and $= \cdot 5772$, &c.), so that instead of (116) we may put

$$E \equiv A\left(\tfrac{1}{2}\gamma + \log 2\right) - \frac{A^3}{3}r_3 + \frac{A^5}{5}r_5 - \frac{A^7}{7}r_7 + \&\text{c.} \quad \text{......(119).}$$

It must be remembered in using the formulae (117) and (118) that in them $A \not> 1$.

44. We are now in a position to complete the case of $A = 0$. We see from (103) and (105) that in order to find what X_1 and X_2

become in this case, we have to find the values of I_5, I_6 and dI_5/dA, dI_6/dA when $A = 0$. To do this, it is best perhaps to expand the right-hand sides of (117) in powers of A for a term or two, to differentiate with regard to A, and then put $A = 0$. We shall get $I_5 = I_6 = \pi^{\frac{1}{2}}/4$,

$$\frac{dI_5}{dA} = \frac{\pi^{\frac{1}{2}}}{4}\left(\frac{\gamma}{2} + \frac{3}{2}\log 2 - \frac{\pi}{4}\right) \text{ and } \frac{dI_6}{dA} = -\frac{\pi^{\frac{1}{2}}}{4}\left(\frac{\gamma}{2} + \frac{3}{2}\log 2 + \frac{\pi}{4}\right),$$

$$C_1 = C_2 = \pi^{-\frac{1}{2}} \text{ and } C_3 = 2\pi^{-\frac{1}{2}}\left(-\frac{\gamma}{2} - \frac{1}{2}\log 2 - \frac{\pi}{4}\right),$$

$$C_4 = 2\pi^{-\frac{1}{2}}\left(-\frac{\gamma}{2} - \frac{1}{2}\log 2 + \frac{\pi}{4}\right).$$

These values of C_1, C_2, &c. satisfy condition (104). Suppose now the reflector to be a sound-*sending* one and OL (Fig. 2) to be a line of sources, and that we wish to find the whole strength of the source. It is shown in Art. 34 (only now N.B. using u and v for the original pu and pv, see Art. 6), that the strength belonging to a length $2du/p$ at point U (Fig. 2) is the limit when $v = 0$ of $\dfrac{2\pi du}{p} \times v dF/dv$ where F is the velocity-potential. We will suppose (see Art. 5) that

$$F \equiv [(E_4 V_1 - E_2 V_2)\cos 2pat + (E_1 V_2 - E_3 V_1)\sin 2pat] \times U_1,$$

where V_1, V_2 are got from (15) and (16), Art. 7, by putting v for u and changing the sign of A, and E_1, E_2, &c. are got from C_1, C_2, &c. in (105) by changing the sign of A there and also in I_5 and I_6, from which suppositions it follows that

$$E_1^2 + E_2^2 = \frac{1}{\pi}(e^{\pi A} + 1) \quad\text{..................(120)},$$

and also that, when v is large and A/v small,

$$\left.\begin{aligned} E_4 V_1 - E_2 V_2 &= \frac{\cos\left(v - \dfrac{A}{2}\log v\right)}{v^{\frac{1}{2}}} \\ E_1 V_2 - E_3 V_1 &= \frac{\sin\left(v - \dfrac{A}{2}\log v\right)}{v^{\frac{1}{2}}} \end{aligned}\right\} \quad\text{.........(121)}.$$

From the above it follows that the strength of source belonging to the length $2du/p$ of OL is = the limit, when $v = 0$, of

$$\frac{2\pi U_1 du}{p} \times v[(E_4 V_1' - E_2 V_2')\cos 2pat + (E_1 V_2' - E_3 V_1')\sin 2pat]$$

$$= \frac{2\pi U_1 du}{p}[-E_2 \cos 2pat + E_1 \sin 2pat].$$

Therefore the greatest value of the strength at U (Fig. 2)

$$= (E_1^2 + E_2^2)^{\frac{1}{2}} \times \frac{2\pi U_1 du}{p} = \pi^{-\frac{1}{2}} \times (\epsilon^{\pi A} + 1)^{\frac{1}{2}} \times \frac{2\pi U_1 du}{p},$$

therefore the whole strength in the line-source LO (Fig. 2) is

$$= \frac{2\pi^{\frac{1}{2}}}{p} (\epsilon^{\pi A} + 1)^{\frac{1}{2}} \int_0^{pl} U_1 du \;\ldots\ldots\ldots\ldots\ldots(122),$$

where U_1 is given by (15) in Art. 7. Now suppose $A = 0$, then by (15) U_1 becomes $J_0(u)$ and we will further suppose pl to be so large that we can practically put ∞ in the upper limit of the Integral. (For the possible range of values of p see Art. 5.) (If, for a smaller value of pl, a more exact value of $\int_0^{pl} J_0(u)\, du$ is required, it can be got by integrating the semi-convergent series that, for large values of u, represents $J_0(u)$.) Then, since by Art. 450 of Todhunter's *Laplace's Functions* it is known that $\int_0^{\infty} J_0(u)\, du = 1$ we have the whole strength of source in LO

$$= 2^{\frac{3}{2}} \pi^{\frac{1}{2}} / p \text{ nearly } \ldots\ldots\ldots\ldots\ldots\ldots\ldots(123),$$

and by (40 a) of Art. 19 and by Art. 5, at a point V on the axis distant $v/2p$ from the geometrical focus of the reflector (Fig. 2) (that distance being supposed great) the velocity in the axis of the reflector $= 2p dF/dv$ (u, v being here used N.B. for the original pu, pv, Art. 6). But by (121), putting $A = 0$, at a distant point V we have

$$F = \frac{\cos(v - 2pat)}{v^{\frac{1}{2}}},$$

so that the velocity at $V = -2p \dfrac{\sin(v - 2pat)}{v^{\frac{1}{2}}}$ nearly, v being large. But here it must be remembered that v is used for $p \times 2OV$. We will, in Fig. 2, for clearness put $OV \equiv v_0$, so that the maximum velocity at V now becomes

$$(2p/v_0)^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(124).$$

From (123) and (124) we see that the ratio of the maximum velocity at V to the whole strength of the source in LO (which we might consider a fair measure of the sound magnification in the case of a sound-*sending* reflector)

$$= \left(\frac{p^3}{2\pi v_0}\right)^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(125).$$

We see from this that if we suppose V to be a fixed point (and therefore v_0 constant) and if we make experiments with different *large* values of p, then the magnification of sound produced by the reflector varies as $p^{\frac{1}{2}}$, so that if we suppose the strength of source constant, *the magnification would vary as* $p^{\frac{3}{2}}$. Perhaps, remembering the linearity of the fundamental equation of sound, we may put the last statement better thus. We have proved that if we have in LO a source of sound whose total strength

$$= C \times \frac{2^{\frac{3}{2}}\pi^{\frac{1}{2}}}{p} \text{ (where } C \text{ is a constant),}$$

it will produce, at a distant point V, a velocity whose maximum value

$$= C \times \left(\frac{2p}{v_0}\right)^{\frac{1}{2}}.$$

But here we may put $C = \mu p$ where μ is an arbitrary constant, so that a source in LO, whose strength $= \mu \times 2^{\frac{3}{2}}\pi^{\frac{1}{2}}$, will produce at V a velocity

$$= \mu \times \left(\frac{2p^3}{v_0}\right)^{\frac{1}{2}} \text{.......................(126).}$$

This justifies the above statement about magnification. But perhaps a still better measure of the sound magnification may be arrived at by finding the ratio between the actual maximum velocity given by (124) and what the maximum velocity at V would be, if the reflector and source in LO were annihilated, and there were at O a single *free* source whose strength is (123). We shall find this ratio

$$= \frac{pv_0^{\frac{1}{2}}}{4(2\pi)^{\frac{1}{2}}}.$$

We might adopt the coefficient of $v_0^{\frac{1}{2}}$ here as a fair measure of the magnification. Whichever measure we adopt, we see that the higher the pitch of the note (or tone) experimented on the greater will be the magnification.

The Abortive Development of the Pollen in certain Sweet-Peas (Lathyrus odoratus). By R. P. GREGORY, M.A., St John's College.

[*Read* 14 May 1905.]

Among the plants produced from the self-fertilization of a certain race-hybrid of *Lathyrus odoratus* Mr Bateson obtained, during 1903, a certain number of individuals whose anthers were contabescent*.

In the plants with coloured flowers the sterility was, with a very few exceptions, correlated with a somatic character, the sterile plants possessing a green leaf axil, while the fertile plants, with the rare exceptions noticed, had red axils.

The same phenomenon was repeated in 1904, with every indication that the sterility occurs in a definite proportion of the plants, both in those with coloured and in those with white flowers. That is to say, the sterility is a character which undergoes segregation in accordance with Mendelian principles†.

Mr Bateson kindly allowed me to take material from his plants for the purpose of making an examination into the cytological phenomena which accompany the sterility of the male organs. The fixing fluids used were (1) a mixture of glacial acetic acid, 1 part; absolute alcohol, 2 parts; and (2) a 1 °/$_0$ solution of chromic acid in water. It was found that anthers preserved in the early morning yielded a rather larger proportion of favourable preparations than did those taken later in the day.

From an examination of fresh material by various methods it became clear that the abnormal forms taken by the cells of the sterile plants were somewhat variable. In most cases the anthers contained a semi-fluid mass, in which a few cells of irregular outline could sometimes be distinguished; others contained a few pollen grains, which however appeared to be bad. This observation was confirmed by the detailed study, which revealed a considerable range of divergence between the behaviour of the nuclei of the pollen-mother-cells, not only of different plants but sometimes of different anthers of the same plant. The appearance presented by the pollen-mother-cells of any one loculus is generally the same, except for slight differences corresponding with the slightly more or less advanced stages of development reached by some of the cells. But it seems probable that the degree of abnormality exhibited by pollen-mother-cells produced in different loculi of one anther is not necessarily the same.

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* Bateson, Saunders, and Punnett, *Roy. Soc. Evol. Com., Rep.* II. p. 91.

† *Ibid.* pp. 91, 92.

Fertile Plants.

Before the phenomena observed in the spermatogenesis of the sterile plants are described, an account will be given of the process normal in fertile plants. The material used for this purpose was taken from pure bred plants of the race known as "Emily Henderson"—one of the parent races of the sterile plants*.

In these plants the active division of the archesporial cells, to which the pollen-mother-cells owe their origin, is succeeded by a period of rest and growth. Towards the end of this period, at the time when the nuclear reticulum is about to become transformed into the coiled spireme, the threads connecting the nucleolus with the peripheral reticulum become very prominent, taking the stain more deeply than those which form the network (Fig. 1). In the nucleolus there appear small vacuoles, which increase in size while the reticulum shows an increasing affinity for the chromatin stain. In correspondence with its increased vacuolation, the nucleolus, which hitherto has been very deeply coloured, takes the stain to a degree which decreases with each successive stage. The numerous threads by which it was connected with the reticulum disappear, apparently being broken off. In some of the later stages the nucleolus was found to be connected to the reticulum by only one rather prominent thread. This single connexion is, I believe, finally broken off; at any rate no connexion is made visible by the stains which have been used†. The nucleolus is now a spherical body, which consists of a very faintly stainable shell enclosing a large vacuole, which may be central or excentric. There can, I think, be no doubt that these changes indicate a transference of chromatin (or of some substance closely allied to chromatin) from the nucleolus to the nuclear thread‡. The subsequent fate of the nucleolus is very difficult to observe; I have been able to find no trace of it at any stage later than that at which the spindle becomes conspicuous. In the sterile plants the nucleolus often fragments, and, shortly after the breaking down of the nuclear wall, I have seen in a few cases two or three small spherical bodies lying in the extra-nuclear protoplasm. Both in colour§ and texture these bodies resemble exactly that portion of the nucleolus which remains in the clear area occupied by the nuclear thread (see Fig. 12). This fact would appear to suggest that the remains of the nucleolus may finally be absorbed by the protoplasm of the cell.

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* *l.c.* p. 91.

† Iron-alum haematoxylon, and Flemming's triple stain.

‡ See Wager, *Annals of Botany*, 1904, Vol. 18, p. 29.

§ These preparations were stained with iron-alum haematoxylon.

While these changes have been taking place in the nucleolus the reticulum has given rise to a closely coiled spireme thread, which, at first evenly distributed throughout the nuclear space, subsequently contracts towards one side of the nucleus, undergoing a longitudinal fission throughout its entire length. At this stage the nucleolus is often seen to be saucer-shaped, the convex side being applied to the nuclear wall in the region toward which the contraction of the thread takes place (Fig. 2)[*]. The polarity of any nucleus has no apparent relation with that of its neighbours.

The contraction of the spireme, with its consequent formation of a series of loops, is followed by the segmentation of the thread into chromosomes. The whole process of the formation of the chromosomes is essentially identical with that described by Farmer and Moore[†]. In certain Ferns, in which a similar process was found to obtain, the large number of small chromosomes made it difficult definitely to ascertain that each individual loop stood in a definite relation to one chromosome[‡]; in the Sweet-Peas the positions assumed by the chromosomes at their first separation must lead to the conviction that, not only does each loop give rise to a chromosome, but that the apex of the loop is the actual point at which, in metaphase, the limbs of the chromosome are destined to separate from one another (Figs. 5 and 6).

The chromosomes shorten up very much prior to metaphase, and each appears as two short thick rods lying parallel with one another. No instances of ring-shaped chromosomes have been observed.

The form of the chromosomes at metaphase renders their structure difficult to observe, but it may be referred to the typical $\overset{\Lambda}{V}$-shaped figure obscured by the shortness and thickness of the arms of the V (Fig. 20). In anaphase, a section transverse to the spindle shows the chromosomes arranged in a ring. They are V-shaped, but very thick at the apex of the V, which is turned toward the centre of the spindle (Fig. 21).

The second (homotype) mitosis is of the usual type, and need not be described in detail.

The reconstitution of the daughter nuclei from the chromosomes can be followed with great clearness in the young pollen grains.

Wager has described the fusion of the chromosomes first of all into a number of small nucleolar masses, connected together by a deeply stained network, and then a further fusion into the large nucleoli found in the mature cells*. This description is not entirely applicable to the process which I have observed in the reduced nuclei of the pollen grains. The nuclei certainly pass through a stage in which there is a single continuous thread (Fig. 23), which is directly formed by the fusion of the chromosomes end to end. Almost immediately after this stage is reached the coils of the thread are seen to be connected with one another by fine branches. From this stage the reticulate condition is attained.

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[*] Cf. Zimmermann, *Morph. u. Physiol. des Pfl. Zellkernes*, Jena, Gustav Fischer, 1896, p. 69.

[†] *Roy. Soc. Proc.*, 1903, Vol. 72, p. 104, and *Q.J.M.S.*, 1904, Vol. 48, p. 489.

[‡] *Annals of Botany*, 1904, Vol. 18, p. 447.

Attempts made to trace the origin of the nucleolus met with no more success than those made to trace its ultimate fate in mitosis. One or two nuclei are first observed as small, spherical, faintly-staining bodies lying in close proximity to the chromatin thread with which they are connected by a fine thread or threads. The single large nucleolus of the later stages is formed by the fusion of the smaller ones which, in the intermediate stages, are seen closely applied to one another. The nucleolus is connected with the nuclear reticulum, to which the chromatin thread has now given rise, in the early stages sometimes by only one connecting thread (Fig. 24), but later by more.

These observations appear to me to indicate that in the process of reconstitution of the daughter nuclei exactly the same changes occur as in the prophases of division, but in the reverse order; and, further, that the nucleolus receives and stores the bulk of the chromatin during the resting periods which intervene between successive mitoses†.

Sterile Plants.

The somatic mitoses of these plants (including those which take place during the development of the archesporium) are quite normal, and differ in no way from those of the fertile individuals of the pure race. The rapid growth of the archesporium is, however, followed by an abnormally long period during which no divisions of the pollen-mother-cells take place. This is to be seen, not only from the large number of preparations which are obtained in this condition, but also from the large growth of the anther itself which takes place. Since there is no corresponding increase in the sporogenous cells, the pollen-mother-cells are found lying quite loosely, and in the later stages almost scattered, in a large cavity. A deceptive appearance is thus produced which is suggestive of the formation of a smaller number of pollen-mother-cells than is usual, but is in reality due to the fact that a fertile anther of the same size would contain pollen-mother-cells of a much later stage, or even young pollen grains.

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* *Annals of Botany*, 1904, Vol. 18, p. 47.

† Cf. Wager, *l.c.*

The various forms which the irregularities take differ from one another only in degree, so that although the plants fall roughly into two groups—those in which chromosomes are formed and those in which complete abortion takes place at an earlier stage—intermediate conditions are frequently found.

In the types more nearly approaching the normal form, the nucleus of the pollen-mother-cell passes into the spireme stage, the thread undergoes a longitudinal fission, and by its contraction towards one side of the nucleus a series of irregular loops is formed (Figs. 2, 5—8).

The formation of the chromosomes through the segmentation of the looped spireme takes place in the normal way. Fig. 7 illustrates a cell in which only very slight irregularity is present; the chromosomes, however, possess a very much more ragged outline than do those of normal pollen-mother-cells. It is perhaps worth mentioning that in the cells of the loculus adjoining that in which this cell occurred, the abnormality was much more pronounced and the formation of chromosomes completely failed (cf. p. 153). It will be seen that the chromosomes in this early stage give the impression of being of different lengths; later they become much shortened, and in metaphase no distinct differences in size can be recognized (Figs. 19, 20).

In other cases in which the attempted formation of chromosomes is successful there appears to be a great deficiency of chromatin; the chromosomes of the abnormal pollen-mother-cells. staining very feebly as compared with those of the surrounding (normal) somatic cells (Fig. 6).

The great irregularity of the spindle of the heterotype division is a very marked feature in these plants; and in addition to irregularities in the direction taken by the spindle fibres themselves, the chromosomes are irregularly distributed upon the spindle (Figs. 19, 20). No case has been observed of the regular arrangement of the chromosomes in an equatorial plate at metaphase, such as obtains in the fertile plants.

Although many anthers have been examined which, by comparison with the fertile plants, were expected to show mature pollen grains (if any were developed), no stages later than that just described have as yet been obtained.

Cases, such as those described above, are comparatively rare. They are characterized by the definite formation of chromosomes and the development of a spindle, the abnormality being confined to the irregularity and asymmetry of these processes.

Much more commonly the abortion of the pollen-mother-cells is complete and only the early stages of prophase are passed through before degeneration sets in. The first signs of irregularity appear in the formation of the spireme thread; the chromatin is distributed unevenly, in such a way that portions of the thread are destitute of that substance. The nucleolus generally becomes irregularly lobed and is often connected with those portions of the spireme which contain chromatin by prominent connecting threads.

The longitudinal fission of the spireme is effected, at any rate in those parts of the thread in which chromatin is present (Fig. 9). The thread now becomes very irregular, both in structure and in configuration, and has a tangled appearance (Figs. 10—12); any indications of looping which may be present show corresponding irregularities, while the threads containing the chromatin are connected with one another to a greater or less degree by irregular threads from which chromatin granules are absent. In some cases the thread now undergoes a partial segmentation, indicating the attempted formation of chromosomes (Fig. 13). In others the chromatin aggregates into irregular masses which are connected with one another by faintly stained threads (Figs. 14, 15). It should be mentioned that these masses generally number about eight, and sometimes give the appearance of being double. The process rather recalls that of the shortening and thickening of the chromosomes of fertile cells just before metaphase.

These changes are accompanied by a drawing-out of the whole nucleus, sometimes in the direction of more than two poles (Fig. 13); at this time the nuclear walls disappear, and in some cases the development of a few striae at the poles of the nucleus indicates an attempted formation of the spindle (Fig. 13).

From this condition almost every form transitional to the curious state shown in Figs. 16 and 17 may be found. In the earlier stages of this form of abnormality, it is quite clear that the masses of deeply staining matter, which are, as it were, left behind by the drawing-out and constriction of the nucleus, consist of a closely tangled thread.

In the later stages of all forms the anthers contain only a matrix in which the cell outlines become less and less distinguishable; embedded in this lie rounded or lobed masses which stain deeply, and, by comparison with the earlier stages, doubtless represent the degenerating nuclei.

The sterility in these plants is confined to the male organs*. The structure of all the embryo-sac mother-cells examined has been quite regular, the contraction of the spireme and the formation of loops resembling exactly that of the fertile plants (Fig. 4).

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* See also Bateson, Saunders, and Punnett, *l.c.*, p. 91.

Such evidence as is obtainable, both from experimental breeding, and more particularly from cytological work, points to the reduction division as the time at which the segregation of allelomorphic characters takes place. It is therefore interesting to notice that, in the sterile Sweet-Peas, abnormalities appear during the processes preparatory to this division, and only then.

The exact bearing of this observation upon our theories of the individuality of chromosomes, and their importance in the transmission of segregated characters, is, however, not very clear.

Sterility, in the case with which we are dealing, is a recessive character, and is almost always confined to those plants which have light-coloured leaf axils—another recessive character. No segregation of the characters in question is therefore taking place at the formation of the gametes. The irregularities do not therefore correspond with defective segregation.

Further, it must be noticed that the irregularity affects the *whole* nucleus and is not confined to any particular pair of chromosomes. Although this may simply imply—what recent cytological work has led us to expect—that individual chromosomes may influence the behaviour of their fellows, it seems more likely that sterility of the male organs is the expression of some deeper-lying phenomenon affecting the physiology of the whole plant. It is moreover almost always coupled with another physiological character—the absence of colour from the leaf axils even in plants with coloured flowers*.

In further support of this view we have the most important fact that the nucleus of the embryo-sac mother-cell, which possesses a series of chromosomes exactly homologous with those of the pollen-mother-cells, shows no abnormality whatever.

The low staining capacity of many of the abnormal nuclei (see p. 152) might indicate a low state of nutrition, although there is no obvious difference between the development of the vascular strands in the fertile and sterile stamens. Possibly a similar deduction may be drawn from the fact that the development of spindle fibres in the cytoplasm is feeble or absent.

If this view be correct, it may be that the sterility of the male organs in these Sweet-Peas is not fundamentally different from that which has been observed, also in the male organs only, in a certain family of *Primula Sinensis*. There is some indication that the contabescence of the anthers in this latter case is an inherited (recessive) character. In these plants the degree of contabescence varies with each anther, and a small amount of good pollen is produced.

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* See also Bateson, Saunders, and Punnett, *l.c.*, pp. 91, 92, on the existence of two classes among the white-flowered plants.

It appears that in all these plants, the reduction division follows the normal course, and young pollen grains are formed. In the contabescent anthers growth ceases at this point, the pollen grains remain small and enclosed within the walls of the mother-cells.

EXPLANATION OF THE FIGURES

illustrating Gametogenesis in *Lathyrus odoratus.*

The figures were drawn with a camera lucida. A Zeiss 2 mm. Apochromatic Objective and Oculars 4, 6, 8, and 18 were used.

Plate I.

Fig. 1. Sterile Pea. Pollen-mother-cell in the resting condition. The reticulum is shown, with some threads joining the nucleolus. The latter is somewhat vacuolated. × 1600.

„ 2. The same. A later stage showing the contraction of the spireme. There are faint indications of the longitudinal fission of the thread. The nucleolus has taken a "saucer-shaped" form. × 1600.

„ 3. "Emily Henderson" (fertile race); showing the looping of the thread, which has undergone a complete longitudinal fission. × 1600.

„ 4. Sterile Pea. Nucleus of the embryo sac, in a stage similar to that shown in Fig. 3. × about 2000.

Figs. 5–8 and 19–20 illustrate sterile plants in which the abnormality is slight.

Fig. 5. Sterile Pea. Nucleus of the pollen-mother-cell, showing a later stage, in which the loops are contracted. The longitudinal fission of thread is shown by the divergence of the halves in one place. The nucleolus has given up most of its stainable matter. × 1600.

„ 6. Pollen-mother-cell from the same anther as that shown in Fig. 5. A rather later stage, in which the segmentation of the thread into chromosomes has taken place.

„ 7. Sterile Pea. Pollen-mother-cell from another plant in which the abnormality was slight. The chromosomes appear to be of somewhat different lengths. × 2000.

„ 8. Two chromosomes from the same, showing the longitudinal fission of each limb. × about 2400.

FIGS. 9–17 illustrate the later stages of sterile plants in which degeneration of the pollen-mother-cells takes place before chromosomes are formed.

FIG. 9. Sterile Pea. Pollen-mother-cell showing the irregular spireme, and its longitudinal fission. × 1600.

FIGS. 10, 11. Sterile Pea. Rather later stage in which the spireme is somewhat tangled. The longitudinal fission can still be traced. There seems some indication of the polarity which is observed in fertile cells. × 1600.

FIG. 12. Sterile Pea. Showing the close tangling of the spireme. The nuclear wall is only visible at the ends of the poles of the rather elongated nucleus. The chromatin thread is surrounded by an area of clear protoplasm. The remains of the nucleolus, now staining very faintly, lie in the centre of the nucleus and are surrounded by a series of threads containing little chromatin, but connecting with the tangled portions of the chromatin thread, of which they really form a part. × 1600.

„ 13. From the same anther as Fig. 12. The nuclear wall has disappeared. Some parts of the thread have separated, representing the attempt to form chromosomes. The longitudinal fission is visible in some places. Cytoplasm of a definitely fibrous structure surrounds the clear area. There is a suggestion of multipolarity. × 1600.

PLATE II.

FIG. 14. Sterile Pea. A later stage showing the aggregation of the chromatin into masses. The nuclear wall has disappeared. The nucleolus is still visible as a faintly staining body. × 1600.

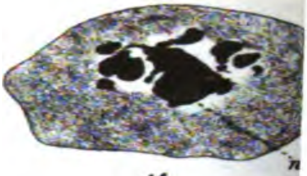
„ 15. The same in a rather later stage. The deeply staining bodies are connected with one another by slightly stained threads. × 1600.

FIGS. 16, 17. Sterile Pea. The incomplete and abnormal division, by constriction of the pollen-mother-cells, which occurs in a few cases. × 1600.

FIG. 18. "Emily Henderson." Metaphase of the heterotype division in the fertile form. × 1600.

FIGS. 19, 20. Sterile Pea. Spindles of irregular form such as occur in those sterile plants which show least divergence from the normal type. × 1600.

Phil. Soc.



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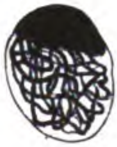

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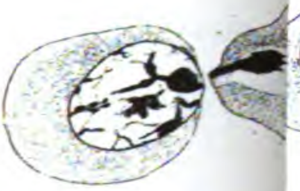

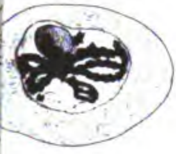

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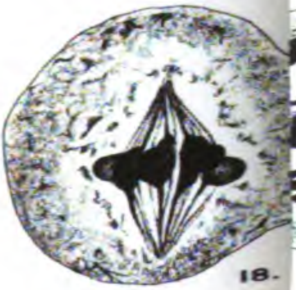

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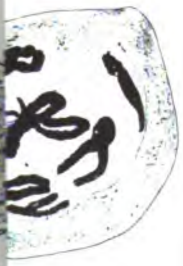

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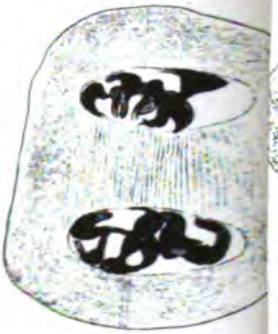

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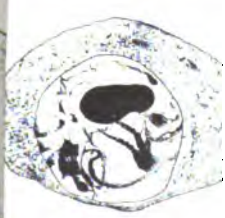

11.

R. P. G., del.

FIG. 21. "Emily Henderson." Late anaphase of the heterotype division, showing the V-shaped chromosomes approaching the poles of the spindle. × about 2000.

„ 22. The same in polar view.

„ 23. "Emily Henderson." Young nuclei of the pollen grains, showing the formation of a continuous thread through the fusion of the chromosomes. Nucleoli are present. × 1600.

„ 24. "Emily Henderson." Young pollen grain, showing the nuclear reticulum and a single nucleolus which is connected with the network at one point only. × 1600.

The seed-bearing habit in the Lyginodendreæ. By E. A. NEWELL ARBER, M.A., Trinity College, University Demonstrator in Palæobotany.

[*Received* 15 May 1905.]

Among fossil plant remains, specimens showing two organs, still in continuity with one another, are as a rule very rare. Thus in the case of *Lyginodendron*, one of the most fern-like of all the Upper Palæozoic plants, while the seed, *Lagenostoma*[*], and other organs are now known in great detail, there has been no evidence, so far, as to the manner in which the seeds were borne. A new species *Lagenostoma Sinclairi* Kidston M.S. has, however, been recently discovered, which throws light on this problem. I am indebted to my friend Mr Kidston for permission to describe this new and interesting seed.

The morphology of *Lagenostoma Sinclairi*[†] agrees very closely with that of *L. Lomaxi*, which has been shown by Prof. Oliver and Dr Scott[‡] to belong to *Lyginodendron*. As in the case of *L. Lomaxi*, these seeds are enclosed in protective organs which have been termed 'cupules'; the 'cupule' of the new species differing only in details from that of the seed previously described. The special interest of the new specimens lies in the fact that, in many instances, the seeds, enclosed in their cupules, are still attached to the axes on which they were borne in the living state. These axes are highly compound structures, the seeds apparently terminating the finer branches. The type of branching is extremely irregular, and no pinnules of the ordinary foliar type are borne on the seed-bearing axes. The highly compound nature of these axes, the irregularity of the branching, and the fact that the seeds terminate both the shorter and longer axes, indicate that the morphology of the branched system is that of a compound frond with reduced lamina. This conclusion is confirmed by the close agreement between this seed and *Lagenostoma Lomaxi*, the latter being now known to belong to a plant possessing highly compound foliage of the *Sphenopteris* type.

At the close of 1904, when the examination of these specimens had been completed, only one case was known in which the seed-bearing habit had been ascertained among the group Pteridospermeæ, to which these Fern-like spermaphytes are now assigned.

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[*] Oliver and Scott, *Phil. Trans. R. Soc.*, Ser. B, Vol. 197, p. 193, 1904.

[†] A full account of the morphology of this seed, and of another new species, will shortly appear in the *Proc. R. Soc.* An abstract of this paper was published in the *Annals of Botany*, Vol. XIX., p. 326, 1905.

[‡] Oliver and Scott, *ibid.*

This was the genus *Medullosa*, in which Mr Kidston[*] has shown that the seeds were borne on ordinary fronds identical with the sterile foliage. Since then two new discoveries have been announced, and in both cases the seeds are borne on fronds. Mr David White[†] has described the seeds of a species of *Aneimites* (= *Adiantites*), an important genus occurring in Lower Carboniferous and Upper Devonian rocks, which are borne on fronds with reduced lamina just as in the case of the new seeds described here.

Still more recently M. Grand'Eury[‡] has made a most interesting discovery, affecting a genus of Palæozoic plants almost universally regarded as true ferns. The genus *Pecopteris*, especially characteristic of the higher beds of the Upper Carboniferous, and of the Permian, contains a large number of species, among which one, *Pecopteris* (*Dicksoniites*) *Pluckeneti* Schl., not perhaps a very typical member, has been shown by M. Grand'Eury to be a seed-bearing plant. In this case the seeds are borne on fronds hardly differing at all from the sterile fronds.

Thus in the four groups or genera, Medulloseæ, Lyginodendreæ, *Adiantites*, and some Pecopterids, in which the seed-bearing habit is known, the seeds are in all cases borne on fronds, which are either little modified and similar to the sterile foliage, or fronds with reduced lamina. In lax arrangement of the fructification, the habit of these Fern-like spermaphytes must have presented a striking contrast to most of the other great Palæozoic groups in which strobili or cones were dominant and conspicuous types of sporangial aggregation.

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[*] Kidston, *Phil. Trans. R. Soc.*, Ser. B, Vol. 197, p. 1, 1904.

[†] White, *Smithsonian Miscell. Collections*, Vol. 47, Pt. 3, p. 322, Dec. 1904.

[‡] Grand'Eury, *Compt. Rend. Acad. Sci.*, Vol. 140, p. 920, April 3, 1905.

Experiments on Penetrating Radiation. By H. L. COOKE, 1851 Exhibition Scholar of McGill University: Emmanuel College.

[*Received* May 1905.]

In a previous paper* the author has shown by experiments carried out at the McGill University, that the natural ionization in a closed vessel is to be partly ascribed to a penetrating radiation. Large masses of lead were used as screens to shield a brass electroscope from this radiation, and a diminution in the ionization of between 30 °/₀ and 35 °/₀ was observed, the radiation inside the laboratory being shown to come from all directions with equal intensity. Experiments were also made to test the ionization outside the laboratory, on the surface of the frozen ground, and on the roof. The experiments showed the existence of a penetrating radiation under these altered conditions, but gave no indication as to its source or direction. The experimental arrangements consisted of taking alternate runs, first with the vessel exposed, and then surrounded on all sides by a lead cage, one inch in thickness. A diminution in ionization was always observed, but the amount of this reduction was not accurately determined, owing to the difficulties of experimenting under the conditions encountered, and on account of the fact that the apparatus was not well adapted for experiments involving great changes of temperature, which were sometimes as great as 30° C.

The primary object of the present research was to detect and measure, if possible, any penetrating radiation coming from the atmosphere. The existence of such a radiation is important from the point of view of atmospheric ionization at high altitudes. The radiation measured in the experiments at Montreal was such that about one-third of an inch of lead screen reduced it to half value, and its effect was not observable after passing through a thickness of 2 inches of lead. This would mean that the effect of this radiation from the earth would be inappreciable at a height of 500 yards, and any ionization above this height would have to be ascribed to radiations arising from the air itself or to radioactive impurities mixed with it.

In these experiments a small portable ionization vessel was used, being designed to work independently of changes in temperature and weather conditions, and capable of withstanding comparatively rough handling, and being turned on its side and inverted. The difficulties of designing such a vessel were considerable, and the form of apparatus shown in the accompanying diagram was gradually evolved, additions and alterations being made to overcome working difficulties as they arose.

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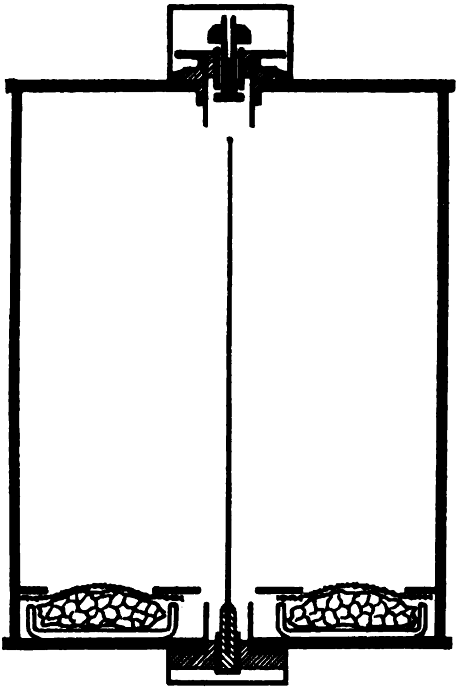
* *Phil. Mag.*, Oct. 1903.



The general description of the apparatus is as follows. A central charged rod of piano wire, with a platinum tip, is fixed in a small rod of block amber insulation, which is supported in a brass cup, kept in electrical contact with the case, except when the insulation leak is being measured. The device at the top is to charge and discharge the insulated rod, a plunger with a platinum disc being employed for this purpose. When not in use a nut may be screwed down to hold it firmly in position, and thus close the air inlet. Outside the plunger is a brass tube, to prevent electrification. This is surrounded by an ebonite tube, outside which is a guard ring, kept earthed during readings. The top of this guard ring is in the form of a disc, to protect the insulation under the flange, on the perfection of which the accuracy of the readings depends. Two brass caps slide on over the top and bottom attachments, and being carefully waxed down, render the apparatus air-tight during runs. The plunger may be connected to the battery by means of a tapered plug, fitting in the hole bored down the centre. The drying agent used is calcium chloride, wrapped in four small linen bags, resting in porcelain trays, and kept in position by means of an electrostatic screen, soldered to the inside of the cylinder. In setting up the vessels, of which there were two, great care was taken in preserving the utmost cleanliness and freedom from contamination with radioactive materials. Except for the central rod, all metal parts were of brass, the sides and top being about 2 mm. in thickness. All metal to metal joints were soldered, the others being made with a marine-glue preparation. The diagram is drawn to scale. The volume of ionized air is 510 c.c.

The method of using these vessels was as follows; the case being supported on a stand connected to earth, the central rod was charged to an accurately measured potential, usually between 400 and 410 volts. The top cap was then replaced and waxed down, and the vessel set in the place where the ionization was to be measured. After remaining there for several hours it was placed on an insulated stand, connected with one pair of quadrants of a Dolezalek electrometer. After separating the quadrants the central rod was charged by means of the plunger to the original voltage. This resulted in the central rod acquiring a quantity of electricity equal to that carried away by the total ionization current during the experiment, and an equal charge of like sign being repelled to the quadrants of the electrometer, and measured by the deflection.

The object in making two similar vessels was to eliminate if possible the effect of any diurnal variation in the natural ionization, by always exposing one under similar conditions, the other being reserved for measuring the leak under varying circumstances. It was soon found, however, that the ratio of the two ionization currents under apparently identical conditions was not more constant than the currents individually. After this was shown to be the case both vessels were utilized in examining varying conditions, the one being a check on the other.

It was originally intended to send the vessels up attached to a balloon or kite, but this plan was abandoned owing to the difficulties and danger of operating in this manner in the neighbourhood of a large town. The method first adopted was to experiment with the vessels on the roof, shielding with lead screens, first from above and then from below. Experiments were carried on in this manner for a considerable time, but no reliable results were arrived at. There were strong indications however that the radiation was more intense from below than from above. The measurements showed a reduction of about 12°/$_{o}$ when the apparatus was shielded from below and from the sides by being placed in a cup-shaped lead-casting one inch in thickness. When this cup was inverted and placed over the top of the vessel no certain alteration in the ionization could be detected.

Although the above experiments did not yield conclusive evidence on the subject of the radiation from the atmosphere, they showed that the radiation on the roof was not the same as that inside the building, being at least 20°/₀ less than the radiation of a similar character inside the laboratory. This increased radiation inside the laboratory is not due to the fact that the building has been used for experiments with radium or similar materials, for the vessels when exposed in the rooms of a private house showed if anything a trifle higher ionization,—certainly not less. The explanation of the results up to this point is clearly that any radiation which may be coming from the atmosphere is less intense than that coming from the materials used in the construction of the building.

The method of shielding the vessel from two directions on the roof not yielding results with sufficient rapidity or certainty, the next plan which suggested itself was to bury the apparatus, and compare the leak under these conditions with that observed when the vessels were exposed on the roof. In the one case the earth surrounds the vessels on all sides; in the other the atmosphere is above the horizontal plane, solid material below. Comparing the ionization under the two conditions should supply data for estimating the ratio of the activity of the earth to that of the atmosphere. Experiments carried out in this way show that the difference in radiation under the two circumstances is very small, if any exists. This however does not prove conclusively that the radiation from the atmosphere is of the same order as that from the earth, as there is always the possibility of the surface of outdoor objects giving forth a penetrating radiation to a greater extent than the unexposed portions. The uncertainty in connection with this point has up to the present prevented definite conclusions from being drawn as to the activity of the atmosphere. It is intended in continuing the work to test this point experimentally if possible.

Experiments were carried out with the apparatus submerged 3 feet below the surface of the Emmanuel College swimming tank, which is about 7 feet deep. The ionization under these conditions is about 12°/₀ less than when the vessels are exposed on the roof. Comparing the results obtained indoors with those observed with the apparatus submerged, they show a reduction similar to that obtained by McLennan[*], and the writer[†], by shielding the ionization vessels with water, working indoors. When the vessels were allowed to float with their tops flush with the surface of the pond, the ionization increased between 5 and 10 °/₀. Suspending the apparatus by a wire over the surface of the pond gave results similar to when the vessels were exposed on the neighbouring lawn, or on the roof of the laboratory. This increased radiation may be due to the banks of the pond, and the presence of a large number of trees in the immediate neighbourhood.

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[*] *Phil. Mag.* v. p. 419 (1903).

[†] *Loc. cit.*

The theory has been suggested that all substances may give rise to a penetrating radiation, and that the ratio of the amount of this activity to the density of the substance may vary in different materials. When an ionization vessel is surrounded on all sides by a thickness of any substance sufficient to absorb all external radiation, the ionization in the enclosed gas assumes a definite value, which depends on the ratio of activity to density of the shielding material, and which is unaffected by additional layers of the screening substance. Under such circumstances the greatest reduction in ionization observed has been 35 °/₀ of the value obtained when the apparatus was unscreened indoors. How much of the remaining 65 °/₀ is due to a penetrating radiation from the screening material, and how much is due to a radiation of a more easily absorbed type from the walls of the vessel, no experiments up to the present have indicated. The results so far show that in water, lead, iron, and possibly the atmosphere, this ratio of activity to density is small, but there is no means of estimating how near zero this ratio is. The results are merely comparative. The experiments up to the present with the apparatus buried have not been sufficiently extensive to draw accurate conclusions from, but there are indications that for earth this ratio is higher than for water, iron, and lead.

The results obtained up to the present may be summarized as follows. The penetrating radiation inside buildings is about 25 °/₀ greater than that outside on the roof. The ionization with the vessel on a lawn and suspended over the surface of a tank is approximately the same as on the roof. Submerging the apparatus to a considerable depth in a spring-water tank further reduces the leak by about 12 °/₀, the reduction not being so great with the apparatus floating on the surface. The ionization with the vessels buried about a foot in the ground does not differ greatly from the results on the roof.

The work is being continued. The author wishes to express his sincere appreciation of Professor Thomson's unfailing interest and many useful suggestions during the course of the work.

A suggestion as to the nature of the "walnut" comb in Fowls.

By W. BATESON, M.A., F.R.S., St John's College, and R. C. PUNNETT, M.A., Gonville and Caius College.

[*Received* 3 June 1905.]

In Report II to the Evolution Committee of the Roy. Soc., p. 109, we discussed the problem of Malay or "walnut" combs in fowls on the supposition that the four types of comb, viz. single (*s*), pea (*p*), rose (*r*), and "walnut" (*rp*), are allelomorphic to each other.

The facts are (1) that $r \times p$ gives in F_1 a zygotic combination *rp*, which differs in essential features from any other kind of comb, especially in the presence of feathers or hairs on its surface usually disposed as a transverse band.

(2) That the gametes of this F_1 are of four types, *s*, *r*, *p*, *rp*, in equal numbers, which is proved by the result of breeding the F_1 with a single. Similarly $F_1 \times F_1$ gives an average of 9*rp*, 3*r*, 3*p*, 1*s*.

(3) The *rp* birds in F_2 may then be pure *rp*; or they may give off only *r* and *rp*; or only *p* and *rp*; or again they may bear gametes of all four types. We have thus far failed to find any individuals that give only *rp* and *s*; or to distinguish any class as the product of $rp \times s$ from those which are the product of $r \times p$.

The *rp* gametes were regarded as true examples of gametic synthesis, and the appearance of *s*, absent from both pure parents, was treated as due to a resolution or break-up of either the *r*, the *p*, or both. A much simpler and probably more correct account has since suggested itself, which it is the object of this note to consider.

This suggestion will be made more intelligible by describing the way in which it was reached. Certain experiments on *Primula sinensis* made by R. P. Gregory in conjunction with W. Bateson had shown that short style, or thrum, is dominant to long style, or pin. In addition to these two types a condition is known which, following Darwin, we have called *homostyle*. In it the anthers are at the level proper to pin, but the style does not pass through them, and the stigma is at the level of the anthers, the whole style having the length proper to the thrum type. The pollen of homostyles is of the pin type. The corollas, however, are peculiar in that they always have a yellow flush which extends the "eye" far up the petals (whence the best known homostyle strain derives the name of "Primrose Queen").

In the course of the work thrum was crossed with homostyle, giving F_1 ordinary thrum without the yellow flush. But F_2 consisted of four types: (1) thrum without yellow flush; (2) thrum with yellow flush; (3) *pin* without yellow flush; (4) homostyles with yellow flush. The ratios were evidently 9 : 3 : 3 : 1. The form pin, not introduced as an original parent, thus appears in F_2 as a novelty.

The interpretation is obvious. The allelomorphic pairs are clearly two. (1) Thrum (D) and pin (R). (2) No yellow flush (D) and yellow flush (R); and the homostyle is merely a zygotic state occurring when yellow flush and pin coincide. The pin therefore appears as a novelty by the meeting of pin and "no yellow flush."

This observation naturally suggested that the single comb in F_2 from $r \times p$ might have a similar mode of origin. Applying this hypothesis the original parents, r and p, are regarded as respectively rose + no-pea, and pea + no-rose. The allelomorphs are (1) rose and absence of rose; (2) pea and absence of pea. The rose and the pea characters belong to distinct allelomorphic pairs. When therefore r meets p the zygote would be walnut, and the *single* is regarded as constituted by the meeting of gametes bearing neither rose nor pea. In this way all the phenomena are brought into simple relation with each other and the case is exactly comparable with that of any other dihybrid. This suggestion has the merit that it obviates any appeal to resolution, and that the gametic synthesis apparently occurring in the case of the rp character becomes an ordinary phenomenon of dihybrid inheritance. Finally, the fact that $r \times p$ cannot be distinguished from $rp \times s$ finds a ready explanation.

If we do not look beyond the special case of the walnut combs, this account seems so complete as to make further discussion unnecessary. We can only express surprise at our failure to perceive it sooner. This failure was of course due to the preconception that the types of comb were definite entities alternative to each other, while, as is now evident, the *presence* of a given modification must be regarded as allelomorphic to the *absence* of the same modification. The essential feature of the rose comb is its papillosity, while the ridges mark the pea. The single comb is thus to be regarded as a comb into which no such additional element is introduced, being neither papillose nor ridged, and is thus a factor common to all the other types, rose, pea, and walnut.

Previously we had regarded the allelomorphism as existing between the various factors which caused the comb to be either rose, pea, walnut, or single. The critical point of difference between the two views lies in the way in which single is regarded. While formerly we treated it as a positive condition segregating from the others, on what we may call the presence-and-absence hypothesis it is to be recognized as the original state into which the factor—or pangen, if we use de Vries' term—for rose, pea, etc., has been introduced.

Since the ratios, both of characters and hereditary powers, are identical in both cases, simple experiment does not discriminate between the two possibilities; though certainly if all the different kinds of elaborated combs*, such as Houdan, Crève-Coeur, etc., are found to give singles in F_2 from rose or pea, the fact will tell distinctly in favour of the presence-and-absence hypothesis.

In spite of the great simplification which this conception effects, there are certain difficulties besetting it. In the first place it is not readily conceivable that rose and pea are in reality nothing but rose elements on a basis of single comb, or pea elements on a basis of single comb. This difficulty is especially great in the case of the Indian Game hen and the Malay hen, for in each of these the comb is so low that it can scarcely be regarded as formed merely by the moulding of even a low single comb into the pea or walnut shape. An extreme difficulty of a similar kind is presented by the fact that the walnut comb has special characteristics not present in either rose or pea, notably the hairs or feathers on its surface. We must recognize, therefore, that if these forms are produced by the addition of shape-elements such as rose, or pea, or both, the result is not merely that which would be expected as the consequence of simple additions.

If we take a somewhat wider range and consider the possibility of applying such presence-and-absence hypotheses to the phenomena of heredity at large further difficulties are encountered, to which it may be well to make reference now. For example, the phenomenon of dominance on the part of a *negative* character is difficult to reconcile with this conception. Such a case occurs even in the combs, for the Fowl × Pheasant is declared to be always without any comb at all†. Striking cases have been recorded by Biffen‡ in Barley crosses, where the absence of female organs in the lateral florets, and even absence of the lateral florets altogether was found to be dominant over the presence of fully developed flowers. Doubtless the statement could be inverted, and it could be suggested that the absence of the florets, etc., was due to the *presence* of some element which prevented their growth, but that would be to abandon all judgments based on the actual appearances, and the terms would become meaningless.

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* Mr C. C. Hurst has suggested to us that a particularly interesting experiment on this point could be made by crossing the Breda fowl with rose or pea. The Breda is described as having *no* comb, but a concavity in its place. In it the wattles are represented as of the size proper to an ordinary single-combed fowl.

† See for instance Tegetmeier, *The Poultry Book*, 1867, p. 165.

‡ Biffen, R. H., *Jour. Agr. Sci.*, 1905, I., p. 255.

The next question is whether hypotheses of presence-and-absence provide a substitute for the conception of compound allelomorphs. In many cases it is likely that characters at first sight reckoned as compound will prove to be made up, like the walnut comb, by the co-existence in one zygote of elements belonging to distinct allelomorphic pairs.

For instance, the colour of a flower, made up of both sap-elements and plastid-elements, might pass for a compound character. Here a disproof is easily obtained from the fact that such a flower when crossed with white will give in F_2 the four types, sap-colour alone, sap-colour + plastid-colour, white alone, plastid-colour alone. Conversely a red crossed with cream (viz. plastid-colour) will give some whites in F_2, as in Sweet-peas and Stocks*. But that there are limits to the extension of this principle is suggested, *e.g.* by the class of cases in which F_2 contains a number of new types, though one of the original pure types may not reappear. Such a case is that of Brown Leghorn × White Leghorn, which gives in F_2 cuckoo, slaty, pile, etc., but the Brown Leghorn colour has not reappeared in about 500 chicks.

Other cases with which any hypothesis of the mere presence and absence of characters cannot apparently deal are to be found among the phenomena of reversion and the peculiarities seen in the behaviour of extracted types; but until the evidence on this part of the subject is more complete it can scarcely be profitably discussed.

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* *Roy. Soc. Evol. Com. Rep.*, 1905, II., pp. 27 and 86.

The Absence of Isomerism in substituted Ammonium Compounds. By H. O. JONES, M.A., Clare College.

[*Received* 15 July 1905.]

Recent work on quinquevalent nitrogen compounds has shown that no stable isomerides, other than optical isomerides, exist except in the case of the α and β phenyl-benzyl-methyl-allyl-ammonium compounds discovered by Wedekind. The α compound is formed in two ways by the addition of benzyl or allyl iodide to the corresponding tertiary amine, while the β compound is formed by the action of methyl iodide on benzyl-allyl-aniline. It has of late become impossible to reconcile the existence of these stereoisomerides with the absence of isomerism in a number of similar compounds, and the writer has examined the β iodide in the hope of gaining further knowledge as to the nature of the isomerism.

If the β iodide be stereoisomeric with the α compound and, further, if it behave similarly in chloroform solution, namely, dissociate partially into benzyl iodide and methyl-allyl-aniline, then in chloroform solution β should be transformed gradually into α, since this is formed by the union of benzyl iodide and methyl-allyl-aniline. It was found that a chloroform solution of β smelt strongly of benzyl iodide, and yet that the β iodide could be recovered unchanged even after the solution had been boiled and allowed to stand for a week.

It is therefore extremely probable that there is a structural difference between these compounds.

Hantzsch and Horn (*Ber.* 1902, **35**, 883) state that the β compound is unsaturated because its chloride unites directly with bromine and decolourizes potassium permanganate solution, neither of which reactions is given by the pure β compound.

The writer has found by analysis (in previous analyses the nitrogen and iodine only had been estimated, and the latter wrongly) and a careful examination of its properties and reactions that the so-called β phenyl-benzyl-methyl-allyl-ammonium iodide is really identical with phenyl-benzyl-dimethyl-ammonium iodide previously described (*Proc. Camb. Phil. Soc.* 1901, XI. 113. *Trans. Chem. Soc.* 1903, **83**, 1409). The reaction between benzyl-allyl-aniline and methyl iodide in which allyl iodide is removed is therefore abnormal.

Since the isomerism observed in certain hydrindamine salts by Kipping has been shown to be due to the acid, there is no known case of stereoisomerism, again excepting optical isomerism, in substituted ammonium compounds containing only one nitrogen atom; and, judging from the large number of negative results, it is extremely probable that such isomerism does not exist.

The rate of recombination and the size of gaseous ions. By J. J. THOMSON, M.A., Cavendish Professor of Experimental Physics.

[*Read* 1 May 1905.]

If two oppositely charged ions start from rest, the attraction between them will cause them to run together, they will form a permanently connected system and cease to act as ions. If, however, the two ions instead of starting from rest start with high velocities, then though they will approach each other, their momentum will cause them to separate again; they will not form a single system, but will continue to act as ions. Now, if the charged ions possess on the average the same kinetic energy as is possessed by the molecules of the gas in which the ions occur, they will be moving with very considerable velocities. Consider two ions the one positively the other negatively electrified; they will attract each other with a force varying inversely as the square of the distance between them; we know from the theory of central forces that if when the ions are at a distance r apart, the kinetic energy due to their relative motion is greater than e^2/r (where e is the charge on an ion), the ions will not describe closed orbits round each other, but will separate and ultimately get an infinite distance apart, they will therefore not recombine. If, however, the kinetic energy T is less than e^2/r, the particles will revolve round each other and recombine. Thus for recombination to take place the ions must approach within a distance r, such that $T < \frac{e^2}{r}$ or $r < \frac{e^2}{T}$. Since T only depends upon the temperature, the limiting distance will be the same for all ions.

Since T is the average kinetic energy of two ions, we have, by the Kinetic Theory of Gases, if N is the number of molecules in a cubic centimetre of gas at a pressure p,

$$p = \tfrac{1}{3}NT, \text{ thus } T = 3p/N \text{ and } e^2/T = Ne^2/3p.$$

At atmospheric pressure and 0° C.,

$$p = 10^6,\ Ne = 1{\cdot}22 \times 10^{10},\ e = 3{\cdot}5 \times 10^{-10};$$

thus

$$Ne^2/3p = r = 1{\cdot}4 \times 10^{-6} \text{ cm.}$$

Thus for recombination to take place the two ions must approach within a distance $1{\cdot}4 \times 10^{-6}$ cm. This distance is exceedingly small compared with the average distance between the ions in any known case of ionization.

To find how long an ion will be before it comes within this distance of an oppositely charged ion, let V be the average velocity of translation of one ion relatively to the other; in the time t the ion will move through a distance Vt relatively to the oppositely charged ion, and every point inside the cylinder, whose volume is $\pi r^2 Vt$, will be at a distance not greater than r from the ion at some point of its path. Thus, if there are n positive and n negative ions per unit volume, the ion in the time t will come within combining distance of $n\pi r^2 Vt$ oppositely charged ions. Thus the number of recombinations per ion per second is $n\pi r^2 V$, and since there are n ions per unit volume, the number of recombinations per unit volume per second is $n^2 \pi r^2 V$. But if α is the coefficient of recombination, this number is by definition equal to αn^2, thus $\alpha = \pi r^2 V$, or substituting the value of r, $6{\cdot}1 \times 10^{-12} V$. If the ions in hydrogen had the same mass as a hydrogen molecule, V at 0° C. would be $\sqrt{2} \times 1{\cdot}8 \times 10^5$ and α $1{\cdot}5 \times 10^{-6}$. This is the right order of magnitude for α, as at atmospheric pressure α for hydrogen, air and carbonic acid is about 10^{-6}. We should expect the value of α determined by the equation $\alpha = 6{\cdot}1 \times 10^{-12} V$ to be too small, for in determining it we have neglected the effect of the surrounding gas on the motion of the ion; this gas will act like a resisting medium and will cause some ions to fall together which would otherwise have escaped from each other's action. We should expect that the effect of the surrounding gas would be greater at high pressures than at low ones, so that α would diminish with the pressure, a result shown very clearly in Langevin's experiments. Since the equation $\alpha = 6{\cdot}1 \times 10^{-12} V$ must give too small a value for α, while the value got by this equation on the supposition that the mass of the hydrogen ion is equal to the mass of the hydrogen molecule is slightly too large, we infer that V for the hydrogen ion must be less than V for the hydrogen molecule, this requires the mass of the hydrogen ion to be greater than that of the molecule; the near agreement of the theory and experiment indicates that the mass of the ion is not a large multiple of that of the molecule.

We have assumed that the kinetic energy of the ion was determined by the temperature of the gas, this will be the case when the electric field is weak, so that the velocity acquired by an ion under the field is small compared with the average velocity due to temperature. In electric fields strong enough to produce discharge the velocity due to the field is much greater than that due to temperature, and the kinetic energy is much greater than that assumed in the preceding calculation. The value of α in this case will be very much less than the value we have calculated, and we see that the value of α will diminish rapidly as the strength of the field is increased.

Let us now find the rate at which a charged molecule would combine with an uncharged one to form a complex ion containing two molecules. In order to get numerical results we shall assume that both the charged and the uncharged molecules are conducting spheres of radius a. If c, the distance between the centres of these spheres, is a considerable multiple of a, then the work required to separate the spheres to an infinite distance is approximately $\frac{1}{2}\frac{e^2a^3}{c^4}$, and if the molecules are to remain paired this must be greater than their kinetic energy. We have seen that mV^2, the kinetic energy of two molecules at 0° C., is equal to e^2/r when $r = 1{\cdot}3 \times 10^{-6}$, hence for union to take place c must be less than $(\frac{1}{2}a^3r)^{\frac{1}{4}}$. If we put $a = 10^{-8}$ cm., c must be less than $2{\cdot}7 \times 10^{-8}$. Thus, if N is the number of molecules, n the number of ions per unit volume, the number of complex ions formed per unit time per unit volume is $\pi Nn(2{\cdot}7 \times 10^{-8})^2V$. The factor $\pi(2{\cdot}7 \times 10^{-8})^2$ is about $1/2{\cdot}5 \times 10^3$ of the factor in the expression for the rate of recombination of oppositely charged ions. The smallness of this factor is, however, far more than compensated by the enormous excess of molecules over ions; the ionization would have to be very intense indeed for the number of ions to amount to $1/10^{12}$ of the number of molecules; thus N will be at least 10^{12} times n, and therefore the number of combinations between a charged and an uncharged molecule will in a given time be at least 5×10^8 times the number of combinations between oppositely charged ions; hence at the very beginning of its career the charged molecule will recombine and form a complex ion; thus a charge will be carried by the complex ion during practically the whole time of its existence as a free charge.

Limit to the size of a gaseous ion. After the charged molecule has increased in size by attracting another molecule, the work required to drag an additional uncharged molecule away from it, starting from a given distance, will be less than the work required to drag a molecule from a single charged molecule. The more complex the ion becomes the less is the work required to drag an uncharged molecule away from it; at a certain stage of complexity the work required to drag a molecule from the ion is less than the kinetic energy possessed by the system in virtue of its temperature, when this stage is reached the ion will cease to attract fresh molecules.

The work w required to separate an uncharged sphere of radius a from a charged sphere of radius b, the spheres being all but in contact, is (see Maxwell's *Electricity and Magnetism*, vol. I. p. 275) given by the equation

$$w = \frac{1}{2}\frac{e^2}{a+b-\left\{2\gamma+\psi\left(\frac{b}{a+b}\right)+\psi\left(\frac{a}{a+b}\right)\right\}} - \frac{1}{2}\frac{e^2}{b},$$

where
$$\gamma = \cdot 57712,$$
$$\psi(x) = \frac{d}{dx} \log \Gamma(1+x).$$

Tables by which we can calculate $\psi(x)$ are given in De Morgan's *Differential and Integral Calculus*, p. 587.

From these tables I find the following values for the work w,

$$a = b, \quad w = \cdot 14 \frac{e^2}{a},$$

$$2a = b, \quad w = \frac{1}{40} \frac{e^2}{a},$$

$$3a = b, \quad w = \frac{1}{156} \frac{e^2}{a}.$$

Now the process of aggregation will stop when w becomes less than the kinetic energy of the system, which at 0° C. is equal to e^2/r, when $r = 1 \cdot 3 \times 10^{-6}$.

Thus, if $a = 10^{-6}$, the work required to separate a molecule from an ion whose radius is $2a$ is greater than e^2/r, but the work required to separate a molecule from an ion whose radius is $3a$ is less than e^2/r; hence in this case the radius of the ion cannot exceed three times the radius of the molecule. If $a = 10^{-7}$ cm., the work required to separate a molecule from an ion whose radius is $2a$ is less than e^2/r; hence in this case the radius of the ion cannot exceed twice the radius of the molecule. The larger the molecule the smaller will be the ratio of the size of the ion to that of the molecule, with very large molecules it is probable that the molecule and the ion are identical. The ions in different gases will thus not differ so much in size as the molecules of the gases. Since the kinetic energy is greater at a high temperature than at a low one, the process of the aggregation of molecules in the ion will stop at an earlier stage the higher the temperature, so that the ions will be simpler at high temperatures than at low ones.

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1904—1905.

ANNUAL GENERAL MEETING.

October 31*st*, 1904.

In the Optical Lecture Room.

DR BAKER, PRESIDENT, IN THE CHAIR.

The following were elected officers for the ensuing year:

President:

Professor Marshall Ward.

Vice-Presidents:

Professor Thomson.
Professor Liveing.
Dr E. W. Hobson.

Treasurer:

Mr H. F. Newall.

Secretaries:

Mr A. E. Shipley.
Mr G. B. Mathews.
Mr P. V. Bevan.

Other Members of the Council:

Mr H. J. H. Fenton.
Mr A. Berry.
Dr J. E. Marr.
Mr D. Sharp.
Professor Larmor.
Mr W. C. D. Whetham.
Mr G. H. F. Nuttall.
Mr F. H. Neville.
Mr S. Ruhemann.
Dr Anderson.
Mr A. Hutchinson.
Mr F. F. Blackman.

The names of the Benefactors were recited.

The following was nominated an Associate of the Society:

H. N. Russell, King's College.

The following Communications were made:

1. On the Dimorphism of the English species of *Nummulites*. By J. J. LISTER, M.A., St John's College.

2. A problem concerning Wood and lignified Cell-walls. By H. MARSHALL WARD, Sc.D., Professor of Botany.

3. The Pine-Apple Gall of the Spruce: Note on the early stages of its development. By E. R. BURDON, B.A., Sidney Sussex College.

4. Note on some peculiar Features in Seedlings of *Peperomia*. By A. W. HILL, M.A., King's College.

5. On certain Quintic Surfaces which admit of Integrals of the First Kind of Total Differentials (Second Paper). By A. BERRY, M.A., King's College.

November 14*th*, 1904.

In the Cavendish Laboratory.

PROFESSOR MARSHALL WARD, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

N. R. Campbell, B.A., Trinity College.
J. H. Field, B.A., St John's College.

The following Communications were made:

1. The charge of the α rays from Polonium. By J. J. THOMSON, M.A., Cavendish Professor of Experimental Physics.

2. On the dynamical significance of Kundt's Law of selective Dispersion in connexion with the transmission of the energy of Trains of Dispersive Waves. By J. LARMOR, M.A., Lucasian Professor of Mathematics.

3. The chlorination of α Picoline. By W. J. SELL, M.A., Christ's College.

4. An attempted synthesis of Uric Acid. By H. J. H. FENTON, M.A., Christ's College.

5. The diffusion of Hydrogen through Palladium. By O. W. RICHARDSON, M.A., Trinity College.

6. Optically active Nitrogen Compounds. By H. O. JONES, M.A., Clare College, and Miss M. B. THOMAS.

November 28*th*, 1904.

In the Optical Lecture Room.

PROFESSOR MARSHALL WARD, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

J. J. E. Durack, B.A., Trinity College.
W. G. Fearnsides, M.A., Sidney Sussex College.

The following was elected an Associate of the Society:

H. N. Russell, King's College.

The following Communications were made:

1. Remarks on *Piroplasmosis* with exhibition of specimens. By G. H. F. NUTTALL, M.A., Christ's College.

2. Exhibition of new and rare Arachnids taken near Cambridge. By C. WARBURTON, M.A., Christ's College, and N. D. F. PEARCE, M.A., Trinity College.

3. The Inheritance of Tortoiseshell and other Colours in Cats. By L. DONCASTER, M.A., King's College.

January 30*th*, 1905.

In the Cavendish Laboratory.

PROFESSOR MARSHALL WARD, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

B. Hopkinson, M.A., Trinity College, Professor of Mechanism and Applied Mechanics.
S. A. McDowell, B.A., Trinity College.

The following Communications were made:

1. On the non-electrification of γ-rays. By J. J. THOMSON, M.A., Cavendish Professor of Experimental Physics.

2. Are metals made radio-active by the influence of Radium radiation? By J. J. THOMSON, M.A., Cavendish Professor of Experimental Physics.

3. Note on the positive leak from hot Platinum in Air. By O. W. RICHARDSON, M.A., Trinity College.

4. Are metals made radio-active by the influence of Radium adiation? By Professor BUMSTEAD. (Communicated by Professor Thomson.)

5. Some methods of increasing the Spark length of the Wimshurst Machine. By B. J. PALMER. (Communicated by Professor Thomson.)

February 13*th*, 1905.

In the Optical Lecture Room.

PROFESSOR MARSHALL WARD, PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

K. Lucas, B.A., Trinity College.

The following Communications were made:

1. Orthogonal and other special systems of Invariants. By Major P. A. MACMAHON.

2. Reduction of generating functions by means of complex integration. By G. B. MATHEWS, M.A., St John's College.

February 27*th*, 1905.

In the University Chemical Laboratory.

MR F. H. NEVILLE IN THE CHAIR.

The following were elected Fellows of the Society:

R. H. Biffen, M.A., Emmanuel College.
E. H. Hankin, M.A., St John's College.

The following Communications were made:

1. Soluble sodium dihydroxytartrate. By H. J. H. FENTON, M.A., Christ's College.

2. Studies on unsaturated Ketonic Compounds. By S. RUHEMANN, M.A., Gonville and Caius College.

3. Some Compounds of Guanidine with Sugars. By R. S. Morrell, M.A., Gonville and Caius College, and A. E. Bellars, B.A., Magdalene College.

4. The influence of strong electromagnetic fields on the spark spectra of some metals. By J. E. Purvis, M.A., St John's College.

March 13*th*, 1905.

In the University Chemical Laboratory.

PROFESSOR MARSHALL WARD, PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

H. T. Ferrar, M.A., Sidney Sussex College.

The following Communications were made:

1. On the relation in size between the megalosphere and the microspheric and megalospheric tests in the Nummulites. By J. J. Lister, M.A., St John's College.

2. The Penguins of the Antarctic. By E. A. Wilson, M.B. (Communicated by Mr A. E. Shipley.)

3. The old Moraines of South Victoria Land. By H. T. Ferrar, M.A., Sidney Sussex College. (Communicated by Mr A. E. Shipley.)

4. Notes on a collection of Parasites from the Museum of University College, Dundee. By A. E. Shipley, M.A., Christ's College.

5. On the Maturation of the Egg and early development in certain Sawflies (*Tenthredinidæ*). By L. Doncaster, M.A., King's College.

6. Densities of the Earth's Crust beneath Continents and Oceans compared. By Rev. O. Fisher, M.A., Jesus College.

May 1*st*, 1905.

In the Cavendish Laboratory.

PROFESSOR LIVEING, VICE-PRESIDENT, IN THE CHAIR.

The following Communications were made:

1. On the striation of the positive column in discharges through vacuum tubes. By J. J. Thomson, M.A., Cavendish Professor of Experimental Physics.

2. On the rate of recombination and the size of gaseous ions. By J. J. THOMSON, M.A., Cavendish Professor of Experimental Physics.

3. Some physical properties of Sodium Vapour. By P. V. BEVAN, M.A., Trinity College.

4. A null method of measuring small ionizations. By N. R. CAMPBELL, B.A., Trinity College.

5. The reflexion of Sound at a Paraboloid. By Rev. H. J. SHARPE, M.A., St John's College.

May 15*th*, 1905.

In the University Botanical Laboratory.

PROFESSOR MARSHALL WARD, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

T. G. Bedford, M.A., Sidney Sussex College.
C. Chittock, B.A., Trinity College.
J. Aldren Wright, M.D., Sidney Sussex College.

The following Communications were made:

1. Exhibition of Lantern Slides of Fungi. By H. MARSHALL WARD, Sc.D., Professor of Botany.

2. Infection phenomena in various species of Uredineæ. By I. B. P. EVANS, Sidney Sussex College. (Communicated by Professor Marshall Ward.)

3. The abortive development of the Pollen in certain cross-bred Sweet Peas. By R. P. GREGORY, M.A., St John's College.

4. Crosses between fully fertile varieties of Barley and varieties bearing unisexual and sexless flowers. By R. H. BIFFEN, M.A., Emmanuel College.

5. The Seed-bearing Habit in the Lyginodendreæ. By E. A. N. ARBER, M.A., Trinity College. (Communicated by Professor Marshall Ward.)

6. Experiments on penetrating Radiation. By H. L. COOKE, Emmanuel College.

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PROCEEDINGS

OF THE

CAMBRIDGE PHILOSOPHICAL SOCIETY.

VOL. XIII. PART IV.

[MICHAELMAS TERM 1905.]

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31 *January*, 1906.

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3. Complete sets of the *Proceedings*, Volumes I.—XIII., Part 1—4, may also be obtained on application to the Secretaries o the Society.

4. Letters and Communications for the Society should b addressed to one of the Secretaries,

Mr A. E. Shipley, Christ's College. [Biological.]
Rev. E. W. Barnes, Trinity College. [Mathematical.]
Mr P. V. Bevan, Trinity College. [Physical.]

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New Museums,
Cambridge.

6. Authors of papers are informed that the Illustrations an Diagrams are executed as far as possible by photographic "process" work, so drawings should be on a large scale and on smooth whit Bristol board in Indian ink.

7. Members of the Society are requested to inform th Secretaries of any change of address.

Reprinted in The Netherlands

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

On a Well-sinking, at Graveley, near Huntingdon. By the Rev. O. FISHER, M.A., F.G.S., Hon. Fellow of Jesus College.

[*Read* 30 October 1905.]

THE position of this well on the six-inch Ordnance Map is half-way between the "R" in "Rectory" and the "B" in "B.M. 154·5," which figure may be taken as the height in feet of the locality above Ordnance datum. It seems desirable to put on record the strata which have been proved in this an extreme western corner of the county of Cambridge.

The work was started in the spring of 1905, under the advice of a water-finder. The sinking was begun in Boulder clay, and after passing through 50 feet of this the Oxford clay was encountered. This was dug into to a depth of 94 feet from the surface, and then bored further to the depth of 302 feet from the surface. A bed of stone, probably Cornbrash, was now reached, and was punched through for 1½ feet. A foot more was then bored in clay, when a hard rock was encountered, and the work was abandoned.

The section then was,

	feet
Boulder clay	50
Oxford clay.....................	252
Cornbrash	1½
Clay	1
Rock (thickness not proved)	
	304½

A little water came into the bore hole, and it was salt.

I have visited this well twice, once on July 5, when the Boulder clay had been pierced, and the boring was in progress in Oxford clay at the depth of 95 feet; but my son, the Rev. O. P. Fisher, who became Rector of Graveley after the work was commenced, had previously from time to time brought me specimens.

I visited the well again October 14, a few days after the work was stopped, and looking over the stuff that had been thrown out found little that was new. But I brought away some of the mud obtained by punching at the bottom of the bore-hole, thinking that it might afford evidence of the nature of the rock at that depth. It is, however, necessarily mixed with Oxford clay scraped off the bore-hole higher up by the action of the rods. This mud I have washed, and think that the residue proves that the rock did not consist simply of indurated Oxford clay, but was Oolitic. A few small fragments which my son obtained from the workmen point in the same direction, and are very similar to the matrix of some Cornbrash fossils in the Sedgwick museum*. This identification is important, as giving the thickness of what remains of the Oxford clay at the locality.

The Boulder clay is of a dark blue colour when wet, and somewhat sandy, full of rounded and scratched pebbles of chalk, mostly small, a few larger pieces like potatoes, bits of sharply broken black flint, but no flint pebbles. Red chalk was not uncommon, and bits of Kimmeridge shale, highly inflammable. The rocks were almost entirely cretaceous or Jurassic. Mr Harmer, however, reported carboniferous limestone among some specimens sent to him. Some larger stones were found towards the bottom of the Boulder clay which were probably derived septaria.

Many boulders are to be seen scattered about the village. I noticed a block of a basaltic rock, and a well-scratched piece of carboniferous limestone, at the entrance to Mr Ashcroft's farm.

The Oxford clay contained many rather undersized shells of *Gryphea dilatata* with both valves united.

The Cornbrash consisted chiefly of comminuted shells with a few Oolitic grains. Occurring, as it does, 150 feet below sea level, the colour is dark instead of the brownish-yellow one commonly finds in Oolitic stone. This is, I think, the first instance of a member of the lower Oolites being met with in East Anglia.

I was told that water had been reached at the depth of 60 feet at Yelling, a mile to the south of Graveley. I think it is possible that the surface of the Oxford clay may sink downwards in that direction, forming a shallow basin so as to hold up water in the base of the Boulder clay, which there seems to be 75 feet thick. There is at Yelling a natural chalybeate spring called "Nill well," which rises at about 100 feet above Ordnance datum.

---

* From Turvey 8 miles W.N.W. from Bedford, on the western confines of Bedfordshire.

I have hitherto been inclined to think that the Boulder clay of East Anglia has been deposited from floating ice, having formed my opinion mainly from what I have seen in the contorted drift of the Norfolk cliffs. But the Boulder clay at Graveley has a different character; and it is difficult to account for the rounded and scratched lumps of hardened chalk, of Kimmeridge shale, and other rocks impacted in it, except on the hypothesis that it has been shoved about in all directions in mass. The smallest enclosures bear scratches pointing in every direction upon their surfaces. This is very noticeable on the scraps of Kimmeridge shale.

A somewhat corroborative piece of evidence is that the Boulder clay reposed upon an indurated layer in the Oxford clay, the Boulder clay having apparently sheared off the softer clay till it was arrested by the harder rock.

On a Portable Gold-leaf Electrometer for Low or High Potentials, and its application to Measurements in Atmospheric Electricity.

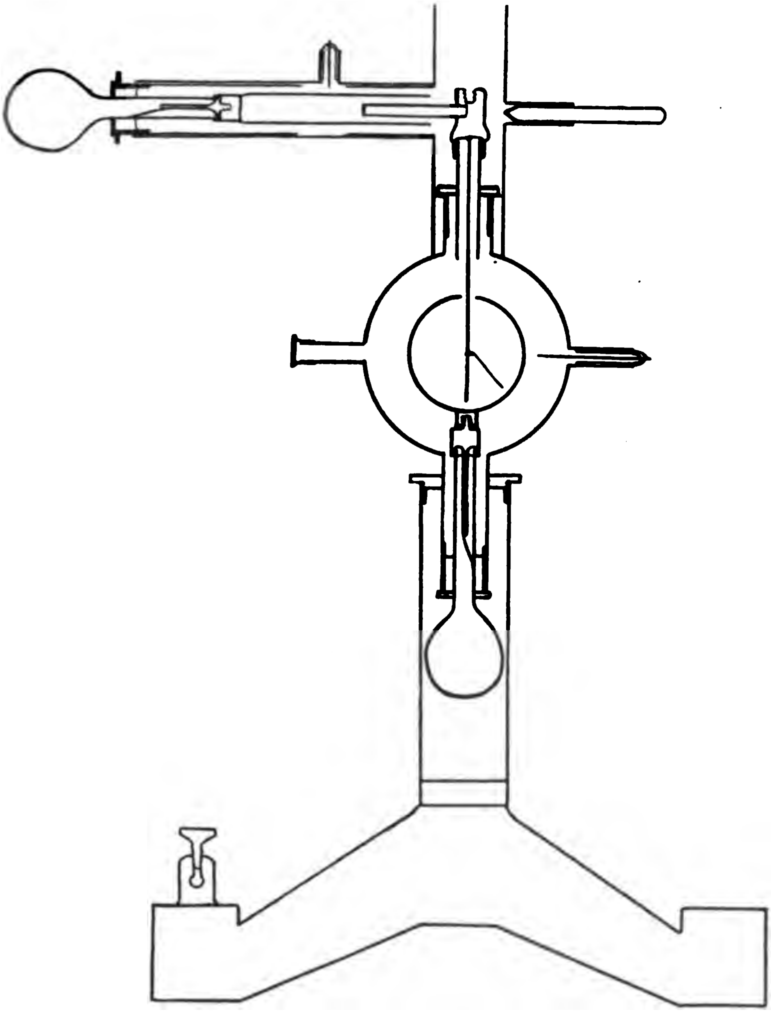
By C. T. R. WILSON, M.A., F.R.S., Sidney Sussex College.

[*Read* 30 October 1905.]

THE instrument has an outer earth-connected case and an inner insulated case supported on the neck of a small quartz Leyden jar, with the inner coat of which it is connected, the outer coat being connected to the outer case of the instrument. For ordinary use the inner case is charged to a positive potential of about 50 volts. The insulating power of the Leyden jar is very high, so that in spite of its comparatively small capacity (less than 100 cms.) the potential does not fall more than a small fraction of a volt in 24 hours.

When the gold leaf is earthed its potential differs from that of the inner case by 50 volts, and is therefore in a deflected position. If we raise its potential the deflection will diminish (the charge of the inner case being positive), if we give it a negative potential the deflection will increase. If we adjust the observing microscope so that the gold leaf is near the centre of its micrometer scale when its potential is zero, the instrument is in condition to measure potentials in the neighbourhood of zero up to about 5 volts positive or negative. For greater potentials the leaf will be outside the field of view of the microscope; over the range available the scale is sensibly uniform. If the gold leaf be raised to a gradually increasing positive potential it will continually fall and will pass beyond the scale of the microscope when a potential of a few volts has been attained. When the potential has risen to 50 volts, that of the surrounding case, the deflection is a minimum, and beyond this point it will again increase. The gold leaf will come into sight again in the microscope when the potential reaches within a few volts of 100, and at 100 volts will be the same as for zero potential, the difference of potential between leaf and case being now the same as at first, the sign being, however, reversed. The instrument is then available over a second range of potentials, say from + 95 to + 105 volts. The displacement of the leaf for an increase of potential of one volt will be the same as in the neighbourhood of zero, the direction of the movement, however, being opposite.

The actual form of the instrument is shown in the figure, which is drawn approximately to scale.



The outer case is a short brass cylinder 4·6 cms. in diameter, the ends being closed by vertical glass plates; the distance from back to front is 3 cms. The inner case is concentric with the outer, and is 2·5 cms. in diameter and 2·2 cms. from back to front. The ends are again closed by glass plates. The glass plates of both inner and outer plates are silvered internally, a horizontal strip of the silvering being, however, removed from each plate to enable the gold leaf to be seen. The inner case screws on to the top of a brass rod passing through the neck of the quartz Leyden jar and connected with its inner coat. The jar is enclosed in the hollow pedestal which supports the instrument. The top of the pedestal, with the outer case of the electrometer which it supports, can be screwed off; and into a tube about 1 cm. wide projecting from the under side of this cover a tube cemented round the base of the quartz tube slides stiffly. This admits of the inner case of the electrometer being adjusted to be concentric with the outer.

The quartz bulb is rather less than 2 cms. in diameter, the tube 4 or 5 mm. in external diameter. The bulb and tube are silvered internally, contact being made between the inner coat and the brass rod by means of a strip of tinfoil gummed at the ends. The outer coat is of tinfoil. I have found such Leyden jars, which have an electrical capacity of from 50 to 100 cms., extremely useful on account of their almost perfect insulation. The bulbs were obtained from Messrs Baird and Tatlock, and have been made with the blowpipe

The gold leaf is about 1·1 cms. long and very narrow. That in use at the present time is about $\frac{2}{10}$ of a mm. wide, but it is fairly easy to cut leaves of this length $\frac{1}{10}$ of a mm. or less in width. The point of attachment of the leaf is a little to one side of the centre of the inner case, being at the tip of a small projection on the supporting wire. This wire is about 1 mm. in diameter, the projection reaching a distance of about $\frac{1}{2}$ a mm. from its surface. The wire passes through an aperture at the top of the inner case about 5 mm. in diameter, then through a quartz tube, by means of which it is insulated from the outer case; it is attached to a brass terminal closing the top of this tube. The quartz tube fits into a short tube projecting from the bottom of the terminal, so that a very small quantity of cement (sealing-wax) is necessary to fix it tightly. It is also cemented about the middle of its length within a hole through the flat end of a brass tube about 1 cm. wide and 1 cm. long. This slides stiffly in a short tube at the top of the outer case of the electrometer. It is thus easy at any time to remove or replace the gold leaf.

For the purpose of charging the inner case and Leyden jar a side tube is attached to the outer case; into this is fixed a glass tube with a platinum wire sealed through its outer end and projecting from its inner end to within a few mm. of the wall of the inner case. The inner case is then charged simply by sparking in from an electrophorus or electrified rod till the desired deflection is obtained with the gold leaf earthed. To another side tube a drying bottle may be attached.

Fitting tightly over the vertical tube attached to the top of the outer case of the electrometer is a brass tube 1·5 cm. wide in internal diameter and 5½ cms. long. On one side is attached a small tube containing an earthing rod for connecting at will the gold leaf terminal to earth. Opposite it is a wider tube containing a sliding condenser to be described later. There is in addition a third side tube (not shown in the diagram) for the attachment of a drying bottle. In experiments on ionisation or radio-activity the testing vessel would be arranged to fix directly on to the top of the wide vertical tube, the testing electrode being screwed into the top of the terminal inside this tube.

One of the annoyances attending the use of an ordinary gold leaf electroscope for measuring leaks is the difficulty of charging it up to the desired initial potential. One of the uses of the sliding condenser mentioned above is to give complete control over the charging of the gold leaf system. This condenser consists of an inner rod fixed at right angles to the terminal of the gold leaf, and surrounding it a concentric tube which can slide parallel to its length to give variable capacity. The rod is about 2½ cms. long and its diameter is about 2½ mm. The tube has an internal diameter of about 6 mm. and has a range of movement of 3 cms. The tube is kept at a negative potential by being attached to the inner coat of a quartz Leyden jar like that already described. The displacement of the tube of the cylindrical condenser can be read on a scale outside the wider tube in which the moving part slides. The condenser is charged by an ebonite rod through a platinum wire, the method being the same as that adopted for charging the inner case of the electrometer itself. The movement of the tube of the sliding condenser is limited by stops, so that when fully in or out its position is definite. For the purpose of charging the gold leaf system the condenser is used as a kind of electrophorus. The slider is pushed in, the gold leaf terminal earthed by the earthing rod, and the slider pulled out against its stops. The leaf is thus left with a positive charge. It is convenient to give the condenser such a charge that the operation of making the earth connection when it is fully in and then drawing it out to its limit leaves the gold leaf at a convenient part of the field of view of the microscope, its potential being the higher potential corresponding to this position, *e.g.* 105 volts if the inner case of the electrometer is at 50 volts. When this has once been done the operation of charging the gold leaf to this definite potential of, say, 105 volts may be repeated indefinitely, by simply pushing the condenser in up to its stops, earthing the gold leaf and pulling the condenser out to the extremity of its range again. Thus in measuring the leak from an electrode initially at 105 volts the above operation would be repeated between the successive observations. To charge to a low potential, say 5 volts, the slider would be moved in until the gold leaf had moved across in the negative direction over the proper number of scale divisions, the earth connection would then be made and the slider pulled fully out again. It is convenient to make the actual measurements always with the slider completely out, as in this position the condenser tube is too far removed from the rod to have any influence upon its potential. The gold leaf reading is then entirely uninfluenced by any possible failure of the condenser insulation.

The sliding condenser is also useful as a compensator. We may use it, for example, to measure charges too large to be directly measured by the displacement of the gold leaf. For we may always proceed by sliding in the condenser (originally fully out) till the potential is reduced to zero. A reading of the scale attached to the slider will give us the charge, when the instrument has once been standardised, if the potential of the condenser tube be kept always the same. The charge acquired by the gold-leaf system has in fact been compensated by that which has been abstracted into the rod of the condenser on account of increased capacity, the difference of potential remaining constant. The change of capacity per mm. is found to be sensibly constant over about 2 cms. of the scale, while if the condenser be moved further out it falls off very rapidly.

The electrometer stands on a substantial iron base. On one of the three feet of this base is fixed an upright iron rod, to which is clamped a cradle which carries the microscope.

The displacement of the gold leaf for a change of potential of one volt depends of course on the particular gold leaf used; it amounts to about $\frac{1}{10}$ of a mm. per volt. With the magnifying power at present attached, 6 eye-piece scale divisions = 1 volt, but it would be easy to increase considerably the magnification. The capacity of the gold leaf and its connections does not much exceed 2 cms.

The readings of the instrument are very steady; owing, no doubt, to the double case there is apparently a complete absence of disturbances due to convection currents. The gold leaf takes up its position of equilibrium within a small fraction of a second, so that very rapid potential changes can be followed.

The zero remained steady when the instrument, mounted on a camera tripod, was tested in the open air in a fairly high wind*.

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* I have to thank the Cambridge Scientific Instrument Co. for their help in the construction of the instrument.

Application of the Instrument to Measurements in Atmospheric Electricity.

Let us fix a horizontal conducting plate, a few square cms. in area, by means of a vertical rod to the terminal of the electrometer. Let it be surrounded by a considerably large plate lying in the same plane and forming a guard ring for it. On this guard plate place a conducting cover, of which the roof is some cms. above the flat plate. Let the electrometer be momentarily earthed by means of the earthing rod and let the cover be then removed. If this operation has been performed in an open field, the electrometer will, under normal fine weather conditions, have its potential considerably raised, and the gold leaf will go out of the field of view. The potential can be at once brought back to zero by sliding in the tube of the condenser as described above. The testing plate being now at zero potential, the charge on its exposed surface is the same as if it were earth connected. This charge is given at once by the reading of the sliding condenser if it has been previously standardised. The charge per unit area of the plate will be proportional to the strength of the earth's field.

Let us maintain the potential at zero for some minutes, compensating for any change in the earth's field by the proper movement of the slider. Then let us replace the cover and draw out the condenser tube to its standard position. The rise of potential indicated by the gold leaf multiplied by the capacity of the system gives the charge gained by the plate in a known number of minutes when kept at zero potential, that is, under the same conditions as when earth connected.

We have thus obtained by these observations a measurement of the charge on our horizontal plate when earth connected and of the current through its surface. The ideal condition would of course be to have the plate on a level with the surface of the ground. We would then have both the charge per unit area of the earth's surface $= \sigma$ (and therefore also the magnitude of the electric field $= 4\pi\sigma$) and also the vertical current per square cm. of the earth's surface at the place of observation.

It should be noticed that the earth-air current, as obtained by the above method, includes not only the current carried by ions under the action of the electric field, but also any charge brought down by falling dust particles or rain drops, but not by any current that might be carried by corpuscles with sufficient penetrating power to traverse the testing plate unabsorbed.

A Series of Optically Active Nitrogen Compounds containing the Allyl Group. By Miss M. B. THOMAS, Girton College, and H. O. JONES, M.A., Clare College.

[*Received* 27 November 1905.]

IN an earlier communication (*Proc.* 1904, XIII. 33) an account was given of the preparation of some salts of substituted quaternary ammonium bases and their resolution into their optically active constituents, which was undertaken with the object of determining whether the relation between rotatory power and constitution would be more evident in the case of ions than it is in complete molecules. The salts then examined were those in which three of the hydrogen substituting groups were phenyl, methyl and benzyl, the fourth atom of hydrogen being successively replaced by the isopropyl and isoamyl radicals. A comparison of the molecular rotatory powers of the basic ions with those obtained for the corresponding ethyl compound shows that a maximum is reached when the isopropyl group is substituted, the rotatory power of the ion containing this radical being greater than that of the ions containing the ethyl or isoamyl radicals.

A more complete homologous series of five compounds has been prepared, each of which contains the radicals phenyl, methyl, and allyl, while the fourth ammonium hydrogen atom is replaced successively by the ethyl, propyl, isopropyl, isobutyl, and isoamyl radicals.

The compounds have been resolved into their optically active components by the method of Pope and Peachey. The d.-camphor-sulphonate or d.-brom.-camphor-sulphonate of the base is made by the interaction of the quaternary substituted iodide with the silver salt of the acid in acetone or other suitable solvent, and the resulting salt is recrystallised until a constant rotatory power is obtained. A tabulated list of the results obtained is given below. The values of the molecular rotatory power are for dilute aqueous solutions at 15° C.; the values for the ions all decrease slightly with increasing temperature.

Base	M.P. of iodide	Salt prepared	M.P. of salt	Solvent	$[M]_D$ for salt	$[M]_D$ for basic ion
Phenyl methyl allyl ethyl ammonium	—	*d.* brom camphor sulphonate	138–9° C.	acetone	+291°	+14°
Phenyl methyl allyl propyl ammonium	110° C.	,,	169–170° C.	,,	+381·5°	+104·5°
Phenyl methyl allyl isopropyl ammonium	172° C.	*d.* camphor sulphonate	167–8° C.	mixture acetone and toluene	−49·3°	−102°
Phenyl methyl allyl isobutyl ammonium	143° C.	,,	173° C.	acetone	−6·26°	−58°
Phenyl methyl allyl isoamyl ammonium	135° C.	,,	155° C.	methylal	+33·5°	−18·2°

The value of $[M]_D$ for the d.-brom.-camphor-sulphonic ion has been taken as 277°, and that of the d.-camphor-sulphonic ion as 51·7°.

In this, as in the benzyl series of compounds, there is a large increase in the molecular rotatory power of the basic ion when the isopropyl is substituted for the ethyl group. Here the values for the propyl and isopropyl compounds, though not identical, are very close together, while the substitution of the isobutyl and isoamyl radicals for propyl or isopropyl results in a lowering of the rotatory power. The rotatory powers are all less than that of the phenyl-benzyl-methyl-allyl ammonium ion 166°. Other similar series of compounds are being investigated, in which the methyl group is replaced by ethyl and isopropyl, in the hope that the same kind of relation may also be observed in these.

The Effect of Hydrogen on the Discharge of Electricity from Hot Platinum. By O. W. RICHARDSON, M.A., Trinity College.

[*Read* 27 November 1905.]

IT has been known for some time that hydrogen has a very marked power of increasing the negative ionisation from hot metals. The effect in the case of platinum has been investigated in detail by H. A. Wilson*, who came to the conclusion that the increase was due to the hydrogen dissolved in the metal. The present paper is a preliminary account of experiments made to shed further light on the matter.

The method adopted was to measure the saturation current from the outside of a hot platinum tube to a surrounding cylinder in air, both when hydrogen was diffusing through the tube from inside and when it was not. The surrounding air would of course oxidise the hydrogen as fast as it came out, and thus keep the surface in a constant state, so that presumably any change in the leak would be due to the hydrogen, and not to a change in the platinum surface.

Even when hydrogen was allowed to diffuse through the tube at a rate equal to 2 c.c. at 76 cms. pressure per sq. cm. of surface per minute, not the slightest change could be detected in the value of the negative leak. This result points strongly to the conclusion that the increase in the ionisation observed in an atmosphere of hydrogen is not due to the presence of the hydrogen in the metal as hydrogen, but to some alteration it produces in the superficial regions of the metal.

Two experiments which support this conclusion may be cited. As is well known, a platinum wire which has been heated for a long time in oxygen gives small but definite ionisations both positive and negative. On replacing the oxygen round such a wire by hydrogen the writer found that, though the positive ionisation was increased somewhat at first, it soon began to fall, whilst the negative leak increased simultaneously. Both these effects seemed far too slow to be accounted for by the rate at which hydrogen entered the wire (which was only ·1 mm. thick), though that explanation is not excluded with absolute certainty.

The other experiment consisted in putting a big negative potential on the wire just mentioned. This was found to rapidly bring down the leak in hydrogen to a small value. If the negative potential on the wire was now reduced it recovered its ionising power, slowly at first and then more rapidly, until it settled down to a steady value. Charging the wire positively did not appear to reduce its ionising power.

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* *Phil. Trans.* A, Vol. CCII. p. 243.

These results certainly seem to indicate that the hydrogen acts by producing a change in the wire, which takes a long time to complete itself. They appear to be capable of explanation if we assume that the positively charged hydrogen forms a double layer at the surface of the platinum which helps the corpuscles out. Further experiments are in progress, which, it is hoped, will decide whether or not this is the correct explanation.

Experiments were also made with the platinum tube described above to see whether the diffusion of hydrogen through the tube had any effect on the positive ionisation produced by the outer surface. The hydrogen was found to increase the positive leak at all the temperatures tried, the relative increase being greater the higher the temperature. At 1200° C. the ratio of the leak with hydrogen at atmospheric pressure to that with air inside the tube was very nearly 2·2. At constant temperature the increase in the leak was found to be proportional to the square root of the pressure of the hydrogen inside the tube, *i.e.* directly proportional to the amount of hydrogen diffusing through. The effect of the hydrogen diffusing out of the metal on the positive leak offers, therefore, a very marked contrast to its effect on the negative leak.

These results tend to indicate that the hydrogen inside the metal, which is known from other considerations to be in the atomic state, is positively charged. Only a small fraction (10^{-7}) of the hydrogen comes out in the ionic form, but we might expect most of it to be discharged in escaping from the metal. In any case, charged hydrogen would have much greater difficulty in escaping from a metal at lower temperatures, which may account for the hydrogen set free from palladium not being ionised. These considerations are also in agreement with the fact that electrolytic hydrogen, which is positively charged, is capable of diffusing into some metals—for instance, iron—at ordinary temperatures.

Experiments on the retention of an electric charge by gases.

By W. A. DOUGLAS RUDGE, M.A., St John's College.

[*Read* 27 November 1905.]

IF an ordinary vacuum tube or incandescent lamp is partially coated with tinfoil, it will act as a condenser, the tinfoil acting as one conductor, and the residual air in the tube as the other.

In the following account of the experiments the apparatus used will be generally referred to as the condenser.

The experimental condenser was made by partially coating a Wurtz distillation flask with tinfoil, and closing the mouth with an india-rubber stopper, through which passed a short wire having a knob at one end. See Fig. 1. The flask could be conveniently exhausted by the side tube. When this was done, and the knob brought near one of the terminals of a Wimshurst machine, the condenser could be charged and discharged in the ordinary way, a bright spark passing on discharge and a glow filling the tube just for a moment. The air in the flask at a pressure of about 1 cm. acts as a conductor, so that, on bringing a wire from the

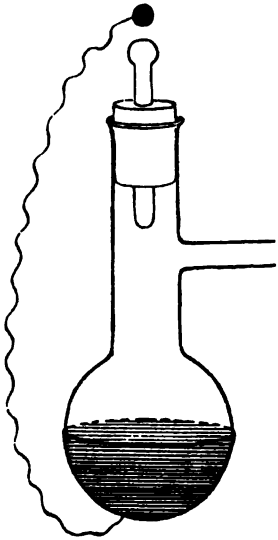

FIG. 1.

tinfoil to the knob, the condenser can be discharged. While an unexhausted vessel does not act as a condenser (any slight charge upon the knob being removed by earthing) it was found that if exhausted after charging, a discharge could be obtained. The knob of the unexhausted condenser was put in connection with the Wimshurst for a few seconds, then earthed to remove any free charge. It was then rapidly exhausted to about ½ cm. pressure, and on joining the tinfoil to the knob, a discharge almost as strong as though the vessel had been exhausted at first was obtained. Several residual discharges could be obtained.

If the pressure were not rapidly reduced, the charge leaked away, but with large condensers, and by working in a very dry room, a discharge could be obtained after many hours. If the vessel was exhausted before charging, then the charge has remained more than twelve hours.

Since a vacuum would thus act as a conductor, it was thought that a condenser might be constructed without any metallic coatings. This was realized by making a tube of the form of Fig. 2. The two chambers have no connection with each other, and could be exhausted simultaneously by one pump

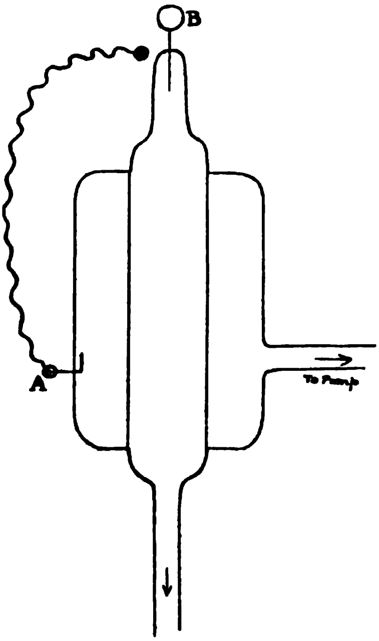

Fig. 2.

or separately by two pumps. To use the apparatus, one of the electrodes *A* or *B* was put in connection with the earth, and the other presented to the Wimshurst. If the pressure was below 1 cm., then, on bringing a wire connected with *A* near to *B*, a bright spark passed, and both chambers were filled with a glow. "Residual" charges could be obtained as with an ordinary condenser. After the condenser has been charged, the air may be removed from it, fresh air admitted, then re-exhausted, repeating this time after time, discharging by earthing the knob after each exhaustion, and still some slight charge will be left. An endeavour was made to ascertain whether the air itself inside the flash was charged, so with this object, the air on the way from the flash to the pump was made to pass through an insulated metal tube (which was sometimes packed with iron filings) connected to an electroscope, but no signs of electrification resulted. Exploring electrodes were also inserted in the condenser, without any definite result. The next essay was to find out whether the air removed from the flask during exhaustion possessed any conducting power, and for this purpose, it was drawn through a glass tube containing two conductors separated a short distance, one being in connection with a charged electroscope, the other in connection with the earth. Only a very slight leak of the charge from the electroscope was observed, so that the air had not become appreciably conducting, or rather, those portions removed did not possess conducting power. Experiments were undertaken with gases other than air, viz. hydrogen, oxygen, carbon-dioxide, carbon-monoxide, and sulphur-dioxide, when it was seen that with the exception of hydrogen, the behaviour of the gases was similar to that of air. Hydrogen acted somewhat differently from the rest, for the charge would not remain as long as with the other gases.

Another set of experiments was conducted in a somewhat different manner. After placing the unexhausted condenser in connection with the Wimshurst, a wire was used to connect permanently the knob and the tinfoil, and then the apparatus was exhausted. When the pressure fell to about 8—10 cm. a faint glow was seen to fill the bulb, and as the pressure was further reduced, occasional bright flashes were observed. The glowing effects lasted for quite a long time (several minutes), and if air were admitted to destroy the glow, it reappeared after reexhaustion, without any further charge being given to the apparatus. A curious effect followed the use of hydrogen, viz. it was difficult to get the apparatus to work with air afterwards. Until, in fact, all traces of hydrogen had been removed by repeated exhaustions and fillings with air, the glow discharge could not be obtained.

Polarization phenomena in the Eclipse 1905 August 30. By H. F. Newall, M.A., F.R.S., Trinity College.

[*Read* 13 November 1905.]

Descriptions and illustrations of the visual observations and photographic records obtained at Guelma in Algeria were given with a view of showing the kind of material collected for the detailed study of the phenomena. It was mentioned that the visual observations combined with the photographic records proved that the plane of polarization of the light diffused by the earth's atmosphere during the eclipse was very nearly horizontal. (This evidence conflicts with the conclusion of M. Salet, who observing at Robertville, about 37 miles from Guelma, had found that the plane of atmospheric polarization was vertical.) Photographic records, obtained with a Savart polariscopic camera for the purpose of quantitative measurements of the relative amounts of polarized and unpolarized light in the corona, showed that the polarized portion of the atmospheric light was equal in intensity to the radially polarized portion of the coronal light at a distance of about 1¼ diameters from the sun's limb. Thus the phenomena of a radially polarized corona seen through a plane polarized atmosphere are somewhat complicated. The results obtained by photographing the corona through a large Nicol prism, which was set to transmit successively the vertical component and two components which were inclined on each side at 45° to the vertical and consequently perpendicular to one another, not only show the strong radial character of the polarization of the corona, but also seem to suggest that there is a selective action and that the prominent streamers of the corona are markedly polarized. A photograph taken with a new form of polarizing spectograph shows a very marked difference in the intensities of the tangential and radial components; but a curious feature in it is that the Fraunhofer lines are not detected in either spectrum, though the conditions are such as must be regarded as very favourable for their detection. Unfortunately a second photograph taken with an exposure six times as long for the sake of getting further information about more distant parts of the corona was spoilt by the admission of sunlight through the slit of the instrument after the total phase of the eclipse was passed.

The Effect of the Lunar Deflection of the Vertical on Latitude Observations. By B. COOKSON, M.A., Trinity College.

[*Received* 30 November 1905.]

The maximum tide-raising force of the moon on a point of the earth's surface amounts to about 1/8,500,000 of gravity at the point, and the maximum horizontal component of this force is nearly 1/12,000,000. Thus, owing to this force, the plumb line may be deviated through an angle of $\frac{1}{12}10^{-6} \times \operatorname{cosec} 1''$, that is, $0''{\cdot}017$, or say $0''{\cdot}02$. The latitude of a point on the earth's surface is defined by the direction in space of the plumb line at the point, and the question arises whether it might not be possible to detect its deviation by examining some of the large series of observations which have been made in recent years with the zenith telescope.

Since the latitude is found from the meridional zenith-distances of stars, we require only the southward component of the moon's tide-raising force: its value is

$$\begin{aligned} L = \frac{3}{4}\frac{Ma^3}{ED^3}\{ & \sin 2\phi \cos^2\delta \cos 2h \\ & - \cos 2\phi \sin 2\delta \cos h \\ & + \sin 2\phi (1 - 3\sin^2\delta)\} \quad \text{...............(1)}, \end{aligned}$$

where

M = mass of moon,
E = mass of earth,
a = earth's radius,
D = distance between centres of earth and moon,
ϕ = latitude of place,
δ = moon's declination,
h = moon's hour-angle.

For the present purpose we may take D as constant: we have then three terms, the first with a period of half a lunar day, or nearly 12·4 hours, the second with a period of one lunar day, and the third depending on the moon's declination. Now suppose that the latitude is found from the observed zenith distances of groups of stars whose mean right ascensions and declinations are $\alpha_1\delta_1$, $\alpha_2\delta_2$, &c. We have

$$\phi_0 + \Delta_1\phi = \delta_1 + z_1 + L_1,$$
$$\phi_0 + \Delta_2\phi = \delta_2 + z_2 + L_2,$$

where L_1, L_2 are the deviations of the plumb line at the sidereal times α_1, α_2; ϕ_0 denotes the mean latitude, and $\Delta_1\phi$, $\Delta_2\phi$ the variation of latitude. In Küstner's method of determining the aberration constant from latitude observations with a zenith telescope, four groups of stars are generally chosen at intervals of 6 hours, and two consecutive groups are observed on every observing night. The variation of latitude is eliminated by subtracting the equations given by the two groups, and we get four equations of the form

$$0 = \delta_1 - \delta_2 + z_1 - z_2 + L_1 - L_2,$$

or putting in the term depending on the correction to the adopted constant of aberration,

$$0 = \delta_1 - \delta_2 + z_1 - z_2 + (a_1 - a_2)\,\Delta k + L_1 - L_2.$$

For most observatories in the northern hemisphere, we can neglect the second term in equation (1) for L, since $\cos 2\phi$ is less than $\frac{1}{8}$. The star groups are separated by half the period of the first term, so that $L_1 - L_2$ has its maximum value, which is

$$\frac{3}{4}\frac{Ma^3}{ED^3}\, 2 \sin 2\phi \cos^2 \delta \cos 2h,$$

where h is the moon's hour-angle at the sidereal time α_1. The maximum value of this is $\frac{3}{2}\frac{Ma^3}{ED^3}$, which is about $\frac{1}{12.10^6}$, or 0″·017, and the whole range of the deviation of the plumb line in the observed quantities is 0″·034. This is a small quantity, but its determination does not seem impossible. The probable error of one determination of the latitude may be taken as ± 0″·12: each group of stars contains as a rule 10 pairs of stars, and accordingly the probable error of the difference of two groups is

$$\pm\, 0''{\cdot}12 \frac{\sqrt{2}}{\sqrt{10}} = \pm\, 0''{\cdot}054.$$

The problem then is to determine the amplitude of an oscillation of known period from observations of which the probable error is about three times the quantity sought.

Besides the moon's tide-raising force there is also the sun's, which is nearly half that of the moon, and the effects of the two are superposed. But since, in Küstner's method, the observations are made symmetrically about midnight, *i.e.* one group is observed at the same interval before midnight as the other is after, it follows that the effect of the solar tide is the same on both groups: the solar terms, therefore, disappear from $L_1 - L_2$ and the lunar terms alone are left. There is also an indirect lunar effect due to the direct attraction of the ocean tides; this might be avoided by using only observations made at inland stations. Its amplitude at a station 100 km. from the coast would be only about $0''\cdot005$, and its period is half a lunar day (see Darwin's *Tides*, 1901, p. 129). The range of the direct and the indirect effect, if they were in the same phase, might amount to $0''\cdot04$. It should, however, be pointed out that an amplitude of $0''\cdot017$ corresponds to the case of a rigid earth, and that in the case of the actual earth it must be less than this.

To see whether such a lunar term exists I have examined the observations made at Philadelphia by Mr Doolittle between 1898 (Sept. 6) and 1901 (Aug. 30). The mean right ascensions of the four star-groups are $6^h\cdot3$, $13^h\cdot8$, $18^h\cdot4$, and $22^h\cdot5$; and the lunar term for the combinations I.—II. and IV.—I. may be written $1\cdot80\, K \cos(2h + 1^h\cdot7)$, whilst for II.—III. and III.—IV. it is $1\cdot80\, K \cos(2h - 1^h\cdot7)$. For a rigid earth K would be $0''\cdot0085$, but its value might be considerably modified at Philadelphia, which is near the coast, by indirect lunar effects. The observations were arranged (according to the value of the angle $2h \pm 1\cdot7$) into six groups, each of 4 hours extent, and the means were taken. In all 173 complete observations were used involving about 3,300 separate determinations of the latitude. The means are

$$-0''\cdot014 \quad +0''\cdot020 \quad +0''\cdot044 \quad +0''\cdot016 \quad +0''\cdot004 \quad +0''\cdot005,$$

or, subtracting $0''\cdot013$ from each to make their sum zero,

$$-0''\cdot027 \quad +0''\cdot007 \quad +0''\cdot031 \quad +0''\cdot003 \quad -0''\cdot009 \quad -3''\cdot008.$$

This appears to indicate an oscillation with a period of half a lunar day and a total range of about $0''\cdot05$, its maximum value occurring 6^h after the transit of the moon. It is worth remarking that high water of the Atlantic tide occurs 7^h after the moon's transit, but the amplitude of the oscillation found above, *i.e.* $0''\cdot025$, seems too large to be attributable to the direct attraction of the ocean tide.

The observations then in Küstner's method are suitably arranged to show a lunar inequality with a period of half a lunar day, but a considerably larger number of observations than the number here discussed is necessary to measure the *direct* effect of the moon on the inclination of the vertical.

Suggestions for a theory of the Milky Way and the Clouds of Magellan. By ARTHUR R. HINKS, M.A., Trinity College.

[*Read* 13 November 1905.]

IN the northern hemisphere the stars in general, the coarse clusters, the gaseous nebulae, and especially the stars with bright line spectra of Pickering's type V, are concentrated upon the plane of the Milky Way, while the spiral nebulae seem to avoid it. In the southern hemisphere the stars and coarse clusters, the gaseous nebulae, and the type V stars are concentrated in the Milky Way, and the greater Cloud of Magellan; the nebulae which are presumably spiral—(photographic evidence of the structure has not yet been obtained)—tend to avoid the Milky Way, as in the North, but crowd into the greater cloud to meet those Milky Way constituents with which they are associated nowhere else.

It is generally assumed that this fact is of the highest significance. Its significance depends, however, upon the view that may be taken of Herbert Spencer's proposition, enunciated in the year 1858, that the stars and nebulae are so definitely separated that they must necessarily be complementary parts of one general scheme of creation. The basis of this proposition is evidently the assumption that the stars are symmetrically condensed upon the plane of the Milky Way, the spiral nebulae symmetrically condensed about its poles. If that were so, it would be necessary to look for some separative influence which had driven those stars and nebulae apart, except in one particular region of the sky; and it would be natural to look in that exceptional region, the greater Magellanic cloud, for some hint as to the nature of the influence which there only had failed to operate.

It appears, however, to the author, that the evidence is very far from being so strong as to compel the adoption of this line of reasoning.

The great masses of the fainter stars are disposed in the irregular but sharply bounded star clouds which correspond very closely to the visual form of the Milky Way, and evidently produce that form.

The stars of the eleventh magnitude and brighter have little connection with the apparent structure of the Milky Way. On the whole they increase in numbers towards its plane, but their distribution in galactic longitude is far from uniform. A study of distribution charts such as those of Stratonoff (*Publ. de l'Observatoire de Tachkent,* 2 and 3) suggests rather that these stars are also arranged in clouds, of greater apparent extent than the Milky Way star clouds, and probably nearer the Sun, but more or less in the same plane with them.

There are not yet available determinations of proper motion for stars fainter than 9½ mag. Stars which have appreciable proper motion are almost uniformly distributed over the sky, and show little or no tendency to aggregation on the plane of the Milky Way. These are, on the whole, the nearer stars. They may perhaps be considered as forming a star cloud of which the Sun is a member.

The author would suggest that these facts of star distribution are consistent with the hypothesis that the stars are distributed in a series of more or less independent clouds roughly in one plane. The cloud in which our Sun is situated would then supply the stars of sensible proper motion which are uniformly distributed; the neighbouring clouds would supply the bulk of the stars down to the eleventh magnitude, which are to a considerable extent distributed about centres in the galactic plane; the more distant star clouds would make the Milky Way proper as seen by the unaided eye.

The smaller cloud of Magellan would on this hypothesis be one of the distant star clouds that happened to lie far out of the general plane. The greater cloud of Magellan may perhaps be more properly considered a nebula and star cluster cloud than a star cloud.

The distribution of the nebulae (other than gaseous) may also be considered a cloud distribution. The most conspicuous nebula cloud in the northern sky has its centre not many degrees from the pole of the galactic plane. But the centre is quite decidedly not at the pole, and the distribution is very far from being symmetrical about the pole. A second, less rich, cloud of nebulae in the northern hemisphere runs up to the borders of the Milky Way.

In the southern hemisphere the distribution of nebulae is dominated by the greater Cloud of Magellan, which is far richer in nebulae than any other part of the sky. The distribution of nebulae outside this cloud has no apparent relation to the southern galactic pole, but is in several large and not well-defined clouds. It appears, in fact, that the idea that nebulae are condensed about the poles of the Milky Way may be due to a fortuitous and not very close approximation of one nebula cloud to the place of the northern galactic pole.

Unfortunately we have at present no knowledge whatever of the distances or proper motions of the nebulae, and conclusions as to their distribution may be profoundly modified when the smaller nebulae which have recently been found in abundance on long exposure photographs are classified and catalogued, and when the already known and catalogued nebulae of the southern sky are classified photographically. It is here suggested that the distribution of known nebulae may be best expressed thus: they are in clouds, which do not lie in the plane of the star clouds, but on the other hand, are not aggregated towards the poles of that plane. In this view the greater cloud of Magellan is a cloud of peculiarly rich constitution, including numbers of Milky Way objects, star clusters, gaseous nebulae, and type V stars, as well as numbers of nebulae which are probably spiral and non-Milky Way objects (though of the spirality of any nebulae in this cloud there is no real proof); but there need be no reason to suppose that it has any special structural significance.

This suggestion that the universe may consist of a number of star clouds more or less in one plane, and a number of nebula clouds out of the plane, is put forward merely as a working idea about which facts may be grouped with some convenience. If it has any advantage over the Spencerian view of the universe as one organic whole, it is this, that we are not concerned to devise an explanation of why stars and nebulae seem as a rule to avoid one another, for we do not admit that the avoidance is so marked as to make it probable that they are complementary members of one system.

It should however be noticed that the statistical methods proper to the trial of the two hypotheses are totally different. If the universe is to be discussed as one system, it is quite evident that the fundamental plane is the plane of the Milky Way, and that the general argument in statistical representations will be galactic latitude; galactic longitude will be of secondary importance, and has, in fact, in many investigations been almost disregarded. But as soon as we admit that the universe may consist of a number of more or less independent clouds, an attempt to find general laws with galactic latitude and longitude as arguments becomes worse than useless, for it probably will obscure the very discontinuities which are most significant. It will be necessary to present the facts, not in the form of tables, but of charts; and the data which are charted must not be smoothed, as has generally been done (notably in the distribution charts of Schiaparelli, Seeliger, and Stratonoff), for the irregularities are probably all-important.

The purpose of this note is therefore twofold: First, to suggest a working hypothesis which seems to have some advantages over that generally, though not perhaps always consciously, adopted in discussions of star and nebular distribution; and secondly, to point out that the statistics of the subject may be most conveniently presented in a form which has been comparatively little used, that of detailed distribution charts.

Preliminary Communication on the Life-History of Pleistophora periplanetae, *Lutz and Splendore.* By W. S. PERRIN, B.A., *Gonville and Caius College.*

[*Received* 19 December 1905].

Pleistophora periplanetae, Lutz and Splendore, is a Myxosporidian parasitic in the Malpighian tubules and intestine of the common bakehouse cockroach *Periplaneta orientalis*. The spores formed by this organism have apparently been seen by other observers, who have, however, failed to refer them to their proper genus. Thus Schaudinn* (1902) describes the presence of the spores "eines Nosema" in the faeces of cockroaches examined by him, and Lutz and Splendore† (1903) mention the occurrence of Myxosporidian spores in the Malpighian tubules and intestine of Brazilian cockroaches. These latter observers refer the spores to the genus *Nosema* and add the species name "*periplanetae*." The descriptions given by these authors of the spores they describe, fit as far as they go the spores of the *Pleistophora*, which is the subject of this paper, and I have therefore thought it best to retain the specific name "*periplanetae*" while referring the parasite to its proper genus.

Methods: For examination of the living parasite the Malpighian tubules were cut into small pieces and examined in normal salt solution.

The parasite is very inactive and the life-history has been chiefly studied by the aid of stained preparations. Films were made of the chopped up Malpighian tubules and either dried and stained with Giemsa's modification of the Romanowsky-Nocht stain, or fixed by immersion in hot corrosive sublimate and alcohol, and then stained with Heidenhain's iron-haematoxylin or Delafield's haematoxylin. The first method of staining is particularly effective for differentiating nuclei from protoplasm, nuclei staining red or violet and protoplasm blue. Sections were found to be practically useless. Every adult *Periplaneta orientalis* examined was found to be more or less infected, very young cockroaches still possessing the yellow colour characteristic of the newly-hatched cockroach alone being free from the parasite.

Vegetative stages: The full-grown vegetative stage of the parasite is an amoebiform nucleated mass of protoplasm, which will be termed the trophozoite. The trophozoites vary from minute specks of protoplasm 2μ—3μ in size with one nucleus, to multinucleated masses measuring in some cases 30μ by 55μ and containing 60 or more nuclei.

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* Schaudinn, F., *Archiv. für Protist.* pp. 306—348.

† Lutz and Splendore, *C. B. Bakt. Pk.* (1), Vol. XXXIII. pp. 150—157.

The protoplasm of the trophozoite is frequently differentiated into a denser, more deeply staining outer layer or ectosarc, and an internal non-fluid layer or endosarc. Pseudopodia occur frequently and are generally lobose, though elongated and pointed pseudopodia are also found. The endosarc may be hyaline or reticular in structure. Vacuoles of varying sizes are of frequent occurrence. Granules appear to be absent, but large mucoid looking inclusions are occasionally to be seen within the trophozoite. In trophozoites stained with Giemsa's eosin-azur mixture two kinds of nuclei are to be distinguished, one kind which stains bright red and another which stains a deep violet. The nuclei staining deep violet are very compact and frequently contiguous to a small area of protoplasm staining bright blue. The nuclei which stain bright red are generally reticular in structure although occasionally compact. With Delafield's haematoxylin the violet nuclei stain very intensely.

Two methods of reproduction can be distinguished in *Pleistophora periplanetae,* one subserving auto-infection, and one resulting in the production of coated spores, which gain the exterior per anum and presumably infect fresh hosts.

Auto-infective methods of Reproduction: Multiplication of the parasite within the host may be effected apparently in four ways: (1) the young uninucleated trophozoite may divide amitotically into two. (2) The nuclei of a young trophozoite may multiply amitotically and proceed to the periphery of the cell, where the protoplasm rounds itself off about each nucleus, the trophozoite finally breaking up into a number of uninucleated spores. This method of reproduction recalls the schizogony of the Telosporidia. (3) A multinucleated trophozoite may divide into two by a process of simple fission. (4) Portions of a large multinucleated trophozoite may separate themselves from the parent cell and begin an independent existence. The fragments budded off may have one or several nuclei, but a trophozoite does not appear to produce buds with one and many nuclei at the same time. Doflein describes a similar process of fragmentation in *Glugea lophii,* and terms it plasmotomy. Division of a trophozoite into two pieces, *i.e.* ordinary binary fission he terms simple, division into more than two fragments, multiple plasmotomy. The production of uninucleated spores from a trophozoite by budding does not appear to have been hitherto recorded among the Myxosporidia.

Spore formation: All the above methods of reproduction subserve auto-infection, but resting spores are also produced, which presumably bring about the infection of fresh hosts. Among the Myxosporidia it is usual, as a preliminary to sporulation, for the trophozoite to produce a number of internal buds, which give rise to a varying number of spores, the production of spores continuing within the trophozoite without any cessation of activity or growth. These internal buds have been styled by Gurley pansporoblasts. In *Pleistophora periplanetae*, however, no such production of pansporoblasts takes place. The whole of the trophozoite simply withdraws its pseudopodia, rounds itself off and makes spores. As, however, the classification of the Myxosporidia is largely based upon the features exhibited by the pansporoblast, the number of spores it produces, etc., it seems advisable to regard the trophozoite of *Pleistophora periplanetae* as producing one pansporoblast, although the point at which trophozoite ceases to be trophozoite and becomes pansporoblast may not be easy to define. The first stage then of sporulation consists in the rounding off of the trophozoite to form a pansporoblast, which is usually more or less oval and almost completely filled with developing spores. The spores are developed from oval masses of protoplasm, the sporoblasts, which rather exceed in size the spores into which they are transformed. In stained preparations each sporoblast appears to lie in a vacuole. This vacuole is not, however, evident in the living cell, and is probably an artifact produced by shrinkage of the protoplasm during preparation. Each sporoblast contains a nucleus, which is at first reticular, and stains bright red with Giemsa's eosin-azur mixture. The interstices between the developing sporoblasts are filled up with protoplasm and nuclei, both of which ultimately disappear.

The nucleus of the sporoblast undergoes various changes which will be described with figures in the full paper, and the sporoblast surrounds itself with a coat of a highly resistant nature. The presence of this coat makes staining the spore very difficult. In its final form as found in the faeces of the cockroach, the spore is a flat oblong structure rounded at both ends, and varying from 5μ—6μ in length, and $2{\cdot}5$—3μ in breadth. In the living state the sporoplasm is finely granular and contains at one end a small highly refractive globule. The relation of the polar capsule, which is almost certainly present, to this globule has not been determined. Application of the usual reagents ether, concentrated sulphuric, nitric and hydrochloric acids, glycerin, iodine and boiling water, etc., has hitherto failed to procure extrusion of the filament of the capsule. The spore coat is apparently unaffected by the above reagents and also by eau de Javelle.

Attempts to produce the emergence of the sporoplasm by treating the faeces with the digestive juices and contents of the alimentary canal of other cockroaches have been unsuccessful.

With the above facts of the life-history of *Pleistophora periplanetae* to hand it is possible to classify the parasite accurately. From the minuteness of the spores, the fact that the pansporoblast produces more than two spores, and the invisibility of the polar capsule in the fresh state, the parasite belongs to the sub-order Cryptocystes of the Myxosporidia and family Glugeidae, which possess the characters of the sub-order. It belongs to the sub-order Cryptocystes Myxosporidia and family Glugeidae, which possesses the characters of the sub-order. It belongs to the section Oligosporogenea (Doflein), because the trophozoite produces a single pansporoblast, and the production of numerous spores by the pansporoblast relegates it to the genus *Pleistophora.* It is this formation by the trophozoite of a single pansporoblast, which removes the parasite from the genus *Nosema,* section Polysporogonea, to the genus *Pleistophora,* section Oligosporogenea. To sum up, the classification of the parasite is as follows:—

Class.	Sporozoa.
Sub-class.	Neosporidia.
Order.	Myxosporidia.
Sub-order.	Cryptocystes.
Family.	Glugeidae.
Section.	Oligosporogenea.
Genus.	*Pleistophora.*
Species.	*Periplanetae.*

There are several points of interest offered by the above scattered details of the life-history of *Pleistophora periplanetae,* both with respect to the Sporozoa in general and the Myxosporidia in particular. In the first place it is seen that there are two very definite phases, a trophic phase characterised by almost excessive multiplication with a view to auto-infection, and a propagative phase, characterised by a cessation of trophic activity and the formation of resting spores. In this sharp division of the life cycle into schizogonous and sporogonous phases *Pleistophora periplanetae* resembles *Thelohania mülleri* described by Stempell. The occurrence of sporulation at the end of the trophic phase is a feature which by no means agrees with Schaudinn's division of the Sporozoa into Telosporidia, *i.e.* Sporozoa in which spore formation occurs at the end of the trophic phase, and Neosporidia, which includes the Myxosporidia, in which spore formation occurs during the trophic phase. As far as the point in the life-history, at which spore formation occurs, is concerned, *Pleistophora periplanetae* is in exactly the same position as a Haemosporidian or a Coccidian. In each case a trophic phase characterised by vigorous multiplication is succeeded by a resting phase characterised by the production of resting spores. As Stempell remarks, Schaudinn's grouping of the Myxosporidia and the Sarcosporidia together in a sub-class separate from the rest of the Sporozoa is probably well grounded, but the difference between the two sub-classes must be expressed in other terms than the period of the life-history at which sporulation occurs. Another point of interest in the life-history of *Pleistophora periplanetae* is afforded by the existence of residuary nuclei and protoplasm, which die off while sporulation is taking place. The meaning and homologies of these nuclei and accompanying protoplasm are obscure. The subject will be more discussed in the full paper.

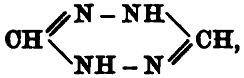
Note on a hitherto undescribed parasite (?) of Periplaneta orientalis.

In the course of this research certain peculiar structures, which occurred in many of the stained preparations of the Malpighian tubules, attracted attention. They resemble the trophozoites and spores of *Pleistophora periplanetae* but are smaller in size. They stain in a similar manner with the same reagents. The structure of the trophozoites is however more delicate than that of the trophozoites of the *Pleistophora*; the nuclei are also smaller and more compact. The spores in the fresh state are highly refringent, resembling the spores of the *Pleistophora* in this respect. They are, however, much smaller, measuring not more than $1\mu - 2\mu$ in length, and about $\cdot 5\mu$ in breadth, instead of $5\cdot 5\mu \times 3\mu$, as in *Pleistophora periplanetae*. In two cases the chylific ventricle was crowded with these bodies. They do not appear to belong to the life-cycle of the *Pleistophora*, and it seems probable that they are the trophozoites and spores of a new Myxosporidian parasite. They stand however in need of further investigation.

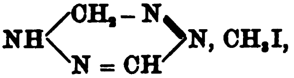
Contributions to the Knowledge of the Tetrazoline Group. By S. Ruhemann, M.A., Gonville and Caius College, and R. W. Merriman, B.A.

[*Read* 27 November 1905.]

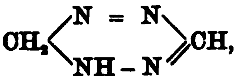
Ruhemann and Stapleton (*Trans. Chem. Soc.*, 1899, **75**, 1131; 1902, **81**, 261) showed that the properties of tetrazoline,



differ most markedly from those of its N-aryl derivatives (see Ruhemann, *Trans. Chem. Soc.*, 1899, **55**, 242; 1890, **57**, 50). Of especial interest is the behaviour of methyl iodide towards tetrazoline, which, as previously (*loc. cit.*) stated, furnishes two compounds, one of them being dark blue, the other colourless. From the analytical results the composition $C_3H_9N_4I_3$ had been deduced for the former, and $C_3H_7N_4I$ for the colourless substance. We now find that the blue iodide contains two atoms of hydrogen less and therefore corresponds to $C_3H_7N_4I_3$, because it is formed directly from the colourless compound by its union with iodine. On further examination of the substance $C_3H_7N_4I_3$ we have been struck by its resemblance to the periodide of diazobenzene chloride. This fact points to analogous formulae, and leads to the view that the iodide $C_3H_7N_4I$ should be represented thus

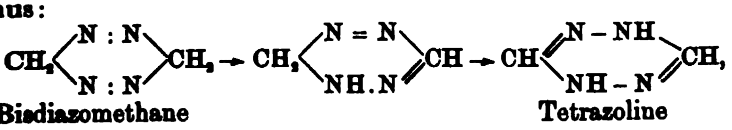


and that the analogous formula has to be assigned to the additive product $C_3H_7N_4I_3$. According to this view, which agrees with the behaviour of the iodides, these compounds would appear as derivatives of a dihydrotetrazine,



isomeric with bisdiazomethane and tetrazoline.

Hantzsch and Silberrad (*Ber.* 1900, **33**, 58) have shown that bisdiazomethane under the influence of hydrochloric acid, changes to tetrazoline. This transformation may be assumed to take place thus:

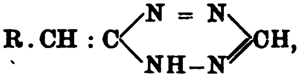


and to be preceded by the formation of the dihydrotetrazine, which we regard as the parent substance of the compounds $C_3H_7N_4I$ and $C_3H_7N_4I_3$. The formation of these iodides, therefore, is due to a regressive change of tetrazoline.

This view is supported by the behaviour of aldehydes towards tetrazoline. Under the influence of piperidine as a catalytic agent, a condensation takes place which is to be represented by the equation:

$$C_2H_4N_4 + R.CHO = H_2O + C_2H_2N_4 : CH.R.$$

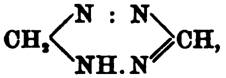
The substances which are thus produced must be represented by the general formula



and their formation necessitates the isomerisation of tetrazoline. We have applied this reaction to a number of aldehydes. The condensation products readily dissolve in mineral acids, although, thus, partial decomposition takes place, yet these solutions yield with platinic chloride, normal salts having the general formula

$$(C_2H_2N_4 : CH.R)_2,\ H_2PtCl_6.$$

The hydrolysis of the compounds which are formed by the action of aldehydes on tetrazoline, is completed on boiling their solutions in hydrochloric acid for a short time; the base, however, which is produced along with the aldehyde, is not the dihydrotetrazine

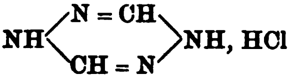
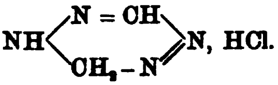


but tetrazoline. It, therefore, follows that the hydrolysis is accompanied by the isomerisation of the base to tetrazoline.

Ruhemann and Stapleton had previously (*loc. cit.*) stated that tetrazoline forms with platinic chloride the sparingly soluble, canary-yellow compound $(C_2H_4N_4)_2$, $PtCl_4$. We now find that the base as well as its hydrochloride, after crystallisation from alcohol, yield a very soluble normal platinichloride

$$(C_2H_4N_4)_2,\ H_2PtCl_6,$$

but that the insoluble additive product is formed from the hydrochloride before re-crystallisation from alcohol. From this fact it follows that the hydrochloride exists in two tautomeric forms:

 and 

This explanation of the phenomenon is supported by the fact that the alcoholic mother liquor of the crystals of tetrazoline hydrochloride yields the yellow additive compound, and that aqueous solutions of the normal platinichloride slowly deposit the insoluble substance $(C_2H_4N_4)_2$, $PtCl_4$.

We have begun already to study, on similar lines, dimethyltetrazoline which is formed by the action of heat on monoacetylhydrazine (see Pellizzari, *Atti R. Accad. Lincei*, 1899 [v], **8**, 327). The properties of this base differ in many respects from tetrazoline; whilst the latter yields with mercuric chloride the compound $2C_2H_4N_4$, $3HgCl_2$, dimethyltetrazoline forms with the metallic chloride the additive product $C_4H_8N_4$, $HgCl_2$. Aldehydes which so readily condense with tetrazoline, have no action on dimethyltetrazoline. On the other hand, platinic chloride yields with dimethyltetrazoline a very soluble normal platinichloride $(C_4H_8N_4)_2$, H_2PtCl_6; its aqueous solution, on standing, slowly deposits a yellow solid which has the composition $(C_4H_8N_4)_2$, $PtCl_4$ and, therefore, is analogous to the additive product of the metallic chloride with tetrazoline. Methyl iodide, also, seems to react with dimethyltetrazoline in a similar manner as with tetrazoline, and yields a substance, crystallising in brown needles, which resembles in properties the compound $C_3H_7N_4I_3$.

We are engaged in the further study of this substance and in examining the action of aldehydes on bisdiazomethane and allied heterocyclic compounds.

Some experiments on Canal-Strahlen. By J. J. THOMSON, M.A., F.R.S.

[*Read* 27 November 1905.]

GOLDSTEIN discovered that when the electric discharge passes through an exhausted tube having a perforated cathode, luminous rays streaming through the perforations can be observed behind the cathode. These rays are deflected by a magnetic field to a small, but only very small, extent and have been shown by W. Wien to consist of positively charged ions whose mass is never less than that of a hydrogen atom and is often considerably greater. Similar streams of ions impinge against the cathode itself, and in modern theories of electric discharge through gases play an important part in the mechanism of the electric discharge, thus in my book on the *Electric Discharge through Gases*, I have supposed that it is by the impact of these positive ions on the cathode, that the cathode rays originate. Again, we know that the cathode disintegrates and the question arises whether this is not due to the impact of the positive ions. The question of ionization by the impact of positive ions against the molecules of a gas, and the nature of the spectra produced by such impacts can also be conveniently studied by means of canal-strahlen: for these reasons I have made a series of experiments on these rays using a discharge tube of the kind represented in Fig. 1.

The cathode C is a large disc nearly filling the tube, with a hole in the middle opening on to a metal tube; the canal-strahlen pass through this tube on their way to strike against the metal plate M at the end of the discharge tube. A wire soldered to this plate passes out from the discharge tube and allows the plate to be charged positively or negatively.

The discharge was produced by a large induction coil, and to prevent cathode rays coming down the tube from the reversal of the discharge, a strong transverse magnetic field was applied to the portion B of the tube, so as to drive any cathode rays against the walls of the tube and prevent them from reaching the metal plate. The experiments were also repeated with a large Wimshurst machine in place of the induction-coil, the results were the same in the two cases.

When the canal-strahlen strike against a solid they cause it to emit cathode rays moving with small velocities. This can easily be shown by charging the metal plate, against which the canal-strahlen impinge negatively, to a potential of say 80 volts, then from the part of the plate struck by the canal-strahlen a pencil of feebly luminous rays can be seen, the appearance being like that shown in Fig. 2; these rays are easily deflected by a magnet in the direction indicating a negative change. That their initial velocity is very small, is shown by the fact that they are completely stopped when the metal plate is charged to a small positive potential.

Secondary ionization is produced in the path of the canal-strahlen. If the metal plate is charged positively, the boundary of the canal-strahlen becomes fluffy and bends towards the positive plate, as in Fig. 3, showing that negative corpuscles are present; the fluffy boundary is very susceptible to magnetic force. These negative ions come out of the beam of the canal-strahlen, which thus ionize the gas. The α particles emitted by radium may be regarded as canal-strahlen of very great velocity, enormously greater than the velocity of those produced in a discharge tube. Now Rutherford has shown that when the α particles, by passing through a gas or thin sheet of metal, have lost about 18 °/$_0$ of their original velocity, they suddenly lose their power of ionization, yet this reduced velocity is

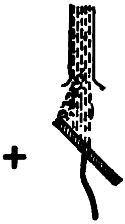


Fig. 3.

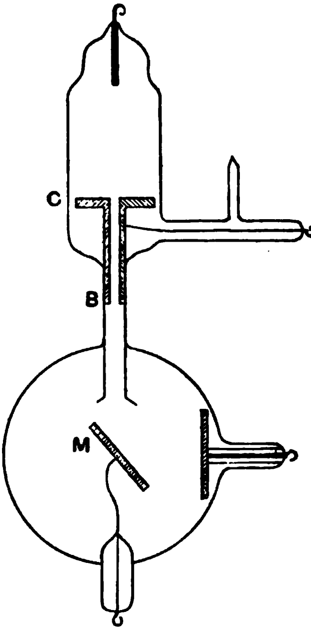


Fig. 1.

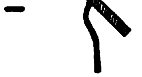


Fig. 2.

enormously greater than the velocity of the canal-strahlen which we have seen possess the power of ionization. The most probable explanation of this difference seems to me to be, that when the velocity of the α particles sinks below a certain value, it will hold fast to a corpuscle against which it may come in collision, and thus have its positive charge neutralized and become an uncharged atom. If v is the relative velocity of the α particle and a corpuscle at collision, when r is the distance between the corpuscle and the particle, m the mass, e the charge on the corpuscle, we can easily prove that the corpuscle will stick to the particle if

$$mv^2 < \frac{2e^2}{r},$$

while the particle will escape if

$$mv^2 > \frac{2e^2}{r}.$$

To find the smallest velocity to enable an α particle to escape, let us give to r the conventional value for the radius of an atom, 10^{-8} cm./sec., we have also in electrostatic measure

$$e/m = 3 \times 10^{17} \text{ and } e = 3{\cdot}5 \times 10^{-10},$$

hence $v = 1{\cdot}4 \times 10^8$ cm./sec. will be the smallest velocity an α particle can possess if it is to retain a positive charge. We need not attach importance to the fact that the limiting value of v found by Rutherford is considerably greater than this, for the value assumed for r has no claim to accuracy. In the case of the α particles the proportion of corpuscles to positive ions is very much greater than in that of the canal-strahlen, so that neutralization is more likely to occur in the former than in the latter case.

The canal-strahlen disintegrate a metal plate against which they strike, for after a long continued bombardment of the metal plate the walls of the tube in the neighbourhood of the plate are found to be covered with a deposit of the metal.

When the canal-strahlen impinge on a salt of sodium, the salt gives out yellow light in which the *D* line is very bright, it is remarkable that this line is not given out when canal-strahlen impinge on the metal sodium itself, a mirror of sodium with specks of oxide on it presents a beautiful appearance when struck by these rays, the spots of oxide shine out brightly with a greenish yellow light, while the surface of the metal itself seems quite dull if no external light falls upon it.

I have much pleasure in thanking my assistant, Mr Everett, for the help he has given me with these experiments.

On the Inheritance of Coat-colour in Rats. By L. DONCASTER, M.A., King's College.

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IN July, 1903, I began breeding-experiments with the various colour-varieties of the Common Rat, with the intention of working out the laws of their inheritance. At that time Cuénot's later papers on Mice had not yet appeared, and my chief object was to investigate the "resolution" which appeared to take place when the wild brown rat is crossed with the albino. In the wild brown (or grey) rat there are three pigments, black, brown, and yellow, as there are in the common mouse; but among the descendants of a wild rat crossed with an albino, black animals containing no yellow pigment commonly appear*. The chief object with which I began the work was to determine if possible how this suppression of the yellow took place, and why the corresponding disappearance of black, which occurs in mice, is not found in rats.

The colour varieties of rats have been described and classified by Crampe†, and again by Bateson (*loc. cit.* p. 78). They are much simpler than in mice, so that a rat which does not fall unmistakeably into one of Crampe's seven categories is extremely rare. In colour all the rats which I have bred have been either brown (wild colour), black, or albino. The brown rat is darkest along the back, has a considerable number of hairs with yellow tips on the sides, and is pale grey below. The black is full black everywhere when the animal is young, but as it grows older, especially when moulting, it tends to a chocolate-brown tint on the sides and ventral surface, but has no trace of yellow. The albino is quite white when young, but inclines to a cream tinge when older. Albinos have pink eyes, coloured rats black. Both the brown and black rat may be quite without white except on the toes, which are never pigmented (Crampe's types 1 and 7), they may have a patch of white of varying size on the chest and belly (types 2 and 6), or they may have a coloured "hood" and stripe along the back, the rest being white (types 3 and 5). Crampe's type 4 is the albino. Types 2 and 6 vary within rather wide limits; the white mark on the chest may be limited to a few hairs, or it may extend into a broad white ventral surface, sometimes spreading slightly out to the flanks. The tail usually has a colourless tip of varying length, and at least the lower part of the fore limb is usually white. Type 6 (black colour) is known in the fancy as "Irish"; a good Irish has only a small white patch on the breast. Types 3 and 5 have the head and neck coloured, and the pigmented hairs may extend on to the fore limb. There is almost always a coloured stripe along the middle of the back from the hood to the tail; the ventral surface, sides, and limbs are white. The back stripe varies considerably; it may be broad, sometimes spreading out irregularly, it may be narrow and even, and in other cases it is broken into a series of spots. Very rarely it is quite absent. When the white patch of types 2 or 6 is very large, the animal approaches the darkest form of types 3 and 5, but only once have I had any hesitation in classing a rat as belonging to one or the other.

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* The distribution of the pigments in the hairs I find to be identical with that described by Bateson in Mice (*P. Z. S.* 1903, II. p. 72); the black rat differs from the brown in the absence of yellow pigment.

† "Kreuzungen zwischen Wanderratten," *Landwirth. Jahrb.* VI. 1877, p. 384; XII. 1883, p. 389; XIII. 1884, p. 692. Also "Die Gesetze der Vererbung der Farbe," *ibid*, XIV. 1885, p. 539.

"Broken-marked" rats (*i.e.* irregularly piebald) do occur in rare instances, but none have appeared in my experiments, nor has there been any departure from the pigmentation described.

It will be seen that pigment distribution in rats is much simpler than in mice, since there are only two colour-types, and two patterns in the coloured-and-white individuals; the complications which arise in mice are therefore absent, and the scheme of inheritance much simpler. From the practical point of view this simplicity is counterbalanced by several disadvantages. Rats breed much more slowly than mice, and take much longer to reach maturity; in my experience they do not often begin to breed when younger than three months. The doe may take the buck immediately after a litter is born, but commonly young appear at intervals of six weeks or two months, often considerably longer. It therefore takes a long time to rear a sufficient number of young from any one pair. They are also very liable to disease, especially of the lungs; if this is to be avoided they require rather large cages and frequent cleaning. Hence they take up a great amount of space, and my work has been delayed owing to want of room in which to rear many families simultaneously. The does do not often eat or desert their young, though this happens occasionally, but on the whole the infant mortality is very small.

The most important work on colour-inheritance of rats is that of Crampe (*loc. cit.*). His papers were written long before the re-discovery of Mendel's work, and are often very obscure, but the following facts can be made out clearly from his results. Brown (grey) is uniformly dominant over black, and both brown and black over albino. The self-coloured varieties, and those possessing small white areas on the lower surface (Nos. 1, 2, 6, 7) are dominant over the coloured-and-white types (Nos. 3 and 5). All seven varieties can probably be obtained pure by continued selection, but this is much more easily effected with the more recessive forms (3 and 5) than with the dominants (1, 2, 6, 7). Albinos bred together gave only albinos. Two points of considerable interest are mentioned with regard to the self-coloured varieties 1 and 7; they are left unexplained, but it seems probable that they are connected and depend upon the same causes. The first of these is that the self-coloured varieties when bred together can give the forms with white marks, but not the pied forms, but that the varieties with white marks bred together may give all the types; that is to say, that type 7 for example can contain type 6, and type 6 can contain both 5 and 7, but that 7 cannot contain 5. This as it stands appears inexplicable, but there are indications of a possible explanation. Type 6 (black with some white below) has two sub-varieties, one with very little white, the other with a large patch, and my work shows that the first of these is closely related to the self-black (type 7), while the second is a heterozygous form always containing type 5 (piebald). The second point of interest arising from Crampe's work is that two kinds of first crosses appear when the wild rat is bred with the albino, one of which has no white (type 1), the other with white below (type 2). These two varieties behave differently as regards their offspring in the same way as types 6 and 7, and the explanation appears to be that some albinos carry the self-character, giving first cross of type 1, others the piebald character, giving type 2.

The animals with which I began my experiments were obtained from several sources, but the majority were sent me by Miss Douglas, of Worcester, to whom I am also indebted for much useful information. I wished to cross the wild *M. decumanus* with the different colour varieties, but failed to get wild animals to breed in captivity. I was fortunate, however, in obtaining from a local ratcatcher two families of young rats from albino does by wild bucks. The man keeps a large cage of white rats, from which two does escaped, and not long afterwards both rats were found in nests in a neighbouring haystack with families of exclusively brown young. I saw them when still with the mothers, and as there are numerous wild rats about the place, I think there can be no doubt that their male parents were common wild rats.

My earlier experiments were somewhat indiscriminate in character, being undertaken partly to test whether the animals used were pure or were giving gametes of more than one kind, partly to determine the proportion in which the different forms appeared among the descendants of the wild rat crossed with albino. Gradually, however, definite problems presented themselves, and my later work has dealt chiefly (*a*) with the power of albinos to transmit colour or pattern when mated with coloured individuals, and (*b*) with the nature and relations of Crampe's types 7, 6, and 5.

It has been shown by Cuénot[*], and others in mice, and more recently by Hurst[†] in rabbits, that albinos are not all similar in constitution, but that when mated with a given coloured individual their offspring differ according to the origin of the albino parent. I find that this is also the case in rats; some albinos carry the determinant for brown, others black, and others again both brown and black. So also with pattern; some carry the piebald character, some the "self" (or "Irish"). An albino can best be tested in these respects by mating it with a black-and-white, since black is recessive to brown and the pied character to "self." Experiments showing the results of such matings are Nos. 32, 33, 41, 45, 58 *a* and *b*, and, with regard to colour only, Nos. 2, 37, 57. Black-and-whites paired together can give nothing but black-and-white and albino, but when paired with an albino carrying brown and "self" determinants, brown self-coloured rats are produced. In Exps. 32 and 41 the albino used contained only black and "pie," so that only black-and-white and albino young appeared, roughly in equal numbers. In Exp. 33 the albino, although of similar ancestry to those used in Exps. 32 and 41, carried only the "self" character as far as pattern is concerned, but both black and brown colour determinants, so that of the coloured young all were self-coloured (with white patch on the underside), 4 being brown and 5 black. In Exp. 45 the albino, derived from two "Irish" parents, carried the black colour determinant and the "self" character, and all the coloured young were "Irish."

Similar results are seen in Exps. 2 and 37, in the first of which the albino bore only black, in the second both black and brown, giving 4 brown young to 6 "Irish." It is therefore quite clear that as in other animals the albinos, although giving nothing but albino when paired together (Exp. 46), differ from one another in constitution, and influence their young both in colour and pattern. In this connexion it is of interest that Haacke[‡] describes an albino rat in which the hair of the hood and back-stripe differed from the rest in texture, although quite without pigment. Such an animal would doubtless have been found to bear the piebald character only, had it been tested by mating with a black-and-white.

The second point to which my attention has been chiefly directed concerns the relations between the different forms of pattern, particularly the nature of Crampe's types 2 and 6 and their relation to the pure self-coloured and pied forms respectively.

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[*] *Arch. Zool. Exp. and Gén.* Notes and Rev. Ser. 4, t. 2, 1904, p. xlv.

[†] *Journ. Linn. Soc. Zool.*, Vol. xxix. p. 283.

[‡] Haacke, *Biol. Centralbl.* xv. 1895, p. 63. "Wesen, Ursachen und Vererbung v. Albinismus etc."

There seems to be no doubt that the brown forms (Crampe's types 1, 2, 3) correspond exactly with the similar black forms 7, 6, and 5, but are less simple to work with since they may contain recessive black. They also have the disadvantage that the extent of the white area in type 2 is much less easily determined at an early age, since the grey of the underside is not at first distinguishable from white. I therefore chose the black forms to experiment with, and have made only a few matings with the brown varieties.

It soon became evident that the two sub-varieties of type 6 ("Irish") are distinct from one another, and in the later pairings recorded in the tables these are distinguished as 6 *a* and 6 *b* (Irish *a* and Irish *b*). Unfortunately these two sub-varieties were not recorded separately in the earlier matings. Where one of the parents was black-and-white (type 5) all the "Irish" offspring were always of the form with a large white patch (6 *b*); this is seen in matings 34 and 45, and was almost certainly also the case in No. 33, where, however, it was recorded only in the last litter. The brown young in No. 33 (type 2) also had much white underneath. When "Irish" rats were paired together, in all the cases where any black-and-white young were produced (matings Nos. 42, 41, 56), both parents had a considerable amount of white on the underside. On the other hand in experiments 35, 35 *a*, and 55, no black-and-white young were produced, and in each case one parent had only a very small white mark. The other parent in these matings had more white, and the coloured offspring were divided between the two sub-types. In Exp. 63 both parents were type 6 *a*, and no young of either 6 *b*, or 5 (blk.-wh.) were produced. It seems, therefore, that type 6 *b* ("Irish" with much white) can be produced by mating black-and-white with black (Exp. 34), or with an albino of "Irish" parentage (Exp. 45), and that when two such individuals are paired together, the offspring consist of black-and-white, "Irish" *b*, and "Irish" *a*. This points to the conclusion that type 6 *b* is a heterozygous form produced by crossing black or "Irish" *a* with black-and-white, which then separate in the next generation in the normal Mendelian manner. Type 6 *a* and black on the other hand do not contain black-and-white, and therefore in matings in which they are concerned no black-and-white offspring appear.

The relations between type 6 *a* and pure black are less clear. Crampe found that blacks (type 7) bred together might give type 6, but they seemed to appear in quite irregular proportions, *e.g.* from some pairs only blacks appeared, in one mating 26 black to 6 "Irish" and in another 6 blacks to 27 "Irish." From Crampe's tables it appears that when "Irish" rats are bred together, pure blacks appear only when they have occurred in the ancestry of the "Irish" used and my experience is in accord with this, although in Exp. 63 there is only 1 black ancestor three generations back. Crampe does not separate the two forms of "Irish," but in all his matings black-and-white appeared, so that when blacks were produced the parents may have been heterozygotes black black-and-white. But in my mating 63 blacks appeared from two parents of type 6 *a*, so that this explanation is impossible, and we therefore find that blacks bred together can throw Irish, and Irish can give black. There is further no sharp line of distinction between black and "Irish *a*," for the latter may have only about half-a-dozen white hairs on the chest, or may have a considerable white patch.

It is practically certain, therefore, that black is only an extreme form of "Irish *a*," and not really a separate variety; either may be bred true by continued selection, but they are not allelomorphic to one another. Both when paired with black-and-white give the heterozygous form "Irish *b*." That this is the true explanation is indicated by Exp. 55, where the ♀ parent was "Irish *b*" made by crossing black-and-white by black, the "Irish *a*" derived from 2 "Irish" parents. If black were dominant over "Irish *a*" the offspring should have consisted of black and "Irish *b*" in equal numbers, but actually 4 "Irish *b*," 2 "Irish *a*" and 2 black were produced. The conclusion must be that as far as pattern is concerned there are only two allelomorphs in rats, viz. "pied" and "self," the pied consisting of coloured hood and stripe, the self either completely coloured or having a small white mark on the ventral surface. When crossed together there is no complete dominance, and a heterozygous form is produced having the whole ventral surface white.

Of the other experiments not much remains to be said. Most of them were begun in a rather indiscriminate fashion before I realized that albinos were not all alike in constitution, and many of the albinos used were dead before they could be tested. Pairings were made between "half-wilds" (children of albino by wild) and albinos in Exps. 14, 21 *a*, 44, 24, and 26, giving a total of 87 young, 44 of which were albino, a close approximation to the Mendelian equality. If the albinos used all bore the black and piebald determinant, the expectation is that the coloured offspring should consist of equal numbers of Crampe's types 2, 3, 5, 6; the actual numbers are 8, 9, 10, 16 respectively. Of the five albinos used in these experiments, 3 were bought, and in my experience albinos obtained from dealers generally carry the black and piebald characters. Unfortunately, however, most of them died before they could be tested in this respect.

In only one experiment (No. 43) did I succeed in getting young from "half-wilds" paired together, 7 out of 25 being albino (expectation 1 in 4). The expected ratio of self-brown, pied brown, self-black and pied black respectively is 9 : 3 : 3 : 1; the actual numbers 12, 0, 4, 2. In this experiment I bred the only brown without white (type 1) which I have obtained.

In Exp. 23, where a half-wild was mated with a black-and-white containing recessive albino, the numbers approximate to the Mendelian expectation, viz. 15 coloured to 6 albino (expectation 15 : 5); and the four colour types 5 : 4 : 2 : 4 (expectation equality).

A point of some interest is the great diversity of different litters from the same pair. Where enough offspring are reared the results are in fair accord with Mendelian expectation, but in individual litters this is often far from being true. To save space the separate litters are not recorded in the tables, but examples can be given. In Exp. 23 (♀ alb. × ♂ blk.-wt.) the first litter consisted of 5 albinos, 1 "Irish," 2 brown (type 2); the second litter of 1 albino, 4 "Irish," 2 brown. In Exp. 37 (♀ alb. × ♂ blk.) three successive litters contained respectively 5 albino, 2 "Irish," 1 brown; 3 albino, 1 "Irish," 2 brown; 1 albino, 3 Irish, 1 brown. And in Exp. 44 (alb. ♀ × "half-wild" ♂) four successive litters consisted of 2 albinos and 6 coloured, 7 albinos and 1 coloured, 3 albinos and 4 coloured, 7 albinos and 3 coloured, the total making up 19 albinos to 14 coloured.

It is evident from such facts that a large number of young must be reared from each pair before the true proportions are known with any accuracy, and unfortunately disease has killed off many of my rats before this could be done.

EXPLANATION OF TABLES.

The tables give a list of all the young bred, with their ancestry as far as it is known.

The columns under "Offspring" include Crampe's seven classes, viz.:—

Cr. 1 = brown (grey) with no white.
Cr. 2 = brown with white on ventral surface.
Cr. 3 = brown piebald.
Cr. 4 = albino.
Cr. 5 = black piebald.
Cr. 6 = black with white ventral patch ("Irish"), with sub-classes—
6*a* = small white patch, 6*b* = large white patch.
Cr. 7 = black.

In the ancestry columns the number of the pairing is given immediately before the bracket which connects ♀ and ♂ of each pair.

Abbreviations used are:—
alb. = albino; blk. = black; brn. = brown (grey); blk. wt. = black-and-white (black piebald); brn. wt. = brown-and-white (brown piebald).

No. of Mating	Great Grand Parents		Grand Parents		Parents		Offspring (Crampe's Numbers)							
							Cr. 1	Cr. 2	Cr. 3	Cr. 4	Cr. 5	Cr. 6 *b*	Cr. 6 *a*	Cr. 7
46	♀ alb. …… ♂ wild. ……	8	♀ brn. Cr. 2 …… ♂ alb. ……	14	♀ alb. (used in Exp. 33) ♂ alb. (used in Exp. 45)	46	—	—	—	12	—	—	—	—
	♀ Irish …… ♂ alb. ……	2												
	♀ blk. wt. …… ♂ brn. (half wild) ……	23	♀ Irish …… ♂ Irish ……	35										
	♀ brn. (half wild) …… ♂ alb. ……	14												
46*a*	……		……		♀ alb. …… (same as in No. 46) ♂ alb. (child of No. 40)	46*a*	—	—	—	2	—	—	—	—

29	....		♀ blk. wt. .... ♂ alb. .... ♀ blk. wt. .... ♂ alb.	25 25	♀ blk. wt. .... (used in Exps. 41, 45) ♂ blk. wt. .... (used in Exp. 33)	29	—	—	—	4	9	—	—	—
32	.... ♀ alb. .... ♂ wild.................... ♀ Irish .... ♂ alb.	 8 2	♀ blk. wt. .... ♂ alb. .... ♀ brn. Cr. 2 .... ♂ alb.	25 14	♀ blk. wt. .... ♂ alb.	32	—	—	—	10	12	—	—	—
41	.... ♀ alb. .... ♂ wild....................	 8	♀ blk. wt. .... ♂ alb. .... ♀ alb. .... ♂ brn. Cr. 2	25 24	♀ blk. wt. .... (used in Exps. 29, 45) ♂ alb. (used in Exp. 36)	41	—	—	—	4	2	—	—	—
33	♀ alb. .... ♂ wild.................... ♀ Irish .... ♂ alb.	8 2	♀ brn. Cr. 2 .... ♂ alb. .... ♀ blk. wt. .... ♂ alb.	14 25	♀ alb. .... ♂ blk. wt. .... (used in Exp. 29)	33	—	4*	—	6	—	5*		—

* The coloured animals probably all had large white ventral patch.

No. of Mating	Great Grand Parents	Grand Parents	Parents	Offspring (Crampe's Numbers)							
				Cr. 1	Cr. 2	Cr. 3	Cr. 4	Cr. 5	Cr. 6 *b*	Cr. 6 *a*	Cr. 7
45	♀ blk. wt. ♂ alb. 25 ♀ alb. ♂ brn. Cr. 2, half wild 24 ♀ blk. wt. ♂ brn. Cr. 2, half wild 23 ♀ brn. Cr. 2, half wild ♂ alb. *ex* 2 14	♀ blk. wt. ♂ alb. 41 ♀ Irish ♂ Irish 35	♀ blk. wt. ♂ alb. 45	—	—	—	6	—	11	—	—
58*a*	♀ brn. ♂ alb. 14 ♀ Irish ♂ Irish 35	♀ alb. (used in Exp. 33) ♂ alb. (used in Exp. 45) 46 Unknown (bought)	♀ alb. ♂ blk. wt. 58*a*	—	2 (3?)	—	10	—	2 (1?)	—	—
58*b*	....	....	♀ alb. sister of ♀ 58*a* above ♂ blk. wt. same as in 58*a*	—	—	—	3	—	2	—	—
53	....	♀ blk. wt. ♂ blk. 34 Unknown	♀ Irish *b* ♂ blk. (used in Exps. 34, 37) 53	—	—	—	—	—	15	6	18*
2	....	Unknown Unknown	♀ Irish *b* ♂ alb. 2	—	—	—	5	1	2	—	—

57	♀ blk. wt. ♂ alb.	25	♀ blk. wt. ♂ blk.	34	♀ Irish *b*		57	—	—	—	7	2	4	—	—
	Unknown														
	♀ alb. ♂ wild	8	♀ brn. ♂ brn.	43	♂ alb.										
	♀ alb. ♂ wild	8													
37	♀ alb. ♂ wild	8	♀ brn. Cr. 2 ♂ alb.	14	♀ alb.		37	—	4	—	9	—	6†		—
	♀ Irish ♂ alb.	2													
			Unknown		♂ blk. (used in Exps. 34, 53)										
34			♀ blk. wt. ♂ alb.	25	♀ blk. wt.		34	—	—	—	4	—	13	—	—
			Unknown		♂ blk. (used in Exps. 37, 53)										
35			♀ blk. wt. ♂ brn. Cr. 2	23	♀ Irish		35	—	—	—	4	—	9		—
	♀ alb. ♂ wild	8													
	♀ alb. ♂ wild	8	♀ brn. Cr. 2 ♂ alb.	14	♂ Irish *a*										
	♀ Irish ♂ alb.	2													
35*a*	♀ blk. wt. ♂ alb.	25	♀ blk. wt. ♂ blk.	34	♀ Irish *b*		35*a*	—	—	—	—	—	4	8	—
	♀ alb. ♂ wild	8	♀ brn. Cr. 2 ♂ alb.	14	♂ Irish *a* (used in Exp. 35)										
	♀ Irish ♂ alb.	2													

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* 5 of these black may have had some white hairs.

† Probably all Irish were "*b*".

No. of Mating	Great Grand Parents	Grand Parents	Parents	Offspring (Crampe's Numbers) Cr. 1	Cr. 2	Cr. 3	Cr. 4	Cr. 5	Cr. 6 *b*	Cr. 6 *a*	Cr. 7
42	....	Unknown .... Unknown	♀ Irish .... ♂ Irish .... } 42	—	—	—	—	4	11*		—
L 1	♀ brn. Cr. 2, half wild ♂ alb. (*ex* 2) .... } 14 ♀ blk. wt. .... ♂ alb. .... } 25	♀ alb. .... ♂ blk. .... } 37 ♀ blk. wt. .... ♂ blk. .... } 34	♀ Irish *b*................ ♂ Irish *b*................ } L 1	—	—	—	1	1	1		—
55	♀ blk. wt. .... ♂ alb. .... } 25	♀ blk. wt. .... ♂ blk. .... } 34 ♀ Irish .... ♂ Irish .... } 42	♀ Irish *b*................ ♂ Irish *a* (nearly blk.) } 55	—	—	—	—	—	4	2	2
56	♀ blk. wt. .... ♂ alb. .... } 25	♀ blk. wt. .... ♂ blk. .... } 34 ♀ Irish .... ♂ Irish .... } 42	♀ Irish *b*................ ♂ Irish *b*................ } 56	—	—	—	7	3	3	1	1

68	♀ blk. wt. ♂ blk. 34 ♀ brn. ♂ alb. 14 ♀ alb. ♂ wild 8 ♀ alb. ♂ wild 8	♀ Irish *b* ♂ Irish *a* 35*a* ♀ brn. ♂ brn. 48	♀ Irish *a* ♂ Irish *a* 68	—	—	—	8	—	—	6	—
64		Unknown Unknown	♀ Irish *a* ♂ Irish *a* 64	—	—	—	—	—	—	10	—
14		♀ alb. ♂ wild 8 ♀ Irish ♂ alb. 2	♀ brn. Cr. 2 ♂ alb. 14	—	—	1	4	2	3		—
21*a*		Unknown ♀ alb. ♂ wild 7	♀ alb. ♂ brn. Cr. 2 (used in Exp. 44) 21*a*	—	2	3	6	1	5		—
44		♀ alb. ♂ brn. half wild 24 ♀ alb. ♂ wild 7	♀ alb. ♂ brn. Cr. 2 (used in Exp. 21*a*) 44	—	5	2	19	3	4		—
24		Unknown ♀ alb. ♂ wild 8	♀ alb. ♂ brn. Cr. 2 24	—	1	2	8	4	3		—
26		Unknown ♀ alb. ♂ wild ...	♀ alb. ♂ brn. Cr. 2 (used in Exp. 23) 26	—	—	1	7	—	1		—

* The Irish were a mixture of *a* and *b*; it is not certain how many of each type.

No. of Mating	Great Grand Parents	Grand Parents	Parents	Offspring (Crampe's Numbers)							
				Cr. 1	Cr. 2	Cr. 3	Cr. 4	Cr. 5	Cr. 6		Cr. 7
									b	a	
36	....	♀ blk. wt. .... 23 ♂ brn. half wild ...... ♀ alb. .... 24 ♂ brn. half wild	♀ brn. Cr. 2 .... 36 ♂ alb. (used in Exp. 41)	—	4	2	4	1	1		—
			Total .... (heterozygous brown × albino)	—	12	11	48	11	17		—
40a	....	....	♀ brn. wt. .... 40a ♂ brn. wt.	—	—	5	2	—	—	—	—
10	....	Unknown .... Unknown	♀ alb. (used in Exp. 8) 10 ♂ brn. wt.	—	—	4	5	—	—	—	—
8	....	....	♀ alb. (used in Exp. 10) 8 ♂ wild........	—	10	—	—	—	—	—	—
48	....	♀ alb. .... 8 ♂ wild........ ♀ alb. .... 8 ♂ wild........	♀ brn. Cr. 2 .... (used in Exp. 14) 48 ♂ brn. Cr. 2 .... (used in Exp. 24)	1*	11	—	7	2	4		—
28	....	Unknown .... ♀ alb. .... 8 ♂ wild........	♀ blk. wt. .... 28 ♂ brn. Cr. 2 .... (used in Exp. 26)	—	5	4	6	4	2		—

* One of the brown young had no white (Cr. 1); some of the others died before this character was determined.

On the Osmotic Pressure of Dilute Aqueous Solutions.

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[*Read* 27 November 1905.]

This paper is an account of a few experiments that were made with dilute solutions during compulsory intervals in work with aqueous solutions of alcohol. The work was thus of an intermittent and preliminary character rather than exhaustive. However, the results that have been obtained are definite in their general bearing.

In experiments with dilute solutions in water there seem to be two reasons why the osmotic pressure set up does not reach its theoretical value. These are due to the impurities absorbed by the outer water from the air and to the slight solubility of the clay of the pot. The pressure set up thus becomes a difference between two osmotic pressures, that of the solution within and that of a solution without. The difficulty arising from contamination of the outer water by the air can be avoided by closing the outer vessel; but the difficulty arising from the solubility of some portion of the pot cannot be overcome for aqueous solutions, where the latter are very weak. For solutions which should give an osmotic pressure above an atmosphere the solubility of the cell is probably too small to interfere appreciably with the theoretical values.

It will thus be seen that this method of direct experiment cannot be used to determine with any accuracy the osmotic pressures of dilute solutions. Sufficient purity of the liquids cannot be ensured under these conditions.

In the experiments given below three cells were used, XIV, XIII and *e*. Each of these had been used shortly before in other experiments and had shown itself capable of bearing internal pressures above three atmospheres. In the interval they were standing in distilled water. Their soundness is assumed from the results of the experiments referred to.

The experiment with cell XIV is with a cane-sugar solution of strength ·056 normal. This is much stronger than the other solutions, but I add it to show that it takes considerable time for a cell to be freed from substances of previous experiments.

Cell XIV. ·056 N cane-sugar solution. Theoretical pressure = 1·26 atmos. The cell after being used for an alcohol solution was washing in distilled water for three days (Oct. 26 to 29). It was then filled with the sugar solution, and after a short time emptied and refilled.

Time (year 1903)	Gauge levels		Temperature (degrees C.)	Calculated pressure corrected for temperature
	narrow-limb	wide-limb		
Thursday, Oct. 29, 4.15	open 202·6	196·8	—	—
	closed 204	196·6	—	—
Friday, Oct. 30, 12.0	231	—	—	—
Friday, Nov. 6, 10.0	335	—	—	—
Tuesday, 12.30	363	168	—	—
Wednesday, 1.50	368·6	167·2	—	—
Friday, Nov. 13, 10.0	375·5	166	12·8	1·57 atmos.
Monday, 12.50	382·6	164·6	12	—
Wednesday, 10.20	386·5	164	12	1·84 atmos.
Thursday, 4.0	388·2	164	13·2	—
Friday, Nov. 20, 10.15	375	165·2	9·2	(Falling)

From these readings it is seen that it took nearly three weeks' time for the pressure to reach the value of 1·84 atmosphere. Both the pressures calculated are above the theoretical value. This is probably due to two causes,—a trace of alcohol from the experiment in which cell XIV was last used, and partial inversion of the sugar during the three weeks of the experiment. If the latter were the *sole* cause of the excess of pressure, we should have

$$i = \frac{\text{observed pressure}}{\text{theoretical pressure}} = \frac{1{\cdot}84}{1{\cdot}26} = 1{\cdot}45,$$

$$\alpha = i - 1 = {\cdot}45.$$

That is 45 °/$_0$ of the sugar was inverted.

I think this is too high to represent the actual state of affairs, and though some of the sugar might be inverted, still most of the excess must be due to some trace of alcohol. This experiment thus affords an example of the length of time that must be allowed in this kind of work for a cell to become clean for osmotic purposes.

The experiments with cells XIII and *e* were performed with more dilute solutions. In each case reference is made to the immediately previous history of the cell.

(1) *Cell XIII.* ·002 N cane-sugar solution. Theoretical pressure = 34 mm. Previous history of cell. On Feb. 9, after washing for more than two months in distilled water which was often changed, the cell was used with a solution of KCl for a week. Then it was washing in water from Feb. 24 to Mar. 15. It was

then used with ·002 N sugar solution. The cell was soaking in this for three days; it was emptied and refilled on Thursday, Mar. 17.

Time	Gauge levels		Temperature (degrees C.)	
	narrow-limb	wide-limb		
Thursday, 11.30	open 263	261	—	
	closed 277	260	10	
Monday	275	260	—	
The cell was emptied and refilled: pressure added.				
Tuesday, Mar. 22	291·6	259	—	Calculated osm. pr. on Apr. 19 = 29·5 − 2 = 27·5 mm. This is below the theoretical value
Tuesday, April 19	288·5	259	11·5	

The cell had been standing through the vacation, and the pressure after 28 days seemed at a steady value. During the interval a higher value may have been reached. The readings on April 19 give a value below that required by theory.

After the above experiment cell XIII was washing for two days, and was then filled with fresh ·002 N sugar solution. After one day, the cell was emptied and refilled with the solution for the experiment, Saturday, April 23, 1904.

(2) *Cell XIII.* ·002 N cane-sugar solution. Theoretical osmotic pressure = 34 mm. at 0° C., Saturday, April 23.

Time	Gauge levels		Temperature
	narrow-limb	wide-limb	
Saturday, Apr. 23 Pressure raised	255·4	219·4	
Monday	247·2	220	Average temperature of the laboratory was 13° C.
Wednesday ...	247	220	
Friday, Apr. 29...	247·2	220	
Friday, May 5 ...	247	220	

The level was thus steady for more than a week, and the osmotic pressure was at the greatest 27 mm. at 13° C.; again below the theoretical value. The pressure in the previous experiment was 27·5 mm. at 11°·5 C.

The experiments with cell XIII where sugar solution was used, and where the cell had been used so long with dilute solutions and had several soakings in water, show that all pressures set up were below the theoretical value. Hence it is most likely that the excess of pressure registered by cell XIV was due to traces of alcohol which had not been got rid of in the time allowed for washing, and was not due to inversion of the sugar.

(1) *Cell e.* ·0012 N solution of potassium sulphate. Theoretical pressure = 20 mm. at 0° C. Previous history of cell. The last experiment for which this cell had been used was one with a 4·6 °/$_0$ solution of cane-sugar to get some values for the rate of rise of pressure. The cell had nearly got up the theoretical value when it was opened, and was still rising. After this it was washing for six days (Jan. 25 to Feb. 1, 1904). On Monday, Feb. 1, it was filled with the above K_2SO_4 solution, closed and standing till Tuesday; it was then emptied and refilled for the experiment.

Time, etc.	Gauge levels		Temperature (degrees C.)	
	narrow-limb	wide-limb		
Tuesday, Feb. 2	open, closed 274	267·6	—	
Wednesday, Feb. 3 ...	277	266·7	10	
Saturday	282·2	265·2	10	
Monday	289·2	264·8	10·2	
Wednesday, Feb. 10 ...	291	264·2	10	
To ensure greater purity of contents, cell refilled; and pressure added				
Friday, Feb. 12	288	265	—	
Saturday	292·6	264·4	10	
Monday	294	264	10	
Wednesday	295	264	9	Maximum pressure
Saturday	295	264	10	
Tuesday	292	—	—	

The maximum readings of the gauge give a difference of level equal to 31 mm. at 10° C. From this has to be deducted the difference of levels when opened to give the excess of internal pressure set up when the cell is closed. Taking this difference to be 6·4 mm. as at the beginning, the osmotic pressure is

$$31 - 6{\cdot}4 = 24{\cdot}6 \text{ mm.}$$

No correction is needed for any capillary depression in the narrow tube, as this will affect both open and closed levels to the same extent.

The theoretical pressure, assuming the gas laws to hold, is 20·7 mm. at 10° C.

Hence
$$i = \frac{24{\cdot}6}{20{\cdot}7} = 1{\cdot}188,$$
$$\alpha = \frac{i-1}{2} \text{ (for three ions)}$$
$$= {\cdot}094,$$
or 9·4 °/$_0$ of the salt is dissociated.

Jones[*], from freezing-point measurements, finds for a ·001 normal solution of potassium sulphate that α = about ·94.

In this experiment the cell may safely be regarded as clean; the result is thus interesting as showing that dissociation can be detected in a direct way, though no satisfactory measure of it can be looked for.

(2) *Cell e.* After the last experiment the cell was washing for four days in changes of distilled water.

On Monday, Feb. 29, it was used with ·00094 N . K_2SO_4 solution. Theoretical osmotic pressure = 16 mm. at 0° C.

Time	Gauge levels		Temperature (degrees C.)
	narrow-limb	wide-limb	
Monday, Feb. 29	open 273	267	
	closed 277	266·1	
Tuesday	277	—	
Thursday	280	266	9
Friday, Mar. 4. Pressure added	292·2	264	
Friday, Mar. 11	280	266	
Wednesday	280	265·6	8

The osmotic pressure set up seems to be 14 − 6 = 8 mm. at 9° C., the steady value for the last five days of the experiment. This is only half the theoretical value at 0° C.

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[*] *Phil. Mag.*, Vol. xxxvi. p. 472.

The solution of this experiment was the weakest used. The low value here set up, compared with that of the experiment before, shows that the effect of impurities which contaminate the outer water is much more marked as the dilution increases.

(3) In the above experiments the outer water in which the cells have been standing was exposed to the air. In the experiment to follow, the outer vessel was closed by a cork, through which the glass of the cell passed. The cork also had a manometer passing through it containing mercury. The whole was rendered air-tight by pouring molten paraffin wax into a mould round the cork, and so covering it with a good layer of wax. That this was air-tight was shown by the manometer steadily standing at different levels under barometric changes. Since the osmotic pressure is the excess of the internal pressure set up over the atmospheric pressure, in this experiment corrections had to be made for the changes in the manometer levels.

Cell e. ·0034 N solution of potassium sulphate. Theoretical osmotic pressure at 0° C. = 57·7 mm.

Time, etc.	Gauge levels		Corrections to difference of levels	Temperature (degrees C.)	Osmotic pressure at given temperature
	narrow-limb	wide-limb			
Thursday, July 21 ...	open 65·2	58·2	—	—	—
Pressure added.. ...	111	53·6	—	18·4	—
Saturday	126·2	51·4	—	18·5	—
Monday	132	51	– 2·5 mm.	18·4	78·5 mm.
Tuesday	131	51	—	18·1	—
Wednesday	126	51·4	+ 4	17·6	78·6
Thursday	119	52·4	+ 12	18	78·6
Thursday (week later)	86·5	56·2	+ 23	19·3	53·3

Followed by steady fall.

From the table it will be seen that the maximum pressure was one of 78·6 mm. at an average temperature of 18° C., and remained steady through four days.

The theoretical pressure at 18° C. = 61·46 mm.

Hence $$i = \frac{78·6}{61·46} = 1·278,$$

$$\alpha = \frac{·278}{2} = ·139,$$

or 13·9 °/ₒ of the salt is dissociated.

According to Jones'[*] cyroscopic determinations, α is about ·90.

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[*] *Phil. Mag.*, Vol. xxxvi. p. 472.

This experiment thus shows that when the influence of the impurities dissolved from the air is provided against, the experimental values for the osmotic pressure approach more closely to the theoretical ones.

It was after the last experiment that a preliminary experiment was made in order to examine the deterioration of the outer water by changes in its electrical resistance.

By the telephone method, the resistance of the water before the experiment was 12875 ohms at 18°·4 (the mean of five readings). After the experiment, after having been in contact with the cell for nearly three weeks, the water was again examined under the same conditions: its resistance was found to be 2014 ohms at 19°·3; or one-sixth of its former value.

Arrangements were then made to carry out such an examination of the water with greater care.

Owing to other work, this examination of the resistance of the outer water at intervals had to be put off for a considerable time, during which cell *e* was washing in distilled water.

The telephone method could not be used with sufficient accuracy for the high resistances offered by the water used at first. A commutator of the kind described and used by Fitzpatrick* and Whetham† was set up. It was driven by a water-motor.

Water redistilled over alkaline permanganate and then over acid potassium sulphate was used. It had an absolute conductivity of $3{\cdot}3 \times 10^{-6}$.

Cell *e* was standing in the glass bottle used for the outer vessel from July 1904 to January 1905. The distilled water was frequently changed. During the last week before the experiment it was standing in and was filled with the redistilled water. The cell was therefore as clean as it well could be. The electrodes were of platinum, covered with platinum black and then heated to redness. They were between the outer vessel and the cell, and about half-way down the latter. The outer vessel was closed to the air by means of a cork covered with molten wax and was standing in a water-bath fitted with a thermostat. The temperature of the bath was fairly steady, and only occasionally was the temperature raised above that of the room in order to get the temperature coefficient for purposes of correction for small changes in the temperature of the water-bath. These corrections seldom affected the observed values above the second place of decimals.

I should say that the constant of the electrolytic cell in which the conductivity of the water was obtained, was found by using a standard solution of potassium chloride. Its diameter was about the distance between the cell *e* and the outer vessel. The actual resistance of the water in the electrolytic cell was about 42200 ohms; that of the water, after arranging for experiment in the outer vessel with the same electrodes, was 42000 ohms. The constant (= ·139) of the electrolytic cell was therefore used in the conductivities given below, from which values Curve I is drawn. These are corrected to 10°·4 C.

---

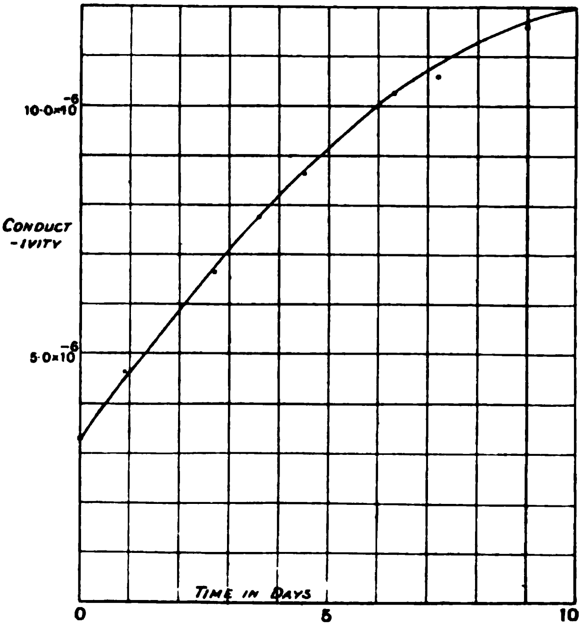
* *B. A. Report*, 1886, p. 328.

† *Phil. Trans.* A, Vol. cxciv. p. 330 (1900).

$$\text{Absolute conductivity} = \frac{\text{cell constant}}{\text{experimental resistance}} = \frac{\cdot 139}{R}.$$

Date	Conductivity at 10°·4 C.	Date	Conductivity at 10°·4 C.
Jan. 27	$3{\cdot}3 . 10^{-6}$	Feb. 1	$8{\cdot}62 . 10^{-6}$
„ 28	4·63	„ 3	10·23
„ 30	6·63	„ 4	10·60
„ 31	7·75	„ 6	11·64
		„ 7	12·00

CURVE I. Change of conductivity of outer water.

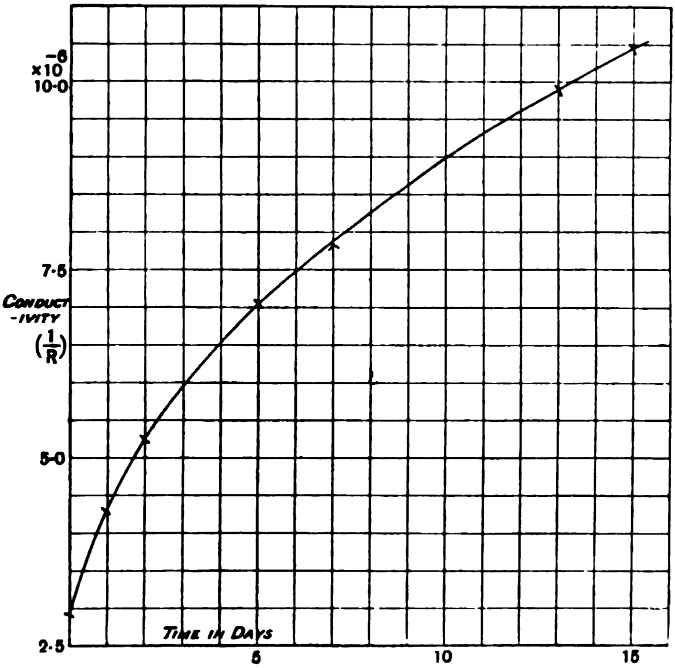


The apparatus was taken down in order to add fresh redistilled water and to examine the working of the commutator. The contacts of this were cleaned and readjusted, and its working tested by using a coil in the place of the cell-resistance. The readings were taken, as throughout, for the resistance and the battery reversed. For the coil, the mean resistance with the commutator still was 979·88 ohms; with the commutator working it was 979·75 ohms. From time to time during the experiments, the coil was put in to see that the commutator was working satisfactorily.

The water used in the second set of readings was rather better than the other, having an absolute conductivity below 3·0 × 10⁻⁶ to begin with. The conductivities are obtained from measurements extending over 15 days; they are corrected to 10°·5 C. The outer vessel was left open to the air.

Date	Conductivity	Date	Conductivity
March 2	$2{\cdot}93 \,.\, 10^{-6}$	March 8	$7{\cdot}48 \,.\, 10^{-6}$
„ 3	4·28	„ 9	7·81
„ 4	5·25	„ 13	9·31
„ 6	6·52	„ 15	9·92
„ 7	7·05	„ 17	10·45

Curve II.



The differences in the general character of these curves are important. In Curve I, where the water was not exposed to the air, the conductivity increases at first uniformly with the time, almost as a straight line, falling off after the sixth day. The flattening at the end indicates that the curve is tending to a constant value. In Curve II, the more curved portion is at the beginning, and the latter part is practically a straight line with the same slope (very little steeper) for the sixth day as for the fifteenth. These differences are caused by the additional disturbing effect of the air in Curve II.

In Curve I, the solution obtained from the soluble portion of the pot, more and more nearly reaches its saturation strength, owing to the very small quantity that will dissolve, and consequently the rate of change in the conductivity of the water falls off. In Curve II, the further portions of the curve are kept from flattening by the impurity which is steadily absorbed from the air. It will take a much longer time for the water to become saturated with very soluble carbon dioxide (which will be the chief impurity) than with the slightly soluble part of the clay; hence the curve becomes practically a straight line and apparently continues as such for some distance beyond the limits of the diagram.

There is one other experiment recently performed, which shows in a direct way that the osmotic pressure is very considerably reduced by the impurities in the outer water.

Cell *e* had been left through the vacation (from March 27 to April 28) with a solution of potassium sulphate that should give a pressure of 219 mm. On March 27, the cell was left with the gauge level at 235·8 (osmotic pressure attained = 208 mm. at this level), the cell being in a small outer vessel. A month later, the level was at 92·6, the cell being well under water outside. The comparatively small volume of the outer water must have become very impure during this month. It was removed and replaced by freshly boiled distilled water at the temperature of the laboratory. The following were the gauge levels during the next few days.

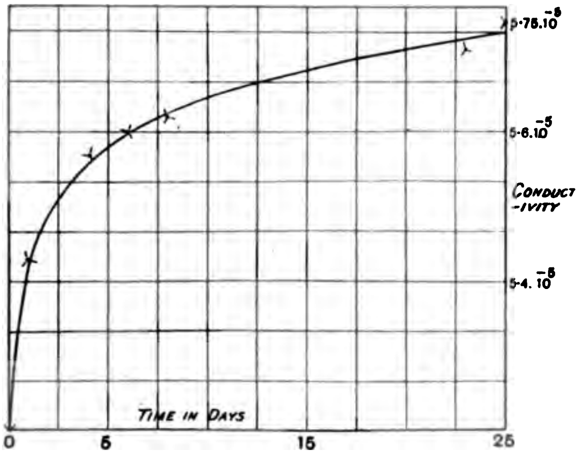
Time, etc.	Gauge level	Temperature of water (degrees C.)	
Friday, April 28	92·6	11·8	
Saturday	109	11·5	
Monday, May 1	127·6	11·4	
Tuesday	131·8	11·6	
Outer water replaced by new water			
Wednesday	136	11·5	
Thursday	138·6	11·5	Rise of 46 mm.
Monday, May 8	136·6	11·6	Falling

With the fresh water present, the level of the gauge rose through 46 mm. in six days, new water being placed outside on the fourth day. On the sixth day, after the second water was added, there were signs of falling, showing that the impurity of the outer water was beginning to have its effect.

Conclusion.

Hence, the general conclusion to which the experiments of this paper point is, that, owing to the impurities received by the outer water from the clay of the pot and the air, the osmotic properties of very dilute aqueous solutions cannot be measured directly by this method, and as formerly we shall have to depend (at least for the present) on freezing-point and boiling-point methods.

CURVE III.



Note. Since the body of this paper was written, I have, at the suggestion of Professor Thomson, examined water containing kaolin for changes in its electrical conductivity. The kaolin used was washing for a week in several changes of distilled water, before being used for the experiment, in order to remove traces of any foreign soluble matter. Finally the kaolin was filtered off and washed with redistilled water. It was then placed in the outer vessel which was used with cell *e* above, and covered to a convenient depth with redistilled water. The vessel was closed to the air and placed in a water bath. The same method of taking the resistance was used. The following values of the conductivity $\left(=\frac{1}{R}\right)$ at 14° C. were obtained, and from them the curve above is drawn.

Date	Conductivity	Date	Conductivity
June 13	5·20 . 10^{-5}	June 21	5·62 . 10^{-5}
„ 14	5·43	July 6	5·71
„ 17	5·57	„ 8	5·75
„ 19	5·60		

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PROCEEDINGS

OF THE

Cambridge Philosophical Society.

The expansion of a gas into a vacuum and the determination of the specific heat at constant pressure for gases. By G. F. C. SEARLE, M.A., F.R.S., University Lecturer in Experimental Physics, Cambridge.

[*Read* 29 January 1906.]

§ 1. *The expansion of a gas into a vacuum.*

In an important thermodynamical experiment, first made by Gay Lussac, gas is forced into a receiver till a high pressure is reached and then the pressure is allowed to fall by opening a communication between the receiver and an exhausted vessel. The experiment was repeated, with modifications, by Joule, who found that, when thermal equilibrium between the parts of the gas had been re-established by means of the water in which the apparatus was immersed, the temperature of this water was not sensibly changed by the expansion. When the change of temperature of the water is *accurately* zero, it follows that the internal energy of a gramme of the gas depends only upon its temperature and not upon its volume. But it may be useful to consider a case in which this ideal experimental result would not be obtained, and to show how the change of temperature of the gas, when the vessels are impermeable to heat, may be calculated for a gas obeying Van der Waals' equation

$$\left(p+\frac{a}{v^2}\right)(v-b)=Rt \quad \text{.......................(1).}$$

Here, and throughout this communication, the symbol t denotes the absolute temperature.

If U be the internal energy and ϕ the entropy of one gramme of gas, and if the heat taken in by the gas be measured in ergs, we have, for isothermal expansion,

$$dU=td\phi-pdv, \quad (t \text{ constant}),$$

since the increase of energy is equal to the heat absorbed *minus* the work done. Thus

$$\frac{dU}{dv_t} = t\frac{d\phi}{dv_t} - p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

By a thermodynamical formula, this may be written

$$\frac{dU}{dv_t} = t\frac{dp}{dt_v} - p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3).$$

But, by (1),

$$p = \frac{Rt}{v-b} - \frac{a}{v^2},$$

and thus

$$\frac{dU}{dv_t} = \frac{a}{v^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

Integrating (4), we obtain

$$U = f(t) - \frac{a}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5),$$

where $f(t)$ is some function of t. This function is closely related to the specific heat at constant volume, for

$$C_v = \frac{dU}{dt_v} = \frac{df(t)}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6).$$

We may notice, in passing, that, according to (6), the specific heat C_v depends only upon the temperature and not upon the volume of unit mass.

In the experiment under consideration no external work is done by the gas, and, when the vessels are impermeable to heat, no heat escapes. Thus, the internal energy of unit mass, after thermal equilibrium has been established, is unchanged by the expansion. Hence, if the volume of one gramme increase from v to v' c.c., and if, at the same time, the temperature change from t to t', we have, by (5),

$$f(t') - \frac{a}{v'} = f(t) - \frac{a}{v}.$$

From (6) we see that

$$f(t') - f(t) = \int_t^{t'} C_v dt,$$

and hence

$$\int_t^{t'} C_v dt = -a\left(\frac{1}{v} - \frac{1}{v'}\right).$$

If the range of temperature be small, we may treat the specific heat C_v as constant over that range, and then we may write

$$t - t' = \frac{a}{C_v}\left(\frac{1}{v} - \frac{1}{v'}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7).$$

This equation, then, shows the fall of temperature $t-t'$ due to the expansion from v to v'.

In the case of CO_2, Van der Waals found

$$\left(P+\frac{0\cdot00874}{V^2}\right)(V-0\cdot0023)=\frac{1\cdot00646}{273}t,$$

where P is the pressure in atmospheres, and V is the ratio of the volume of 1 gramme to its volume (V_0) at normal temperature and pressure. Since one atmosphere is equivalent to $1\cdot014\times10^6$ dynes per square cm. and since V_0 is 505 c.c., Van der Waals' equation, when expressed in C.G.S. units, becomes

$$\left(\frac{p}{1\cdot014\times10^6}+\frac{0\cdot00874\times505^2}{v^2}\right)\left(\frac{v}{505}-0\cdot0023\right)=\frac{1\cdot00646}{273}t,$$

or $$\left(p+\frac{2\cdot26\times10^6}{v^2}\right)(v-1\cdot16)=1\cdot89\times10^6t.$$

Thus $a=2\cdot26\times10^6$ cm.^{5} grm.$^{-1}$ sec.$^{-2}$.

$b=1\cdot16$ c.c. per gramme.

$R=1\cdot89\times10^6$ ergs per gramme per degree.

According to Joly the mean specific heat at constant volume over the range from about 15°C. to 100°C. varies appreciably with the pressure—an indication that Van der Waals' equation does not completely represent the facts of the case. For the present purpose, however, C_v may be taken as $0\cdot17\times4\cdot2\times10^7$ or $7\cdot14\times10^6$ ergs per gramme per degree.

If in the expansion experiment the initial volume of 1 gramme be 50 c.c. and the initial temperature 273°, the initial pressure will be about 10 atmospheres. If the final volume be 100 c.c., we have

$$t-t'=\frac{2\cdot26\times10^9}{7\cdot14\times10^6}\left(\frac{1}{50}-\frac{1}{100}\right)=3\cdot16^\circ.$$

Thus, in the case of CO_2, when the pressure falls from about 10 to about 5 atmospheres there is a cooling of about 3°C.

By equating the energy of the whole of the gas in its final state to the sum of the energies of the separate parts in their initial states, it is easily shown that when two vessels of volumes V_1 and V_2 c.c. containing M_1 and M_2 grammes of gas at temperatures t_1 and t_2 are put into communication, the temperature t' attained when thermal equilibrium has been established, is, for small ranges of temperature, given by

$$\begin{aligned}C_v(M_1+M_2)t'&=C_v(M_1t_1+M_2t_2)-a\left\{\frac{M_1^2}{V_1}+\frac{M_2^2}{V_2}-\frac{(M_1+M_2)^2}{V_1+V_2}\right\}\\&=C_v(M_1t_1+M_2t_2)-\frac{a(M_1V_2-M_2V_1)^2}{V_1V_2(V_1+V_2)}.\end{aligned}$$

§ 2. *The determination of the specific heat at constant pressure for gases.*

In the experimental determination of the specific heat of a gas at constant pressure, the gas is sent through a spiral tube immersed in water in a calorimeter, and the temperature of the gas before and after passing through the tube is observed as well as the rate of change of the temperature of the water in the calorimeter. When the pressure of the gas is the same on both sides of the spiral, the experiment gives the specific heat of the gas directly, but, in practice, there must be a small difference of pressure, and thus some small correction becomes necessary.

The process is really an extreme case of the "porous plug" experiment, and is best considered from the point of view of that experiment.

We may begin with the case of a "perfect" gas, *i.e.* a gas obeying Boyle's law and having its internal energy dependent only upon its temperature. The second of these conditions implies that dU/dv_t is zero. Thus, by (3)

$$t\frac{dp}{dt_v}=p.$$

Integrating, we have

$$\log p+G(v)=\log t,$$

or

$$pF(v)=t,$$

where $G(v)$ and $F(v)$ are functions of v alone. But, since Boyle's law is obeyed, $F(v)=v/R$, where R is a constant. The characteristic equation is therefore

$$pv=Rt \qquad\qquad (8).$$

Since U is a function of t alone, we may write

$$C_v=\frac{dU}{dt} \qquad\qquad (9).$$

It is unnecessary to add a subscript v to the differential coefficient to show that v is constant during the differentiation, because U is independent of v.

If we equate the heat absorbed, when the gas is heated at constant pressure, to the sum of the energy gained by the gas and the work done by it, we have

$$C_p dt=\frac{dU}{dt}dt+p\frac{dv}{dt_p}dt,$$

so that

$$C_p=C_v+p\frac{dv}{dt_p}$$

$$=C_v+R \qquad\qquad (10).$$

Suppose, now, that, as the gas enters the spiral tube, its state is defined by p, v, t and that, on leaving, its state is defined by p', v', t', and that each gramme of gas, as it passes through the tube, absorbs H ergs of heat from the calorimeter. Suppose, further, that the flow is very slow so that the conditions may be treated as "steady" at each instant, and so that the kinetic energy of the gas at the places where t and t' are measured may be negligible. Then we have

$$pv - p'v' + H = U' - U,$$

or, by (8),
$$H = R(t' - t) + U' - U.$$

But, by (9),
$$U' - U = \int_t^{t'} C_v dt,$$

and thus
$$H = R(t' - t) + \int_t^{t'} C_v dt,$$

or, by (10),
$$H = \int_t^{t'} C_p dt.$$

Hence we see that, when the gas is perfect, the heat absorbed by one gramme of gas is equal to the range of temperature multiplied by the mean specific heat at constant pressure for that range. Since, by (9), C_v depends only upon the temperature, it follows from (10) that C_p depends only upon the temperature, and thus it is not necessary to know the pressure of the gas either before or after its passage through the spiral, or to take any precautions to ensure that the fall of pressure of the gas on passing through the spiral should be small.

The text-books lay stress on the importance of ensuring that this fall of pressure is very small; this is illustrated by the following extracts which refer to Regnault's experiments: (A)[1] "It was further necessary to ascertain if the pressure of the gas was the same at entering as at leaving the calorimeter. If that is not the case, the gas will have expanded in the calorimeter, doing work and absorbing heat and a consequent error will be introduced[2]. Water manometers placed at the entrance and the exit showed no more than 1 mm. difference so that the error arising from this cause was quite insensible." (B)[3] "Its pressure at entry and emergence was shown by subsidiary experiments to differ by not more than 1 mm. of water. Hence the pressure was practically constant. We see then that a known weight of gas at constant pressure was cooled in the calorimeter by an observed mean amount."

---

[1] Preston, *Theory of Heat*, 2nd ed., p. 275.

[2] There appears to be some confusion here. A gas necessarily expands when its temperature rises and its pressure remains constant.

[3] Poynting and Thomson, *Heat*, p. 70.

From such statements a student would not unnaturally draw the conclusion that, if the pressure at entry had exceeded that on emergence by the pressure due to a few centimetres of water, the heat absorbed from or given out to the calorimeter while one gramme passed would have differed appreciably from the product of the change of temperature and the mean specific heat. But we have seen that, for a "perfect" gas the difference of pressure is absolutely without effect upon the result and thus we are led to expect that, for gases which are almost perfect, the effects of differences of pressure will be practically negligible.

The correction which is necessary in the case of an imperfect gas can be at once written down, when the results of the Thomson-Joule porous plug experiment for that gas are known.

If H ergs of heat be absorbed from the calorimeter while one gramme of gas passes through the spiral, we have

$$pv - p'v' + H = U' - U \quad \text{.................}(11),$$

where p, v, U refer to the gas as it enters and p', v', U' refer to the gas as it leaves the spiral. Here we are only concerned with the initial and final states of the gas and with the total amount of heat absorbed from the calorimeter during the passage of one gramme of gas through the spiral. We may therefore suppose that the gas passes through a porous plug before it reaches the calorimeter, and that there is no fall of pressure in the spiral itself. If we further suppose that no heat enters or leaves the gas as it passes through the porous plug, the heat absorbed from the calorimeter is the same as in the actual experiment. The gas, which enters the plug at temperature t, leaves it at a temperature t'', the difference $t'' - t$ being calculable from the results of the Thomson-Joule experiment in terms of the fall of pressure $p - p'$.

If p', v'', U'' refer to the gas as it leaves the porous plug, the energy equation, corresponding to the passage of the gas through the spiral at the constant pressure p', becomes

$$p'v'' - p'v' + H = U' - U''.$$

In finding $U' - U''$ we may take the gas from one state to the other by *any* path on the pv diagram, since $U' - U''$ depends only on the two states. The most convenient path is obviously the one in which the pressure has the constant value p'. Then $U' - U''$ is equal to the heat absorbed by one gramme of gas along this path *minus* the work done by it along the same path. Hence, we have

$$p'v'' - p'v' + H = \int_{t''}^{t'} C_{p'}\, dt - p'(v' - v'').$$

Thus
$$H=\int_{t''}^{t'} C_{p'}\,dt,$$

or
$$H=\int_{t}^{t'} C_{p'}\,dt-\int_{t}^{t''} C_{p'}\,dt.$$

Here we must specify that the specific heat is that at the constant pressure p', for in an imperfect gas the specific heat may depend upon the pressure[1]. Since the specific heat C_p depends only slightly upon the temperature in the case of actual gases, we see that the correction to be applied to the change of temperature observed in the actual determination is approximately equal to that which occurs in the Thomson-Joule experiment, when the initial temperature is t and the pressure falls from p to p'.

The results obtained by Thomson and Joule[2] for air show that, when p' is one atmo and t is 373° on the absolute scale, there is a cooling of about one-seventh of a degree for a fall of pressure of one atmo, or about 1/7000 degree for a fall of pressure corresponding to one centimetre of water. Thus, if the difference of pressure correspond to h cms. of water, we have approximately

$$t''-t=-h/7000.$$

Thus, approximately,

$$H=\int_{t}^{t'} C_{p'}\,dt+\frac{h}{7000}C_{p'}.$$

Hence, if the pressure of the air on entering the spiral exceed its pressure on leaving the spiral by so much as corresponds to 10 cms. of water, the correction to be applied to the observed difference of temperature is only about $\frac{1}{700}$ degree. The difference of temperature would probably never be less than 10° C. in an actual experiment, so that the correction is practically negligible.

If the results of the Thomson-Joule experiment be not known for any particular gas, the correction can still be calculated when the characteristic equation of the gas is known. To simplify the calculation in the case of a gas obeying Van der Waals' equation, terms involving the product ab and squares and higher powers

---

[1] Since for any substance $\frac{dC_p}{dp_t}=-t\frac{d^2v}{dt_p^{\,2}}$, it follows that the characteristic equation of a substance, for which C_p is independent of the pressure, is

$$\{v+f(p)\}\,g(p)=Rt,$$

where $f(p)$ and $g(p)$ are any functions of p. If this equation is not satisfied, C_p will depend upon the pressure.

[2] Lord Kelvin, *Mathematical and Physical Papers*, Vol. I. p. 419.

of a and b may be neglected. The equation then takes the modified form

$$pv - pb + \frac{a}{v} = Rt \quad \text{....................(12).}$$

Let the points A, B in the figure represent, on the pv diagram, the state of the gas on entering and leaving the spiral. Then, in finding the difference between U', the energy of one gramme of gas at B, and U the energy at A, we may take the gas by any path we please. The most convenient path is ACB, which is made up of the isothermal AC passing through A and the isopiestic CB passing through B. Let the areas into which the diagram is divided by lines of constant pressure and constant volume be indicated by the letters placed in them. If $C_{p'}$ be the specific heat at the constant pressure p' and if Q be the heat absorbed by one gramme of gas as it expands, at constant temperature, from A to C, we have

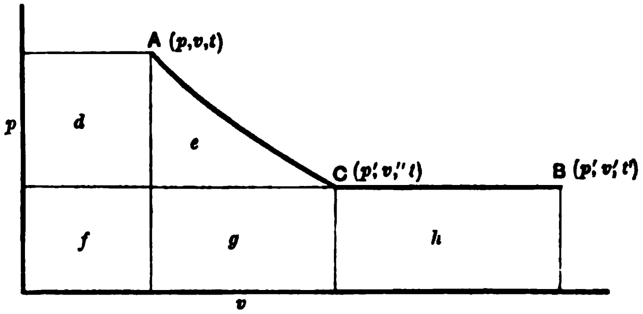

Fig. 1.

$$U' - U = Q + \int_t^{t'} C_{p'}\,dt - \text{work done in path } ACB$$

$$= Q + \int_t^{t'} C_{p'}\,dt - (e + g + h).$$

But $$Q = \int_p^{p'} t\,\frac{d\phi}{dp_t}\,dp = -\int_p^{p'} t\,\frac{dv}{dt_p}\,dp,$$

the temperature having the constant value t in the last two integrals.

Now, if H ergs of heat be absorbed from the calorimeter by each gramme of gas as it passes through the spiral,

$$pv - p'v' + H = U' - U.$$

Hence

$$
\begin{aligned}
H - \int_t^{t'} C_{p'} dt &= p'v' - pv + Q - (e + g + h) \\
&= (f + g + h) - (d + f) + Q - (e + g + h) \\
&= Q - (d + e) \\
&= \int_p^{p'} \left(v - t\frac{dv}{dt_p}\right) dp \quad \text{.......................(13),}
\end{aligned}
$$

the temperature having the constant value t on the right side.

From the modified Van der Waals' equation (12) we have

$$v - t\frac{dv}{dt_p} = \frac{Rt + pb - a/v}{p} - \frac{Rt}{p - a/v^2}.$$

Expanding the second denominator by the binomial theorem, we obtain, to the order of accuracy which we have adopted,

$$v - t\frac{dv}{dt_p} = b - \frac{a}{pv} - \frac{aRt}{p^2v^2}.$$

To the same order of accuracy we may replace pv by Rt, and thus

$$v - t\frac{dv}{dt_p} = b - \frac{2a}{Rt}.$$

Hence

$$H - \int_t^{t'} C_{p'} dt = -(p - p')\left(b - \frac{2a}{Rt}\right).$$

If $p - p'$ be the difference of pressure due to a height of h cm. of water, we have $p - p' = 981\,h$, and thus we find that, when CO_2 enters the spiral at 373° on the absolute scale,

$$
\begin{aligned}
H - \int_t^{t'} C_{p'} dt &= -981\left(1{\cdot}16 - \frac{2 \times 2{\cdot}26 \times 10^9}{1{\cdot}89 \times 10^6 \times 373}\right) h \\
&= 5150\,h \\
&= \frac{8{\cdot}4 \times 10^6}{1630}\,h \text{ ergs per gramme.}
\end{aligned}
$$

Taking $C_{p'}$ as $0{\cdot}2 \times 4{\cdot}2 \times 10^7$ or $8{\cdot}4 \times 10^6$ ergs per gramme per degree, we see that the correction arising from a difference of pressure of h cm. of water amounts to only $h/1630$. Thus when $h = 10$, the correction amounts to 1/163 of a degree. Though this is about four times as great as the correction we found was necessary in the case of air, it is still practically negligible.

Thanks are due to Prof. J. H. Poynting, F.R.S., for his criticism and assistance in connexion with this paper.

A Novel Instrument for Illustrating the Magnetic Properties of Iron. By A. H. PEAKE, M.A., St John's College.

[*Read* 29 January 1906.]

The instrument here described is one which was designed by the author in order to illustrate, with as simple apparatus as possible, the phenomenon of magnetic hysteresis.

This instrument, of which a view is given in Fig. 1, is self-contained, that is to say, no other apparatus or instruments are required in conjunction with it, neither does it require any electric current, the magnetising forces being produced by permanent magnets which form part of the instrument.

Principle of the Instrument.

The principle on which it is based is as follows:—

If a sample of iron, which is very thin in proportion to its length, is suspended in a uniform magnetic field, the magnetising force, to which the sample is subjected, is equal to the resolved component of the field in the direction of the length of the specimen, and the induction produced is proportional to the turning moment experienced by the sample, when the plane of the sample makes but a slight inclination with the plane normal to the lines of force. In this particular instrument the turning moment is balanced by a gravitational control, and the same pointer serves to read on two different scales both the angle of inclination of the specimen to the field, and hence the magnetising force H, and also the angle turned through from the direction assumed when under the action of gravity alone, this latter angle being a measure of the induction B produced. See Figs. 2 and 3.

Details of the Instrument.

The sample used is a strip of sheet iron, or wires arranged side by side so as to approximate in form to a strip; they are three inches in length, and may be of any width up to five-eighths of an inch. They are supported in a cradle, to which a long aluminium pointer and an adjustable brass control weight are attached; the cradle with the pointer and control weight are carried on two steel points, which are pivoted, the one in an agate cup of conical shape, and the other in a V-shaped trough of agate, so that after removing the cradle for the insertion of a sample, the steel points are easily replaced in their bearings in exactly the same position as before. The sample is pushed into position in the cradle up to a stop provided for the purpose, where it is held by a brass spring; when in position its centre of gravity is in line with the two steel points which define the axis about which it may turn, and consequently the weight of the sample in no way affects the gravitational controlling force to which the pivoted portion of the apparatus is subject.

The axis of the portion of the apparatus just described is in the same line with the axis of a turn-table of brass, to which the

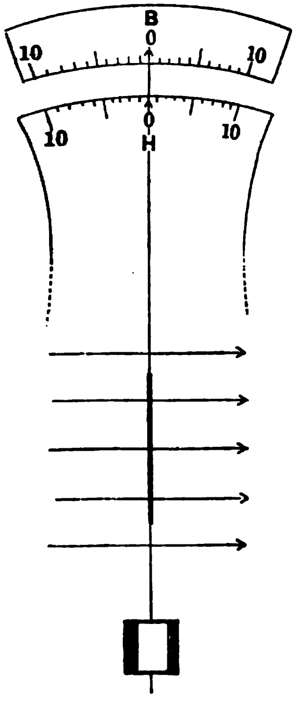

Fig. 2.

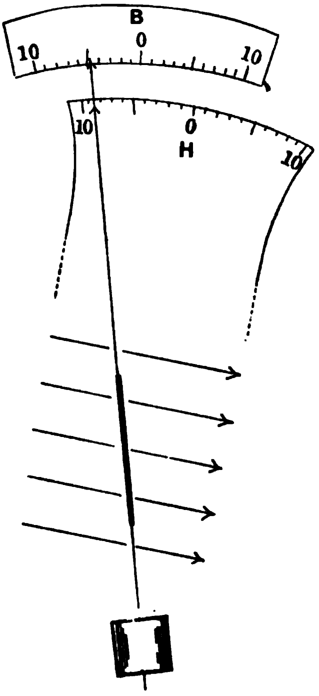

Fig. 3.

permanent magnets providing the magnetic field are bolted; these magnets are built up of sixteen bar-magnets, placed so as to give a strong horizontal field at right angles to the axis of the turn-table. The turn-table is attached to a wooden stand, and may be turned by means of a nut and screw on the right-hand side of the instrument so as to incline the direction of the field a few degrees to the horizontal either way; "back-lash" is taken up by a strong spiral spring, which causes the nut to bear always in one direction against its support. One scale of the instrument, the *H* scale, is rigidly attached to the turn-table, while the other, the *B* scale, is fastened to the back of the stand. A strip of looking-glass is provided to avoid parallax in reading the position of the index; this index consists of a fine wire at the end of the aluminium pointer, stretching across both scales. A damping vane is attached to the end of the rod supporting the control weight, and generally is immersed in an oil dash-pot.

Method of Operation.

The stand is levelled by means of the levelling screw in the base; this is best done by putting the cradle in position without any iron in it, and then turning the levelling screw until the index points to the zero of the *B* scale; the zero of the *H* scale should then be brought into line with the pointer, and the instrument is then ready to receive the specimen. After the insertion of the sample in the cradle, the latter is replaced in position, and if the sample is not magnetised the index will again be over the zero of both scales, the plane of the specimen will be vertical, and normal to the lines of force due to the permanent magnets. Then by gently turning the turn-table, avoiding jerks which would cause the pointer to oscillate, the specimen may be subjected to any desired cycle of magnetising force, and the corresponding readings on the *B* scale may be observed. The whole time required to take the observations for a complete hysteresis cycle is but a few minutes.

Demagnetising of Sample.

To demagnetise a sample of iron that may have been magnetised previous to testing, all that is necessary is to place it in position with the oil dash-pot removed, and start it swinging; this demagnetises the sample by reversals, and it soon comes to rest, owing to the damping action of the hysteresis loss.

It was found by experiment that the *BH* curve obtained by slowly increasing *H* after demagnetisation was the same as that obtained by reversing several times successive values of *H*, and hence, the former method being much more rapid, it was the one used in obtaining the "reversal curve."

Calibration of Instrument.

The calibration of the scales of the instrument has not been carried out with any great refinement, as the instrument is intended for illustrating principles, and is not suitable for determining quantitatively the actual hysteresis loss in the samples tested; nevertheless it was thought advisable to obtain approximately the value of the scale divisions in ordinary units, and also to determine the reduction in the magnetising force, resulting from the actual production of magnetic induction in the specimen.

Correction for Demagnetising Action of Poles.

The correction for the demagnetising action due to the flux produced is proportional to the total flux, and was determined by obtaining the *BH* curves for samples consisting of 2, 4, 6, and 8 similar iron wires; these were plotted with readings of the *H* scale for abscissae, and readings of the *B* scale, divided by the number of wires in sample, for ordinates; it was then seen that by shearing back the axis corresponding to *H* = zero by an amount equal to the *B* scale reading, the curves became nearly coincident, and consequently if this axis is always sheared back by this amount, the resulting curve referred to rectangular coordinates is corrected for the demagnetising action.

Fig. 4 shows the curves thus plotted for the samples consisting of 4 and 8 wires respectively.

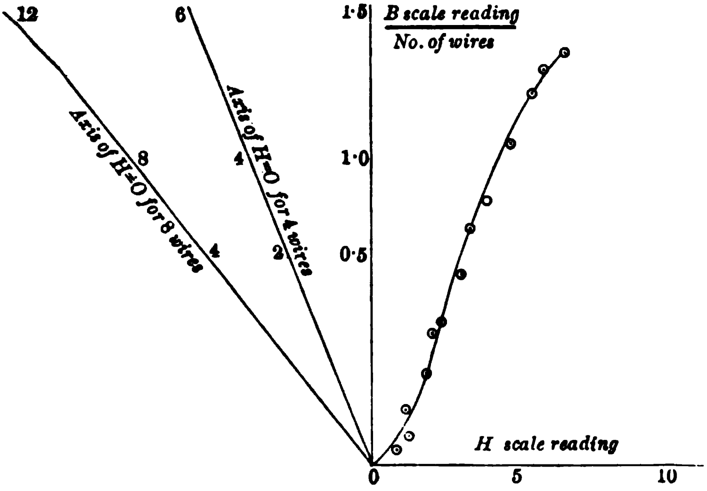

Fig. 4.

Value of Scale Divisions.

The approximate value of the scale divisions in C.G.S. units was determined by obtaining first of all the controlling or restoring couple for a deflection of one division, and then with a sample of known maximum permeability in the instrument a *BH* curve was obtained, these data, together with the dimensions of the sample, being sufficient for the purpose.

The controlling couple was determined both by direct observation of the deflection produced by a known weight suspended at a measured distance from the axis, and also by observing the periodic time of the cradle portion for small oscillations, firstly under normal adjustment and then with the addition of a brass bar, which was lashed securely to the pointer, the change in the moment of inertia and in the gravitational restoring couple of the swinging system being easily calculated from measurements of the dimensions, position, and weight of the added rod.

In the first of these methods a 20 gramme weight when suspended at a horizontal distance from the axis of 3·86 cms. produced a deflection of 12·3 divisions on the *B* scale, hence the controlling force per div. $= \dfrac{20 \times 3{\cdot}86}{12{\cdot}3}$ gramme cms., or

$$\frac{20 \times 3{\cdot}86 \times 981}{12{\cdot}3} = 6{,}160 \text{ dyne cms.}$$

In the second method the time for a complete oscillation under normal adjustment was 0·782 seconds. Let us call this T_0.

Let I_0 and G_0 represent respectively the moment of inertia of the system about the axis of suspension and the controlling couple in dyne cms./radians for very small angles of displacement, under this same adjustment, before the addition of the brass rod.

Let also T_1, I_1, and G_1 stand respectively for the new periodic time, etc., after the addition of the brass rod.

T_1 was found to be 1·18 secs.

Then

$$(1) \quad T_0 = 2\pi \sqrt{\frac{I_0}{G_0}},$$

$$(2) \quad T_1 = 2\pi \sqrt{\frac{I_1}{G_1}}.$$

These equations may be written thus:

$$(1) \quad I_0 = G_0 \left(\frac{T_0}{2\pi}\right)^2,$$

$$(2) \quad I_1 = G_1 \left(\frac{T_1}{2\pi}\right)^2.$$

Subtracting

$$I_1 - I_0 = (G_1 - G_0)\left(\frac{T_1}{2\pi}\right)^2 + G_0 \frac{T_1^2 - T_0^2}{(2\pi)^2},$$

or

$$G_0 = \frac{(2\pi)^2 \{I_1 - I_0\} + (G_0 - G_1)\, T_1^2}{T_1^2 - T_0^2}.$$

$$\left.\begin{array}{l} \text{Now } I_1 - I_0 = \text{change in moment of inertia} \\ \text{and } G_0 - G_1 = \text{change in gravitational control} \end{array}\right\} \text{due to addition of brass rod.}$$

The particulars of brass rod were as follows:

Length 10·75 cms.
Width 1·25 cms. (Negligible in calculating *I*.)
Centre of gravity 8·62 cms. above axis.
Weight 36·74 grammes.

Consequently

$$I_1 - I_0 = 36{\cdot}74\left\{8{\cdot}62^2 + \frac{1}{3}\left(\frac{10{\cdot}75}{2}\right)^2\right\} = 3{,}080 \text{ gramme cms.}$$

and

$$G_0 - G_1 = 36{\cdot}74 \times 8{\cdot}62 \times 981 = 310{,}000 \text{ dyne cms./radians,}$$

hence

$$G_0 = \frac{(2\pi)^2\{3{,}080\} + 310{,}000\,(1{\cdot}18)^2}{1{\cdot}18^2 - 0{\cdot}782^2} = 706{,}000 \text{ dyne cms./radians.}$$

Now one division on the B scale = ·0086 radians, therefore restoring couple per division = 706,000 × ·0086 = **6,090 dyne cms.** Average of results by two methods = **6,120 dyne cms.** per div.

Then, as previously stated, a specimen of iron whose maximum permeability was known, having been determined by the ballistic galvanometer method, was placed in the instrument, and the BH curve obtained for it in the arbitrary units fixed by the scale divisions. This curve, corrected for demagnetisation, is shown in Fig. 5.

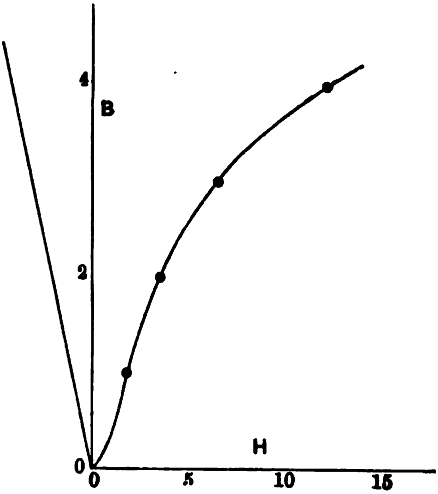

Fig. 5.

It is seen that the permeability is a maximum in the neighbourhood of $H = 2{\cdot}8$ divs. and $B = 1{\cdot}6$ divs.

The approximate values of B and H in C.G.S. units corresponding to these scale readings were then determined from the equations

(1) Moment in dyne cms. $1{\cdot}6 \times 6{,}120 = \dfrac{Bal}{4\pi}\,.\,\mathbf{H}$,

(2) $B = H\mu = \mathbf{H}\theta\mu$,

where $\mathbf{H}$ = strength of magnetic field,
a = cross-section of specimen,
and l = effective length of specimen between magnetic poles.

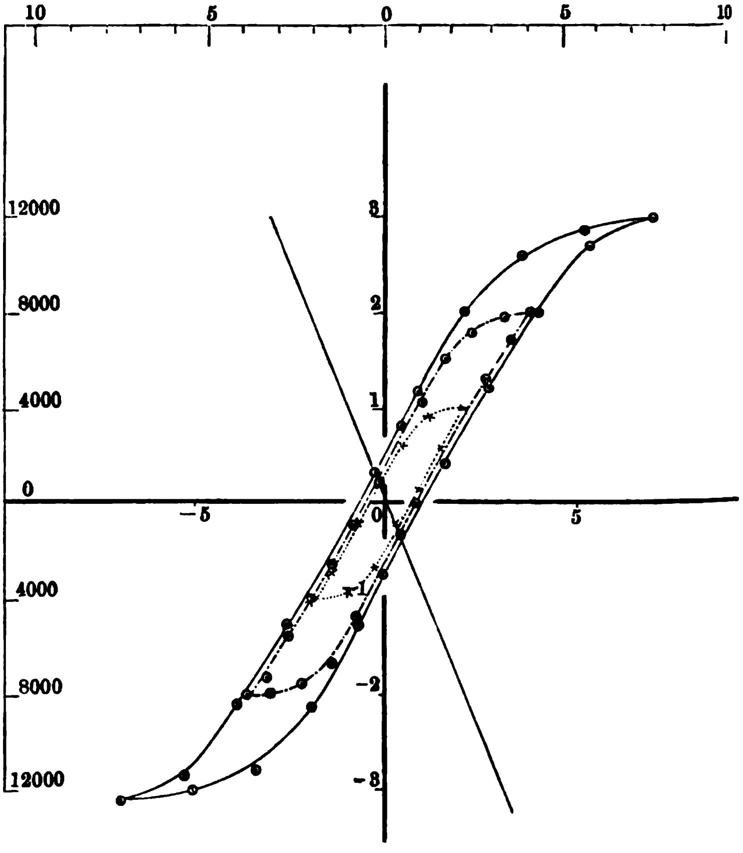

Fig. 6.

One division of H scale = ·00836 radians,
therefore $\theta = \cdot 00836 \times 2\cdot 8$ radians,
$\mu = 2{,}100$, as determined by ballistic method.

Hence from (2) $H = \dfrac{B}{2{,}100 \times \cdot 00836 \times 2\cdot 8} = \dfrac{B}{49\cdot 1}$.

The weight of the specimen was 1·14 grammes, the specimen was of uniform section and of specific gravity 7·78, hence

$$al = \frac{1\cdot 14}{7\cdot 78}.$$

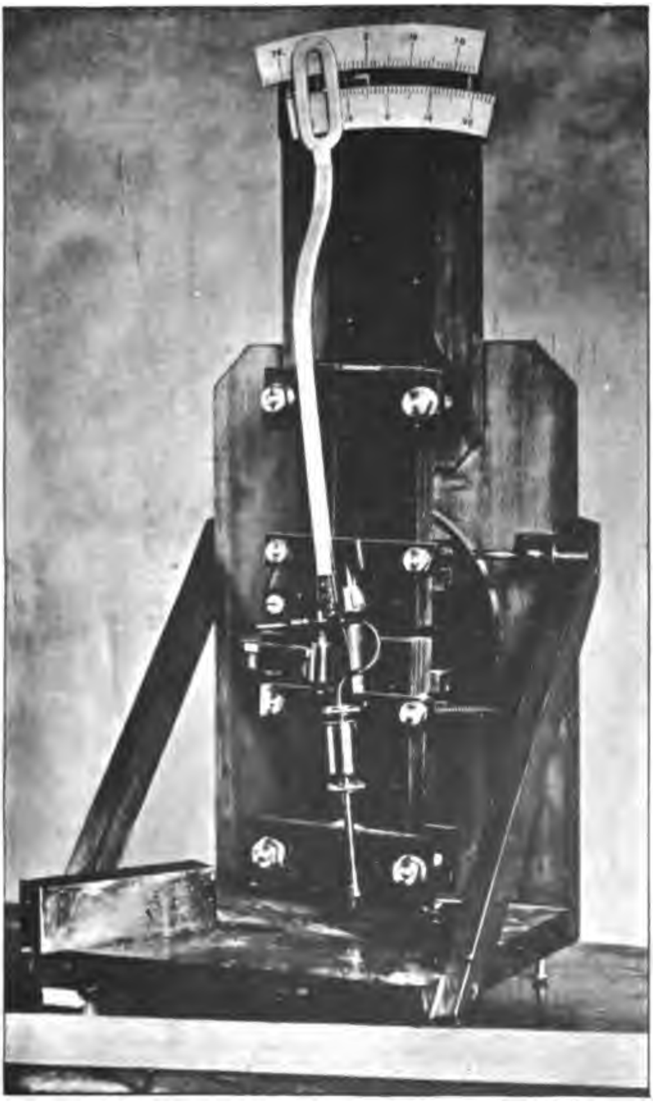

Fig. 1.

This is only approximate, since the assumption is here made that the effective distance between the poles is the full length of the sample.

Inserting these values in (1) we have

$$B^2 = \frac{1{\cdot}6 \times 6{,}120 \times 49{\cdot}1 \times 4\pi \times 7{\cdot}78}{1{\cdot}14} = 41{,}250{,}000,$$

$$B = 6{,}410, \text{ and } H = \frac{6{,}410}{\mu} = 3{\cdot}06, \text{ or } \mathbf{1{\cdot}09 \text{ gauss per div.}}$$

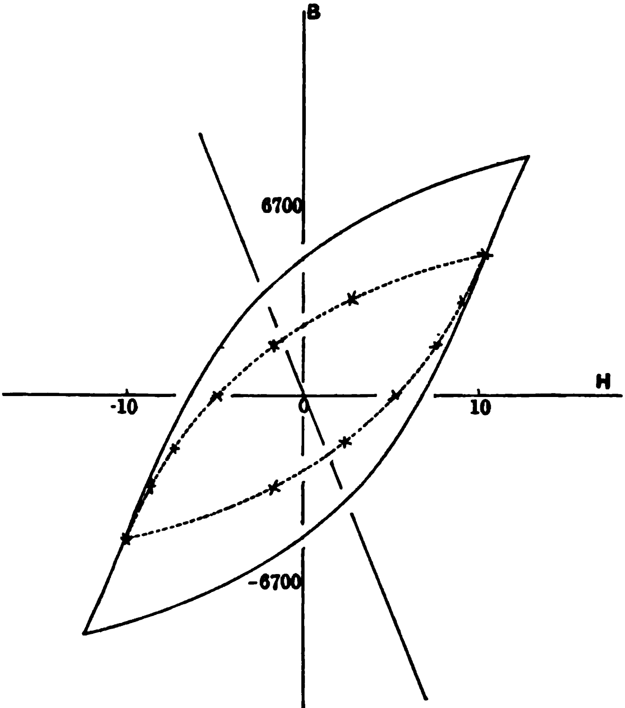

Fig. 7.

The length of the sample was 7·62 cms., hence the cross-section

$$= \frac{1{\cdot}14}{7{\cdot}78 \times 7{\cdot}62} = {\cdot}0193,$$

therefore $Ba = 6{,}410 \times {\cdot}0193 = 123{\cdot}3$, or **77 per div.** of B scale.

Hence to obtain B in any sample, multiply the B scale reading by 77, and divide by the cross-section of the specimen.

Results Obtained.

Figs. 6 and 7 shew examples of hysteresis cycles got with the instrument.

On the Action of Radium and other Salts on Gelatin. By W. A. DOUGLAS RUDGE, M.A., St John's College.

[*Read* 29 January 1906.]

On October 31st I gave a short account of some experiments which I had made on the action of radium salts on gelatin. The present communication is a more detailed account.

If a small amount of a radium salt is sprinkled upon the surface of some sterilised gelatin, a rapid change is seen in the appearance of the latter where the salt touches it. A whitish sort of patch is formed under the radium and this patch extends and becomes nebulous and like an ordinary fungus or mould in appearance. The growth goes on for about two days.

The most convenient means of examination was to place the gelatin upon a glass slip, add a speck of radium preparation, cover with a thin glass and examine with a half-inch power. It was then seen that around the radium the gelatin liquefied and a copious evolution of gas bubbles took place. At the same time a sort of "growth" appeared, quite distinct from the bubbles, and spread round the particle of radium. The growth seemed like a mass of micrococci, and it slowly extended, reaching a maximum in about a day. On examining the growth with a high power, it appeared to consist of a collection of cells resembling starch granules, and after a day or two something like a nucleus could be seen.

The idea has been advanced that these cellular growths were in some ways analogous to living cells.

Many groups of cells could be seen, and especially pairs which looked as though a single cell had divided into two, but very careful watching showed no signs of any fusion actually occurring, and the fact that a "dumb-bell" arrangement was frequently found, appeared to the author to be purely fortuitous.

As radium preparations are composed chiefly of barium it seemed likely that salts of the latter metal might be capable of acting on the gelatin, and on putting the matter to the test this was seen to be the case, and a growth was obtained which could scarcely be distinguished from that caused by the radium.

A close examination was next made with most of the metallic salts available in the laboratory, and besides radium and barium only strontium and lead salts yielded any definite results.

Now gelatin contains sulphur as an essential constituent, and in the course of manufacture is, I believe, treated with sulphur dioxide in order to clarify and bleach it, therefore it seemed likely that the so-called growth originated about a precipitate of an insoluble sulphate, and this idea was substantiated by the circumstance that only those metals which form insoluble sulphates could produce the growth.

The formation of a cell round a particle of precipitate may be explained by an alteration in the degree of hydration either of the precipitate or of the surrounding gelatin, thus forming an empty space. It has been stated by Mr Burke that the cells disappear on heating. This of course would naturally be the case as the gelatin would melt and the vacuole collapse, and the small particle of precipitates might easily escape notice.

Experiments were made with the object of ascertaining whether the gelatin could be rendered truly "sterile" to the action of radium salts. It was found that on adding a solution of either radium or barium salt to liquefied gelatin a precipitate was obtained, which varied in amount with different samples of gelatin. This precipitate was filtered off and was found to consist of barium sulphate in both cases. A clear liquid was secured by filtering through a porous tube under pressure. Drops of this liquid were allowed to set on glass slips, a speck of radium added, and the preparation watched under the microscope. *No growth could be seen.* This was to be expected, for the matter forming the precipitate having been removed the growth naturally could not form. On adding to this sterile gelatin a small quantity of a soluble sulphate, and testing with radium, the growth made its appearance. Solutions of other gelatinous and mucilaginous substances were used, such as sodium silicate, colloidal calcium sulphate, starch, gum, etc. With sodium silicate and calcium sulphate, growths made their appearance, but this was not the case with starch or gum unless a small amount of soluble sulphate was present. If the solutions were made with tap water instead of distilled water, the sulphate present in the former was quite sufficient to cause a very perceptible growth.

From these experiments it seems quite conclusive that radium has no specific action upon gelatin such as to lead to the formation of cells, and that any effect which may be seen is due to the barium, which makes up the greater part of the radium salt, acting upon the sulphur compounds present in the gelatin.

Susceptibility of Iron in Colloidal Solution. By E. F. Burton, Emmanuel College, and P. Phillips, Emmanuel College; 1851 Exhibition Scholars. (Communicated by Professor J. J. Thomson.)

[*Read* 29 January 1906.]

Introduction.

Some years ago Townsend* made an exhaustive series of experiments on the susceptibility of aqueous solutions of various iron salts of both the ferric and ferrous type and was led to formulate two empirical formulae connecting the susceptibility of any such solution with the quantity of iron per cubic centimetre in the solution. If k is the susceptibility of a solution, w the weight of iron per c.c., then, taking the susceptibility of pure water equal to $-7{\cdot}7 \times 10^{-7}$, we have for ferric salts

$$10^7 k = 2660\,w - 7{\cdot}7,$$

and for ferrous salts

$$10^7 k = 2060\,w - 7{\cdot}7.$$

Further Townsend found that, roughly, the magnetic properties of the various salts in the dry state are the same as when they are in solution.

These results show clearly that the magnetization is entirely due to the iron, and is accurately the same whatever the acid radical connected with it so long as the iron remains either in the ferrous or ferric state. Assuming a similar formula to hold for pure iron in a solution, a simple calculation shows that the constant coefficient for 'w' would be much larger than that in the above equations. Again, Townsend found that when iron appears in the acid radical, e.g. in ferrocyanides, its permeability is less than $\frac{1}{100}$ of the value when the iron plays the part of a metal in a salt. From these results we may conclude that, so far as its magnetic properties are concerned, iron exists in four different states: (1) as pure iron, (2) as ferric iron in salts, (3) as ferrous iron in salts, and (4) in the acid radical in salts.

Iron exists in still another form—in colloidal solution—for which determinations of magnetic properties have been wanting. The purpose of the experiments recorded in the present paper was to find the susceptibility of a colloidal solution of iron in methyl alcohol and to deduce the formula analogous to Townsend's for iron in this state. The results show that the iron in the colloidal solution is over thirteen times as magnetic as that in ferric (or ferrous) salts—a conclusion which has an important bearing on the theory of the constitution of colloidal solutions.

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* "Magnetization of Liquids," *Phil. Trans.* 187, A, 1896, p. 533.

Apparatus.

On account of the extreme tenuity of the colloidal solution and the small quantity available, it was necessary to be able to make delicate measurements on small quantities of the solution. The method adopted was that first suggested by Lord Kelvin[*] and afterwards used by Wills[†] in his experiments on the susceptibility of weakly magnetic solids and liquids.

A general idea of the apparatus is given in Fig. 1. A rectangular slab (A) of the material whose susceptibility is to be measured, is so suspended from one arm of a balance (B), that its long axis is vertical and its lower face is in the strongest part of the field between the poles of an electromagnet.

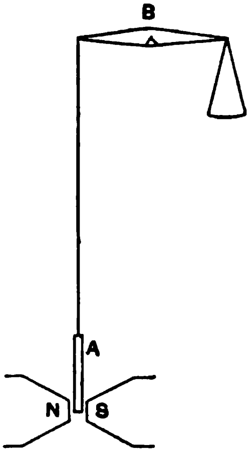

Fig. 1.

If k = susceptibility of the material,
A = area of the horizontal section of slab,
H_1 = magnetic field at bottom of slab,
H_2 = magnetic field at top of slab,

then the pull, P, on the slab due to the magnetic field is given by the equation

$$P = \frac{kA\,(H_1^2 - H_2^2)}{2g} \text{ grams.}$$

In the present experiment the pole pieces were prismatic ones, whose vertical faces were 7·5 cms. by 1 cm., the longer edges being horizontal. These were fixed once for all at a distance of 1·2 cms. apart. As the slab (A) was over 8 cms. long, H_2^2 may be neglected in comparison with H_1, so that the expression for P becomes

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[*] *Brit. Assoc. Rep.* 1890, p. 745.

[†] Wills, *Phys. Rev.*, April, 1898; *ib.* March, 1905.

$$P = \frac{k \,.\, AH_1^2}{2g}.$$

In order to measure k for a liquid, the slab A is replaced by a small rectangular glass vessel, and the difference between the force on the empty vessel and that on the vessel full of liquid, gives the pull on a known slab of liquid. The internal dimensions of the vessel used were 3·01 cms. by ·873 cms. by 8·0 cms., of which only the dimensions of the horizontal section need be known accurately. The vessel was filled through a small hole in the top, which could then be sealed up so as to prevent evaporation.

The colloidal solution was prepared by sparking (40 volts, 6·5 amps.) between two soft iron electrodes beneath the surface of pure methyl alcohol, which was kept surrounded with ice. In order to estimate the quantity of iron per c.c. in such a solution a measured quantity was evaporated in a previously weighed porcelain dish; the residue was then heated to redness so as to convert it to Fe_2O_3 and weighed.

Experimental Results.

The results will be tabulated under the following heads: (1) the measurement of the field between the poles, (2) the susceptibility of pure methyl alcohol, (3) the susceptibility of the colloidal solution, and (4) the deduction of the Constant in the Townsend formula.

1. *Measurement of Field.*

The field was measured by finding the pull exerted by it upon a conductor carrying a known current, and placed horizontally between the poles. A block of wood having the same external dimensions as the glass cell to be used, was made so as to permit it to hang between the poles in exactly the same position as the cell itself. Around the two vertical edges and bottom of this block was pasted a continuous strip of tin-foil; to the upper ends of the foil wires were soldered, and then led out so that any desired current might be passed through the foil. If the current carried by the foil be i ampères, the length of the horizontal portion of the foil be l cms., then the pull, P, in grams exerted by a field (H) on the lower horizontal strip of the foil will be given by the equation

$$P = \frac{Hli}{10g}, \text{ i.e. } H = \frac{10Pg}{li}.$$

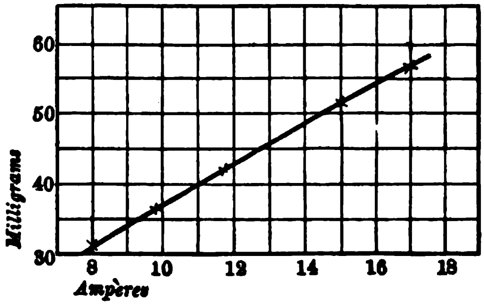
As wood is itself slightly diamagnetic the field will exert a force on the block even with no current through the foil—an effect which must be taken into consideration. Table I gives the weights which were necessary to counterbalance the block of wood, with no current passing through the tin-foil, when different currents were sent through the coils of the electromagnet. In column III are recorded the pulls exerted by the fields of different strengths on the block of wood.

TABLE I.

Susceptibility of Wooden Block; no current in foil.

Current for Electromagnet	Counterbalancing weight	Pull due to the Magnetic Field
0 ampères	39·053 grams	
8 "	39·023 "	– ·031 grams
0 "	39·054 "	
9·9 "	39·016 "	– ·037 "
0 "	39·053 "	
11·8 "	39·012 "	– ·042 "
0 "	39·054 "	
15 "	39·001 "	– ·051 "
0 "	39·052 "	
17 "	38·996 "	– ·056 "
0 "	39·052 "	

In Curve I the numbers given in columns I and III are plotted, the abscissae being currents and the ordinates the weights in milligrams. From this curve we may deduce the pull when any intermediate current is sent through the magnet.



CURVE I.

A measured current was then passed through the tin-foil and similar readings taken, the results being given in Table II.

TABLE II.

Force on Block and Conductor carrying Current.

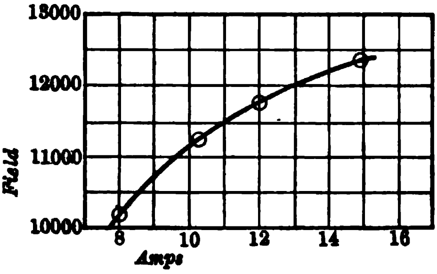
Current in Foil in milli-ampères	Current for Magnet	Weight in Pan in grams	Pull due to Field in grams
40·7	0 amps.	39·054	
40·6	8 „	39·160	+ ·107
40·6	0 „	39·053	
40·6	10·2 „	39·168	+ ·115
40·7	0 „	39·053	
40·7	12 „	39·170	+ ·117
40·9	0 „	39·053	
40·8	14·9 „	39·170	+ ·117
40·8	0 „	39·053	
40·8	17·4 „	39·173	+ ·121
40·9	0 „	39·052	
0	0 „	39·052	

TABLE III.

Calculated Values of Field.

Current in Electro-magnet	Pull on wood with no current through Foil	Pull on block with current through Foil	Pull due to current in Foil	Strength of Field, $10Pg/li$
8 amps.	− ·031 grm.	+ ·107 grm.	+ ·138 grm.	10200 units
10·2 „	− ·038 „	+ ·115 „	+ ·153 „	11300 „
12 „	− ·043 „	+ ·117 „	+ ·160 „	11790 „
14·9 „	− ·051 „	+ ·117 „	+ ·168 „	12360 „

Table III prepared from the two former tables gives in column IV the force due only to the current through the foil and in column V the calculated values for the field, i.e. $\frac{10Pg}{li}$, corresponding to the different values of the current through the electromagnet. Curve II, which shows the values of the field plotted against the current, enables us to deduce the field corresponding to a given current through the magnet. The length of the horizontal portion of the tin-foil strip was 3·27 cms.



CURVE II.

2. *Susceptibility of Methyl Alcohol.*

In order to determine this we have again to make two sets of readings, firstly, the values of the forces exerted on the empty cell by any field, and secondly, the analogous values for the cell full of alcohol.

Table IV, which is exactly analogous to Table I, shows that the empty glass cell is slightly paramagnetic.

TABLE IV.

Susceptibility of Empty Glass Cell.

Current in Electromagnet	Weight in Pan	Pull due to Field
0 ampères	45·8963 grams	+ ·0100 grams
8 „	45·9063 „	
9·9 „	45·9070 „	+ ·0107 „
11·75 „	45·9080 „	+ ·0117 „
14·95 „	45·9095 „	+ ·0132 „
0 „	45·8963 „	

The cell was removed, filled with methyl alcohol, sealed up and a similar series of readings taken for the full vessel. These are given in Table V.

TABLE V.

Force on Cell full of Alcohol.

Current in Electromagnet	Weight in Pan	Pull due to Field
0 ampères	61·8212 grams	–·0772 grams
7·8 "	61·7440 "	
0 "	61·8212 "	–·0922 "
10 "	61·7290 "	
0 "	61·8211 "	–·1040 "
11·9 "	61·7171 "	
0 "	61·8212 "	–·1190 "
14·8 "	61·7022 "	

The results recorded in the Tables IV and V enable us to deduce the pulls exerted by the magnetic fields on the methyl alcohol alone and, consequently, to calculate the values of the susceptibility of the alcohol from the equation given above,

$$k=\frac{2Pg}{AH^2}.$$

These values are given in Table VI.

TABLE VI.

Susceptibility of Methyl Alcohol.

Field in c. g. s. units	Pull on Full Cell in grams	Pull on Empty Cell in grams	Pull on Methyl Al. in grams	Susceptibility of Alcohol, $k=2Pg/AH^2$
10,070	–·0772	+·0098	–·0870	$-6{\cdot}34\times10^{-7}$
11,220	–·0922	+·0107	–·1029	$-6{\cdot}13\times10^{-7}$
11,750	–·1040	+·0118	–·1158	$-6{\cdot}27\times10^{-7}$
12,330	–·1190	+·0132	–·1322	$-6{\cdot}49\times10^{-7}$
			Mean value	$-6{\cdot}31\times10^{-7}$

The mean value is in close agreement with those given by Winkelmann* for methyl alcohol:

Quincke $-6{\cdot}8\times10^{-7}$, Henrichsen $-6{\cdot}1\times10^{-7}$.

A slight difference in the purity of the alcohols would account for such small variation in these values.

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* *Handbuch der Physik*, III. 2, pp. 206–9.

3. *Susceptibility of Colloidal Solution of Iron.*

The cell was next filled with some of the colloidal solution which had been prepared in another portion of the same sample of methyl alcohol. A difficulty soon presented itself because the strong magnetic field soon pulled some of the iron to the bottom of the cell, so that a protracted series of readings could not be made. Weighings were therefore made so as to get exactly the pull due to one certain field—that with 7·9 ampères through the magnet. With the cell freshly filled with colloidal solution the difference between the counterbalancing weight when there was no current in the magnet and that with 7·9 ampères, was determined as quickly as possible. The cell was then emptied, rinsed out and refilled with fresh solution and the counterbalancing weight with no field on was again found. By the help of the former reading the balance could be set approximately at the weight required to counterbalance the cell with the magnetic field on. In this way the required weight was obtained in a few seconds and so the error introduced by coagulation of the particles was nearly obviated.

It was found that the pull due to a field of 10140 units on the cell full of colloidal solution was − ·0565 gram. From Table IV we see that the pull exerted by this field on the glass cell alone is + ·0100 gram. The pull on the colloidal solution is therefore − ·0665, from which, by use of the formula

$$k = \frac{2Pg}{AH^2},$$

we find that $k = -4{\cdot}83 \times 10^{-7}$.

4. *Townsend formula for Colloidal Solution of Iron.*

60 c.c. of the above solution gave ·035 gram of Fe_2O_3, which gives the weight of iron per c.c. to be 44×10^{-6} grams.

Assuming the same form of equation as that given by Townsend for ferric solutions we have

$$10^7 k = C \, . \, w + [\text{susceptibility of solvent } (k_s)].$$

For the particular case here given we have

$$k = -4{\cdot}83 \times 10^{-7},$$
$$k_s = -6{\cdot}30 \times 10^{-7},$$
$$w = 44 \times 10^{-6}.$$

From which we get

$$C = 33{,}400.$$

This value of C is 13 times the corresponding number for ferric solutions and 16 times that for ferrous solutions.

This leads us to enquire as to the force which a given magnetic field would exert on the colloidal solution if the particles were small spheres of pure iron.

If n = number of particles per unit volume,

A = cross-section of the cell,

M = magnetic moment of a small spherical particle,

then the pull dP exerted by the magnetic field H on an elemental horizontal layer of thickness dx would be

$$dP = \frac{nA}{g}\, dx\, M \frac{dH}{dx} \text{ grams.}$$

Now $M = Iv$, where I is the intensity of magnetisation in the sphere and v is the volume of the sphere. Substituting this value of M in the expression for dP and integrating we have

$$P = nv \,.\, \frac{A}{g} \int_{H_1}^{H_2} I\, dH \,.\, \text{grams} = nv \frac{A}{g} (H_1 - H_2)\, 1650,$$

for I may be taken at its limiting value about 1650. vn is equal to the volume of iron per c.c. of the colloidal solution, i.e. $\frac{44 \times 10^{-6}}{8}$ c.c.s. A as before is 2·63 sq. cms. Supplying these values in the expression for P and neglecting H_2 in comparison with H_1 (= 10000) we have $P = {\cdot}243$ grams. Since the actual pull exerted by this field on the cell, full of colloidal solution, was ·0870 − ·0665 = ·0205 grams, it is quite evident that the particles do not consist wholly of pure iron.

These results lead to the following alternative conclusions regarding the state of iron in colloidal solution: (1) the particles in suspension are made up uniformly of iron which is in a condition in which it has magnetic properties different from those of iron in any state of combination which has been studied; or (2) each particle consists of a core of pure iron surrounded by a layer of some compound of iron, e.g. hydroxide.

In conclusion we wish to acknowledge our extreme indebtedness to Prof. Poynting for so kindly placing at our disposal the resources of the Physical Laboratory at the University of Birmingham, and we desire to express our thanks to Mr G. F. C. Searle for some helpful suggestions.

A method of following the course of certain chemical actions, and a period of induction in the action of excess of water on monochloracetic acid. By P. V. Bevan, M.A., Trinity College.

[*Read* 12 March 1906.]

The experiments which are described in this paper arose from an endeavour to trace the initial course of the action of water on monochloracetic acid, with a view to testing the theory that, in actions of this type where a double decomposition takes place, the first stage of the action is an association of the two reacting molecules. In certain other chemical actions, especially the union of hydrogen and chlorine, an induction period, a period during which the action does not go on with the maximum velocity under the conditions of the experiment, is observed. This induction period admits of explanation if we assume that the first stages of the action consist in a union between the reacting molecules, or of these with other molecules, to form a molecular aggregate, in which the individual atoms can come into each other's spheres of action. The action then would go on by a breaking down of these aggregates giving rise to the final products.

It is difficult to form a picture of the process of such actions, unless something of this kind takes place, as for example in the case of hydrogen and chlorine when we have the two perfectly stable systems which in a pure state shew practically no tendency to combine even under the influence of light. And in such an action as that of monochloracetic acid and water it seems probable that, for a pair of molecules to react, they should first get into some more intimate relation than that simply of a dissolved molecule and molecule of solvent. The action takes place according to the equation

$$CH_2ClCOOH + H_2O = CH_2 . OH . COOH + HCl,$$

glycollic acid and hydrochloric acid being formed. This action has been studied*, and it has been found to follow the normal monomolecular course when the water is in large excess. The method adopted by Buchanan was to titrate a specimen of the reacting mixture, at various times after the beginning of the action, in order to determine the acidity of the mixture. Since, as a result of the action, we have two molecules of acid for one at the beginning, these titrations give a measure of the proportion of monochloracetic acid decomposed. It occurred to the author that a determination of the conductivity of the solution in which the action was taking place might be a delicate method of finding the amount of decomposition that had occurred at any instant, and that this method would afford a means of studying the initial stages of the action, a part of the action to which the titration method is by its nature inapplicable. One obvious advantage of such a method is that the action can be allowed to progress steadily and need not be disturbed at all during the course of an experiment.

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* Buchanan, *Berichte*, IV. 340, 1871. Van 't Hoff, *Studies in Chemical Dynamics*, p. 14.

The resistance of the solution was measured by the commutator method of Fitzpatrick, and this was found to give results of more than sufficient accuracy. The regulation of temperature, with the thermostat used, was not sufficiently delicate to justify the accuracy that could be obtained in the resistance measurements. The thermostat was used with a toluol regulator, and the temperature, except in case of accident, did not vary more than one-fifth of a degree centigrade through an experiment lasting some days. The temperature at which the action was conducted was 90° C.

Assuming that the action is a monomolecular one we should have, if x be the quantity of monochloracetic acid decomposed,

$$x = a(1 - e^{-kt}),$$

where a is the initial quantity of monochloracetic acid and k the velocity constant for the reaction. The constant k is independent of the initial concentration provided that the water is in large excess, which was always the case in the present experiments. The proportion of monochloracetic acid was never more than 4 grammes to a litre of solution.

The following table gives the results obtained for an experiment in which the action was allowed to progress nearly to an end. The first column gives the time from the beginning of the action, the second the observed resistance, the third the change of conductivity, and the last the value of k obtained from the expression

$$k = \frac{1}{t}\log\frac{a}{a-x}.$$

The monochloracetic acid was freshly distilled and dissolved in water at 90° C. and as quickly as possible introduced into the conductivity vessel. The initial resistance was obtained by plotting a curve for the first few values of the resistance observed and producing it backwards to the resistance axis. There is of course some uncertainty in the early values of the resistance as the solution at first is probably at a temperature slightly below 90° C.

Time in hours	Observed Resistance	Change of Conductivity $\times 10^6$	$k \times 10^4$
·80	357·0	101	31
2·7	323·5	391	36
3·1	310·5	520	42·2
5·6	278·0	897	41·2
6·6	266·5	1052	41·0
22·2	174·6	3080	38(1)
27·2	152·4	3862	40·1
29·4	146·10	4144	40·4
45·6	114·20	6055	40·8
52·2	105·35	6793	41·3
71	88·44	8600	41·9
75·8	85·50	9000	41·8
101·2	73·53	10920	41·1
124·1	67·27	12100	40·5
144·3	63·91	13100	42·2
166	59·71	14000	43·0
190·4	57·38	14730	42·4
220·5	55·11	15440	43·1
239·9	54·16	15760	43·2
265·8	53·14	16120	43·2
287	52·59	16320	42·6
336	51·81	16600	40·8(2)
364	51·68	16650	39·9(2)
404·3	50·70	17020	42·7

(1) The temperature when this reading was taken had fallen to 88° C.

(2) Towards the end of the experiment a bubble of gas had formed in the resistance vessel and this made the resistance for these two readings too high. This bubble was got rid of before the last reading.

The value of k deduced from these experiments is $42{\cdot}5 \times 10^{-4}$.

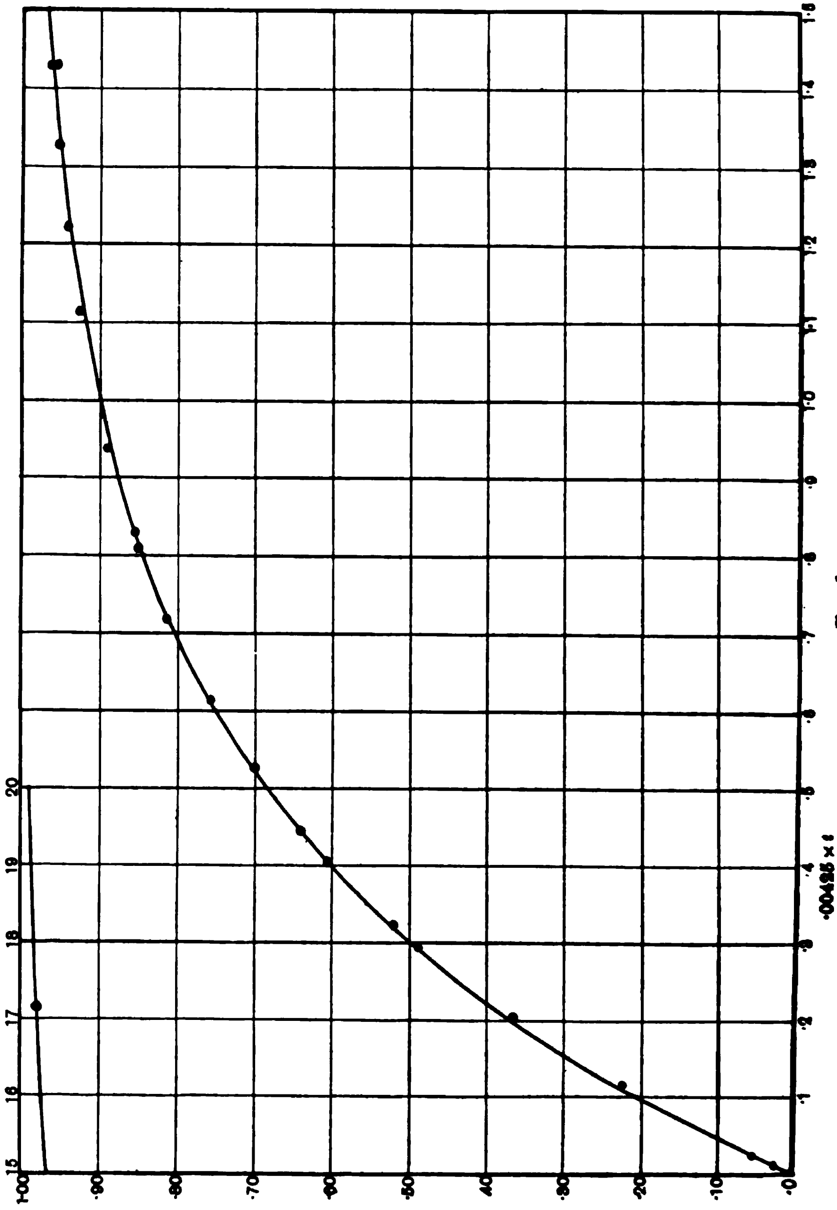
1·5
1·4
1·3
1·2
1·1
1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
·00425 × t
20
19
18
17
16
15
1·00
·90
·80
·70
·60
·50
·40
·30
·20
·10
·0
C divided by 17350

Fig. 1.

In figure (1)* the curve

$$y = 1 - e^{-x} \text{ is plotted.}$$

This corresponds to the observations if we make

$$x = {}^{\cdot}00425\, t,$$

and

$$y = \frac{C}{17350},$$

where C is the change of conductivity multiplied by 10^6.

The points representing the experimental numbers are found to lie very well on the curve.

It is interesting to compare the values for the constant of the action by this method with those obtained by Buchanan. The following table† contains Buchanan's results for the action at 100° C.

t	$k \times 10^5$
2	15·8
3	19·5
4	19·2
6	17·0
10	18·4
13	17·6
19	17·3
25	16·5
34·5	18·7
43	15·5
48	15·9

The constant here obtained is plainly not so good as that on p. 271.

We may conclude then from this experiment that this method enables us to follow such an action, and in virtue of the fact that the change of conductivity is so large we can observe closely the initial stages of the action.

Experiments were made with solutions of different strengths and readings of the resistance were taken as soon after the beginning as possible.

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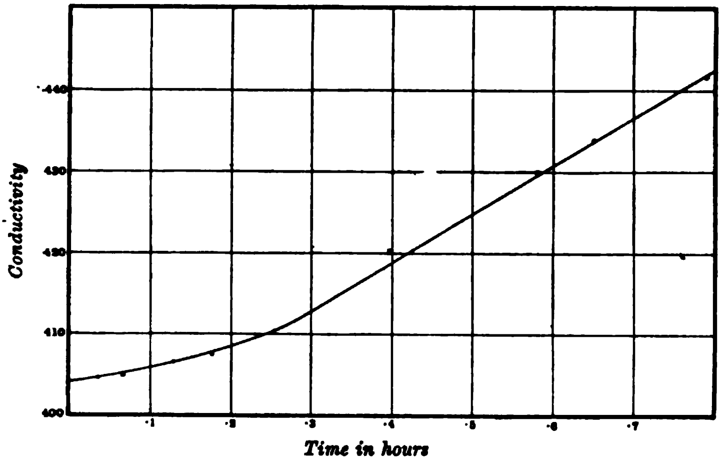
* In Figure 1 the continuation of the curve is transferred to the left-hand side of the figure.

† Ostwald, *Lehrbuch der Allgemeinen Chemie, Verwandschaftslehre*, p. 208, and Van 't Hoff, *Studies in Chemical Dynamics*, p. 14.

The following table contains the results of two such experiments. In the first column is the time from the beginning, in the second the observed resistance, and in the third the conductivity.

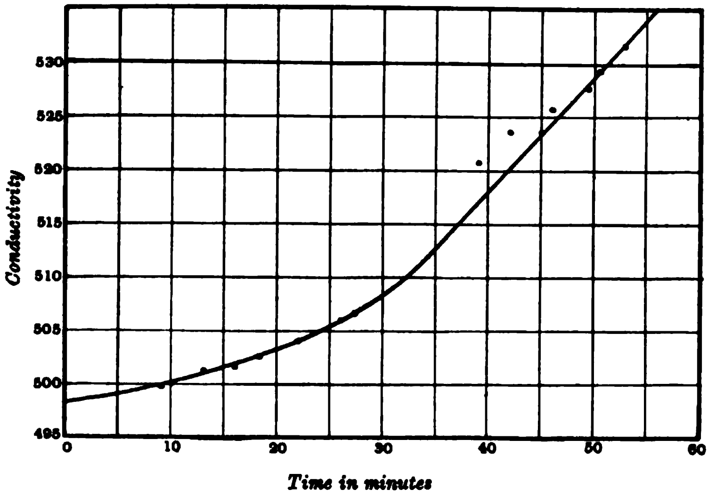
T hours	R	Conductivity $\times 10^6$	T minutes	R	Change of Conductivity $\times 10^6$
·036	247·2	4046	9	200·2	4996
·067	247·0	4049	10	200·0	5000
·129	246·0	4065	13	199·5	5012
·177	245·5	4074	16	199·4	5014
·253	243·7	4103	18·3	199·0	5025
·396	238·0	4202	22	198·4	5044
·484	235·0	4255	23·5	198·1	5048
·580	232·6	4299	26	197·6	5060
·650	230·5	4339	27·25	194·7	5067
·790	226·5	4419	39	192·0	5208
			42	191·0	5236
			45	191·0	5236
			46	190·2	5258
			49·5	189·5	5276
			50·5	189·0	5291
			53	188·0	5319

These results are plotted in figures 2 and 3. It is obvious from these curves that we have not, in the initial stages of the observations, the increase in conductivity to be expected if the action at once begins as a monomolecular one. We have to consider what effects we should expect at the beginning of the action which might be due to the conditions not being the steady ones attained a short time after the action has commenced. Firstly, the temperature of the solution would probably be rather below the steady temperature as the solution has to be made, and in transferring it to the conductivity vessel its temperature would fall slightly. This was avoided as far as was possible by performing all operations in the thermostat. But any effect due to this cause would be in the opposite direction. For if initially the temperature were too low the conductivity would be less than the proper amount, and as the solution attained the steady thermostat temperature the conductivity would shew a faster rate of increase than if the temperature all through were constant. Secondly, we might have an effect due to incomplete mixing of



Conductivity
·440
430
420
410
400
·1
·2
·3
·4
·5
·6
·7
Time in hours

Fig. 2.



Conductivity
530
525
520
515
510
505
500
495
0
10
20
30
40
50
60
Time in minutes

Fig. 3.

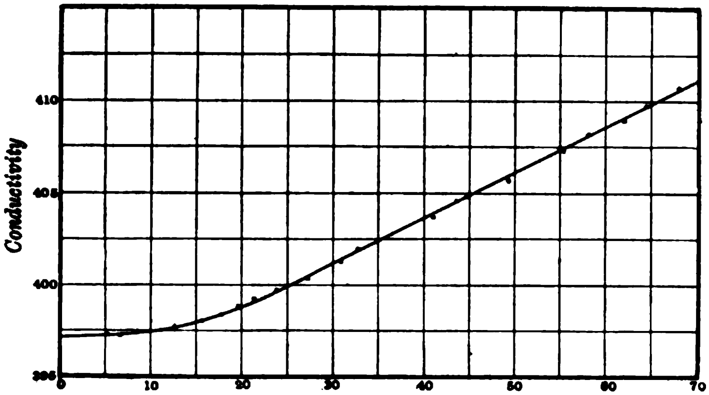
the solution. The solution was made in a beaker in the thermostat, and was well stirred before being placed in the conductivity vessel to have a well-mixed solution when the observations were made. But in this case also any effect would be to make the initial resistance too great, and therefore the initial conductivity too small, so that as the mixing became more complete the initial rate of increase of conductivity would be too great. These two purely practical causes would then lead to a diminution of the induction effect rather than to its production.

There remains another consideration. It is well known that with very dilute solutions the molecular conductivity decreases rapidly with increasing dilution. When the action begins the hydrochloric acid is for some time present in very small quantities, and it might be thought that some effect of this kind might give too slow an increase of conductivity initially. To decide this an experiment was performed in which the solution was made with a dilute solution of hydrochloric acid instead of water. In this case the hydrochloric acid was present already in sufficient quantity for any addition to it to produce an additive effect in the conductivity. The following table gives the results of this experiment.

Time in minutes	Conductivity × 10^6
5·1	3973
6·5	3972
12·5	3978
15·5	3981
17·7	3984
19·5	3989
21·3	3992
23·8	3997
27·1	4004
30·8	4013
32·8	4020
41	4037
43·5	4046
44·8	4049
49·2	4057
55·2	4074
58	4082
62	4090
64·5	4098
68	4107

These results are plotted in Fig. 4 and shew clearly that here also we have the same period of induction, or period when the action has not attained its normal velocity.

We may conclude therefore that the period of induction as observed is a regular part of the action.



Time in minutes

FIG. 4.

The occurrence of this period of induction shews that to regard the action as a simple monomolecular action is not a complete view. If we assume that the first action is an aggregation of an H_2O and a $CH_2ClCOOH$ molecule, and that an unstable compound is thus formed which tends to break down into glycollic acid and hydrochloric acid, we should necessarily have a period of induction; and after a short time the action could progress in the ordinary monomolecular way. For suppose x is the number of molecules of the original acid present, y the number of molecules of the complex, and z the number of molecules of hydrochloric acid or glycollic acid present, then the mass action law gives us

$$\frac{dy}{dt} = k_1x - \frac{dz}{dt},$$

$$\frac{dz}{dt} = k_2y,$$

and therefore

$$\frac{dy}{dt} = k_1x - k_2y.$$

Hence if k_2 is large compared with k_1, that is the rate of decomposition of the aggregate is large compared with its rate of formation, which is quite possible as its breakdown is in a different way to the mode of its formation,

$$\frac{dy}{dt} \text{ is zero when } \frac{y}{x} = \frac{k_1}{k_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1),$$

and $$\frac{dz}{dt} = k_1 x,$$

so that we should have the action progressing after the short time necessary to reach the stage (1) as a monomolecular reaction. Initially $\frac{dz}{dt}$ is zero, as y is zero, and the velocity $\frac{dz}{dt}$ increases until we reach the limiting value of y.

In these experiments, in order to observe the initial stage the action was allowed to proceed for some days to see if the same constant for velocity would be obtained as in the first experiment detailed, and also to see whether the additional hydrochloric acid in the last experiment affected the general rate of the action.

As the action takes a very considerable time to be nearly completed it is necessary to find the value of the final conductivity from experiments reached in a finite time. This can be done by taking two observations at times such that one is twice the other. Suppose that as before a is the final value of the increased conductivity, x_1 the conductivity increase corresponding to time t_1, and x_2 that to t_2, then we have

$$\frac{1}{t_1} \log \frac{a}{a - x_1} = \frac{1}{t_2} \log \frac{a}{a - x_2},$$

and if $$t_2 = 2t_1$$

$$\frac{a}{a - x_2} = \left\{\frac{a}{a - x_1}\right\}^2,$$

whence $$a(2x_1 - x_2) = x_1^2,$$

so that $$a = \frac{x_1^2}{2x_1 - x_2}.$$

We find in this way for the experiment of figure 3 the results in the following table, with $a = 5460$.

Time in hours	Change of Conductivity $\times 10^5$	k
19·6	974	43·5
22	1077	43·4
28·5	1350	43·7
46	1992	42·8
73·1	2708	41·0
97·5	3274	40·8
141·8	3970	40·2
169·7	4283	39·5
193·7	4576	41·2
215·8	4723	40·6

And also for the experiment in which initially hydrochloric acid was present (Fig. 4).

$$a = 15600.$$

T	Change of Conductivity $\times 10^6$	k
21·25	2695	38·7
22·5	2987	41·0
24·8	3246	41·6
45·25	5487	41·7
49·3	5878	41·8
92·75	9207	41·8
100·2	9627	41·7
121·4	10667	41·0
145·6	11770	42·0
165·4	12490	42·5

The values of the constants obtained are practically the same as those found in the first experiment, the results of which are represented in Fig. 1.

We see therefore, as we should expect, that the velocity constant is not dependent on the strength of the solution, and also that the presence of additional hydrochloric acid does not alter appreciably the velocity constant.

This method is applicable also in other cases where the action is characterised by a change of conductivity in the medium of the action. One such action is the inversion of cane sugar under the catalytic action of hydrochloric acid. The amount of water present here is affected by the combination of a molecule of water and a molecule of the sugar, and the conductivity changes in consequence. A change also may be produced by the alteration of the sugar in the solution. But in any case the change of conductivity affords a method of observing the change. In the following experiment the resistance was measured by the Kohlrausch method with a telephone and alternating current. This method is not so accurate as the commutator method, but it gives results which are fairly satisfactory. The author is investigating further some points in connection with this action with a view to understanding this kind of catalytic action, and the experiment here described is a preliminary one to find whether the action could be thus investigated. The results are given in the following table; in the first column the time from the beginning of the action and in the second the observed resistance.

INVERSION OF SUGAR.

Time	Resistance	Time	Resistance
·13	134·2	91	137·80
·5	133·2	92	137·74
1·0	133·9	97	137·87
2·33	134·0	115·5	138·13
4·5	134·0	121	138·00
19·25	135·3	137·5	138·06
23·0	135·3	139·75	138·19
41·25	136·33	142·0	138·25
41·75	136·52	145·0	138·21
43·75	136·60	163·0	138·27
67·5	137·30	186·0	138·36

In Fig. 5 the curve

$$y = (1 - e^{-x})$$

is plotted. The change of conductivity corresponding to $t = \infty$ is 262. In the figure the changes of conductivity are divided by 262 and the times in hours by 10·45. The points are seen to lie irregularly about the curve.

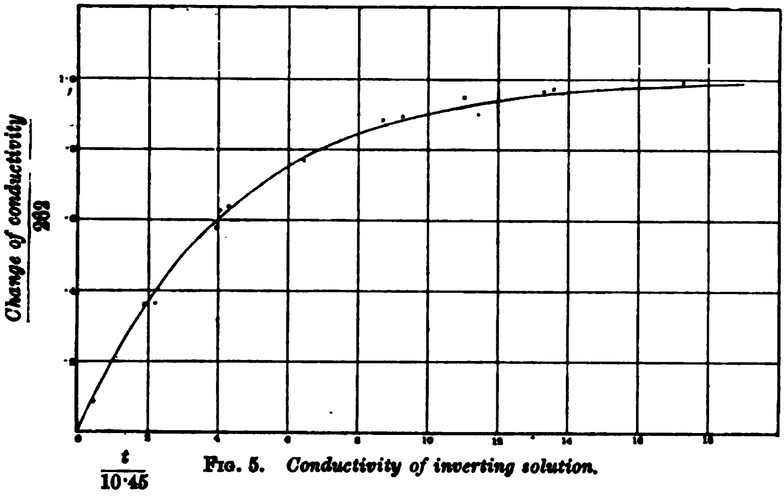


FIG. 5. *Conductivity of inverting solution.*

Summary of Results.

The measurement of conductivity affords a delicate method of tracing the course of certain reactions.

In the case of the action of water on monochloracetic acid, the action as a whole proceeds as a monomolecular reaction, but there is a period of induction.

This induction period may be explained by assuming that the first stage of the action is the formation of an additive compound of a water molecule and a monochloracetic acid molecule, this additive compound then breaking down yielding a molecule of glycollic acid and hydrochloric acid.

The Radio-activity of Metals and their Salts. By NORMAN R. CAMPBELL, M.A., Trinity College.

[*Read* 12 March 1906.]

1. Considerable evidence has accumulated in favour of the view that all elements are truly radio-active. It has been shown that ordinary metals emit a radiation analogous to that emitted by radium, and that the properties of this radiation are characteristic of the metal from which it proceeds. But there is a remarkable property of radio-activity which suggests a question that may be asked and can be answered before the source of the activity of ordinary materials can be regarded as established beyond doubt.

It is known that the activity of radium and uranium is an atomic property: the activity of any substance containing these elements is independent of its chemical and molecular condition, and depends only on the quantity of these elements which it contains. Is this law true for ordinary materials? Could we predict (say) the activity of lead oxide from the known activities of lead and oxygen? The experiments described below are an attempt to answer this question.

We need only take into account the α radiation, for the analogy of radium leads us to suppose that the energy contained in any β or γ rays that may be emitted will be infinitesimal compared to that carried by the α rays.

2. It has been shown* that the ionisation (p) caused by the α radiation from an infinitely thick slab containing a radio-active substance is given by

$$p = CkaA,$$

where C is a constant, k the mass of the radiating material contained in unit mass of the slab, a Bragg's constant for the rays emitted by that material and A a quantity measuring its activity in terms of the energy given off from it. The activity ($p_{uvw}\ldots$) of a slab composed entirely of several radiating elements u, v, w, etc. will be given by

$$p_{uvw}\ldots = C(k_u a_u A_u + k_v a_v A_v + k_w a_w A_w + \ldots),$$

where the suffixes denote the element to which each quantity refers. Now the ionisation caused by a slab composed entirely of one of the elements (u) will be given by the relation

$$p_u = Ca_u A_u,$$

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* *Phil. Mag.* Feb. 1906, p. 210.

since for a slab composed of one material throughout $k = 1$: hence

$$p_{uvw} \dots = C\left(k_u p_u + k_v p_v + k_w p_w + \dots\right).$$

We see then that if we know the ionisation caused by slabs composed entirely of each of the elements we can predict the ionisation caused by a slab containing them in any known proportion, on the assumption that the activities are uninfluenced by their state of chemical combination. The comparison of the activities calculated with those observed will serve to test the truth of the assumption.

3. The chief part of the apparatus used in these measurements is a wooden box, the internal dimensions of which are 40 × 40 × 22 cms.: it is lined with zinc sheet: a drawer 8 cms. deep, lined with aluminium foil, slides into the bottom of the box: in this drawer are placed glass trays 0·15 cm. deep and 36 cms. square, containing the material the activity of which is to be investigated. A sheet of iron wire gauze of 0·15 cm. mesh is fixed across the box at such a height that it just touches the surface of the glass trays when these are in position: in the space above the gauze is an electrode of coarse wire netting, supported by a wire passing through insulation in the side of the box and connected to the measuring device described in *Proc. Camb. Phil. Soc.*, Vol. XIII. p. 132, which proved exceedingly suitable for these experiments. The lining of the top part of the box is kept at a potential sufficient to send a saturation current through the gas: the lining of the drawer is earthed. It is found necessary to keep a rapid stream of air passing continually through the vessel in order to keep the current constant: this air is deprived of its free ions by passage through a tube in which it is subjected to a strong electric field.

4. It is clear that by measurements with such a vessel it is only possible to find the difference of activity of two bodies placed successively beneath the gauze. In order to find the absolute activity of any one material it is necessary to use a material which has no activity at all: but no such material is known. Moreover it is necessary to have some solid body constantly in this position, for ions can be dragged through the meshes of the gauze and the current to the electrode changed thereby. But since it is impossible to use a body without any activity whatever, the best that can be done is to use a body the activity of which is as small as possible. The most satisfactory material for this purpose was found to be carefully cleaned glass plates: no material has been investigated which possesses a smaller activity. The error introduced by assuming that the glass has no activity will be that caused by regarding $\frac{a-c}{b-c}$ as equal to $\frac{a}{b}$, where c is at most one-

fifth of *a* or *b*. Some estimate of the activity of the glass was obtained by comparing the increase in the ionisation which should be caused by the radiation from a lead plate of the size used, calculated from the results given in *Phil. Mag.*, Feb. 1906, p. 217, with the observed difference between the ionisation when the lead plate was in the tray and when the glass tray was empty. The agreement was sufficiently close to show that the activity of the glass could not be a large fraction of that of the lead.

5. The whole of the measuring box was enclosed in a thick screen of iron, sufficient to absorb nearly all the penetrating radiation from outside: no appreciable amount of the total radiation from the materials could be secondary radiation excited from outside.

6. In making the measurements the ionisation in the vessel was observed when the clean glass tray was in position in the drawer: the tray was then filled with the material under investigation and the measurement repeated. These observations were taken alternately several times, and the mean of the differences in the currents through the gas was taken as a measure of the activity of the material. The quantities to be measured are so small that it is impossible to attain great accuracy. The whole increase of current caused by the radiation from lead, the most active material investigated, was only 10 per cent. of the whole current through the vessel. The current could be measured with an accuracy of 1 per cent.: this is 10 per cent. of the effect caused by the lead and a much larger percentage of the effect caused by many of the materials investigated. The experiments require considerable time for their performance to this degree of accuracy: work has been much delayed by the difficulty of avoiding spurious activity derived from the air of the laboratory, but it is hoped that these difficulties have been overcome.

7. The table shows the results obtained. The first column shows the material investigated: the second the activity of a slab of that material in arbitrary units. The third gives the activity calculated on the assumptions that the activity of the metal is correctly given in the second column, and that the whole activity of the compound is due to the activity of the metal contained in it. It will be seen that the only elements occurring besides the metals are sulphur and oxygen: it is known that the activities of these elements are very small—too small to be measured, at least by this method: but their activity must contribute something to the total, and hence it is to be expected that the observed activities will be slightly too high.

TABLE I.

Lead	9·3	—
$PbSO_4$ (1)	6·8	6·4
$PbSO_4$ (2)	7·0	6·4
$PbSO_4$ (3)	7·2	6·4
PbO	8·2	8·6
Tin	4·4	—
SnS (1)	4·1	3·4
SnS (2)	3·9	3·4
SnS (3)	3·8	3·4
Bismuth	6·0	—
Bi_2O_3	5·7	5·4
Mercury	0·9	—
Hg_2O	0·5	0·86
HgO	0·6	0·84

8. I think it will be agreed by those who have had experience of the difficulties of similar measurements that the agreement is sufficiently good to confirm the theory that the activity of the common elements, like that of the radio-active elements, is an atomic property. It must be remembered that an activity represented on this scale by 0·5 can only just be detected: the deviations of observation and theory do not amount to much more than this quantity. Only in the case of mercury are the deviations large in relative, though not in absolute, amount. It should be noticed that the whole activity of mercury is just on the borders of the sensitiveness of the method.

9. Attention must be drawn to the agreement of the values obtained for each of the three samples of lead sulphate and each of the three samples of tin sulphide. In the case of each metal the three specimens were prepared from the same solution by three distinct methods. In the case of lead the samples were prepared as follows:—

(1) By direct precipitation of lead acetate by sulphuric acid.

(2) By precipitation with hydrochloric acid and subsequent conversion into sulphate by boiling with sulphuric acid.

(3) By precipitation with sodium carbonate and conversion into sulphate as in (2).

In the case of tin the three methods were:—

(1) Direct precipitation of tin chloride with hydrogen sulphide.

(2) Precipitation with potash: solution in hydrochloric acid and precipitation with hydrogen sulphide.

(3) Precipitation with sodium carbonate and subsequent treatment as in (2).

As far as is known the solutions from which the salts were derived were not made from the same samples of the metals as those used in testing the activities of the metals. The identity of the properties of samples prepared by different methods seems to be as conclusive a proof as could be given that the activity of the metals is not due to a contained impurity but to an intrinsic activity of the metals themselves. There is only one suggestion that could be made for avoiding this conclusion. It might be that the very small quantity of radio-active impurity that would be required to account for the observed effects might not behave like a true chemical substance but might be dragged down by any precipitate that was formed in the solution. Something of this kind has been observed in the case of the disintegration products of thorium. To ascertain whether such an explanation were possible additional experiments were made. In the case of lead, about one ounce of the sulphate was precipitated from a solution containing four pounds of the acetate: if the activity were due to impurity the whole activity of the solution ought to have been concentrated in the small precipitate. Measurements showed that this was not the case. In the case of tin, calcium chloride was dissolved in the tin solution and sodium sulphate added. The precipitated sulphate showed no abnormal activity.

10. If similar measurements had been made of the activity of thorium prepared in different ways the activities of the samples might not have been the same, for some of them would have contained the thorium *X* and some would have been free from it. Hence the agreement of the samples in these experiments shows that if there is such a thing as lead *X* having an activity different from that of lead, it is present in quantities too small to be separated and detected by this method. This might have been expected, for it is probable that the change of lead is so slow that no sample of it existing on the earth has attained, or nearly attained, radio-active equilibrium[*].

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[*] See *Jahrbuch der Radioactivität*, II. 4, p. 459.

11. It would be interesting to inquire whether the examples of the same element when exercising different valencies would exhibit the same activities. The case of mercury shown in the table yields no certain result, for the activity is too small for any definite statements to be made about it. Further investigation into this question is in hand.

The question also arises naturally whether any relation can be traced between the activity of a metal and its other properties, for instance its position in the periodic arrangement of the elements. No such correlation is apparent at present, and indeed the results obtained seem difficult to reconcile with any connection of the kind. The number of metals examined at present is insufficient to enable any definite statements to be made; but progress will be easier now that it has been established that the activity is an atomic property. Many elements, which it is very desirable to establish from this point of view, such as the alkali metals and the halogens, could not be tested before, because it was impossible to use them in the elemental state. Now they can be examined in the state of compounds. It should be pointed out that the activity of a slab of a substance (p in § 2) does not directly measure the activity of the substance itself: it is a product of this activity by the penetration of the rays given out. This consideration will doubtless complicate the investigation.

The expression of my indebtedness to Professor Thomson for his advice and encouragement is not the less sincere because it is repeated so often.

On a relation between the velocity and the volume of the ions of certain organic acids and bases. By T. H. LABY, Fitzwilliam Hall, and G. A. CARSE, Emmanuel College; 1851 Exhibition Scholars.

(Communicated by Professor J. J. THOMSON, F.R.S.)

[*Read* 12 March 1906.]

Since Ohm's law holds in the case of an electrolyte, at a given temperature the current $i = -k\frac{dV}{dx}$, where k denotes the conductivity of the electrolyte and $\frac{dV}{dx}$ is the potential gradient between the electrodes, and as the current is carried by n univalent ions per cub. cm. with a velocity of u for the positive and v for the negative ion, each having a charge of electricity e, then

$$i = ne(u+v),$$

$$\therefore\ ne(u+v) = -k\frac{dV}{dx}, \text{ or } u+v = -\frac{k}{ne}\frac{dV}{dx}.$$

Thus for dilute solutions of the same equivalent concentration, i.e. n constant, and with a constant potential gradient of say 1 volt per cm., $u + v = k \times \text{constant}$, an equation which may be derived in an independent manner from experiment by using Hittorf's migration ratios. Thus the conductivity of an electrolyte is proportional to the sum of the velocities of the ions. When the values that have been found for the velocities of univalent ions are reduced to the one potential gradient, the velocities are those of the different ions moving under the same force, viz. that arising from this electric field acting on the constant ionic charge.

Kohlrausch* has shown that the fluidity (reciprocal of the viscosity), and the conductivity—and therefore the velocities of the ions of dilute solutions—vary with the temperature in a similar manner, that is for a number of electrolytes the conductivity-temperature and the fluidity-temperature curves are similar. This has been borne out by the work of Bousfield and Lowry†, Lyle and Hosking‡, and others§. The former of these workers found that the temperature variation of the conductivity and viscosity of water can be expressed by one formula and represented by one curve. Kohlrausch's conductivity-temperature curves for aqueous solutions of sodium acetate and sodium valerate (the only two substances, whose anions are common to his and this paper), were sensibly the same as the fluidity-temperature curve of water. Thus in these three cases there is ground for supposing that U (the velocity of a given ion under a constant force) is proportional to the fluidity $\left(\frac{1}{\mu}\right)$ or

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* "The Resistance of the Ions and the Mechanical Friction of the Solvent," *Proc. Roy. Soc.* LXXI. 338 (1903).

† *Proc. Roy. Soc.* LXXI. 42 (1902).

‡ *Phil. Mag.* March 1900, p. 274; May 1902, p. 487; May 1904, p. 469.

§ For further references see H. C. Jones and Bingham, *Am. Chem. Journ.* XXXIV. 481, Dec. 1905.

$$U = \text{constant} \times \frac{1}{\mu}.$$

When the driving force (F) varies

$$U = \text{constant} \times \frac{F}{\mu}.$$

This suggests that the motion of an ion through an electrolyte may be very similar to that of small bodies through a viscous fluid. Sir George Stokes[*] has shown that the velocity (assumed small) of a sphere moving through a homogeneous viscous medium, when no slipping occurs, is

$$v = \frac{F}{6\pi\mu a},$$

where F is the driving force, μ the coefficient of viscosity of the medium, and a the radius of the sphere[†]. If we assume this law to hold for the motion of an ion in an electrolyte, though for bodies of molecular dimensions the medium is not homogeneous as postulated above, the radius of the ion, assumed spherical, may be calculated. The value thus obtained for the radius is sufficiently different from that deducible from the radius of the molecule calculated by Jeans[‡] from the kinetic theory of gases to make the applicability of the law to this case doubtful.

There appeared, however, another way of finding approxi-

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[*] *Camb. Phil. Trans.* IX. 58 (1850).

[†] A more general expression is

$$v = \frac{F}{6\pi\mu a}\left(\frac{\beta a + 3\mu}{\beta a + 2\mu}\right),$$

where β is the coefficient of sliding friction and is infinity for no slipping and zero for infinite slipping, and so $\frac{\beta a + 3\mu}{\beta a + 2\mu}$ may vary between 1 and $\frac{3}{2}$. Whetham (*Phil. Trans.* CLXXXI. A 559, 1890) has shown that for the steady flow through a capillary tube there is no slipping, and H. S. Allen (*Phil. Mag.* 323, Sept. 1900) that for small air bubbles rising in water or aniline, "the velocity of the bubble agrees with that deduced from theoretical considerations by Stokes on the assumption that no slipping occurs at the boundary."

[‡] *Phil. Mag.* Ser. VI. Vol. VIII. pp. 692—699 (1904).

mately the relation between the linear dimensions of an ion and its velocity. The relative volumes of certain organic anions and cathions may be deduced from molecular and atomic volumes. The cube roots of the ionic volumes give a series of numbers proportional to the ionic linear dimensions, and these combined with the velocities of the ions enable the relation sought for to be investigated.

CALCULATION OF IONIC VOLUMES[*].

The calculation of the ionic volumes may be best explained by two typical examples.

(1) Volume of anion of acetic acid.

The molecular volume of acetic acid $CH_3.COOH$ at 20° C.

$$= \text{molecular weight } (M) \times \text{volume of 1 gram acetic acid at } 20^\circ \text{ C. } \left(\frac{1}{\rho}\right)$$

$$= \frac{\text{molecular weight}}{\text{density at } 20^\circ \text{ C.}} = \frac{M}{\rho} = \frac{60}{1{\cdot}05} = 57{\cdot}2.$$

The anion of acetic acid being $CH_3.COO'$, its volume is found by deducting from the molecular volume of $CH_3.COOH$ the atomic volume of hydrogen in carbon compounds at 20°C. Kopp's value of the atomic volume of hydrogen given by Ostwald[†], when corrected for temperature, is 5·3 and this is the value used. Thus the volume of the acetic anion, $CH_3.COO'$, is $57{\cdot}2 - 5{\cdot}3 = 51{\cdot}9$.

(2) Volume of cathion of diethylammonium hydroxide.

The molecular volume of diethylamine $(C_2H_5)_2HN$ at 20° C. is

$$\frac{M}{\rho} = \frac{73{\cdot}1}{{\cdot}706} = 103{\cdot}5$$

$$\text{atomic volume of hydrogen} = h = \quad 5{\cdot}3$$

$$\therefore \text{ ionic volume of } (C_2H_5)_2H_2N^{\cdot} = 108{\cdot}8.$$

It may be pointed out that an error of 10 p.c. in the atomic volume of hydrogen produces at most an error of ·5 p.c. in the cube root of the ionic volume of any substance quoted.

A more strictly applicable value for the ionic volume of acetanion would be obtained if it could be deduced from the density and concentration of the sodium acetate used in determining the ionic velocity, but as zero or negative values are obtained for the molecular volumes of some substances, this method is not trustworthy.

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[*] The meaning of the term "ionic volume" in this paper does not include the watery atmosphere, which the ions are usually regarded as carrying.

[†] *Lehrbuch d. Allg. Chem.* Bd. I.

IONIC VELOCITIES.

The velocities used in the table have been obtained from Bredig's memoir*. To convert the velocities as given by Bredig, into centimetres per second per volt per centimetre, the formula

$$u+v=-\frac{k}{nq}\frac{dV}{dx}$$

must be used, where u and v are the velocities of the cathion and anion respectively in cms. per sec., k the specific conductivity in reciprocal ohms, n the concentration in gram-equivalents per cub. cm., q the electrical charge in coulombs carried by a gram-equivalent of ions, and $\frac{dV}{dx}$ the potential gradient in volts per cm. Bredig's units are the same as these except that the conductivities are in Siemens' mercury unit, and the potential gradient is one volt per cm.; to be applicable to his data the expression becomes

$$u+v=\frac{1\cdot063\,k}{n\times 96440}=1\cdot1\times10^{-5}\frac{k}{n}=1\cdot1\times10^{-5}\mu_{\infty}.$$

Thus the values of the velocities given by Bredig were multiplied by $1\cdot1\times10^{-5}$.

DENSITIES.

These were got from several authorities, chiefly from Clarke's "Table of Specific Gravities." In some cases the original authority does not state the temperature of the water used in the determination of the specific gravity. When it is given, the density in grams per cub. cm. has been calculated.

EXPLANATION OF TABLE.

In the table there will be found the ionic velocities and the data used for deducing the cube roots of the ionic volumes. The densities could not all be obtained at the same temperature, but the temperatures selected were, in as many cases as possible, the same for each series. The variation with temperature of the ionic volume in the range of temperature used is small, e.g. the ionic volume of the propionic acid anion is 67·7 at 0° and 71·7 at 50°, a change of ·1 p.c. per degree. An error in a density affects the product in the last column to about one-third of the extent that the same error in ionic velocity does.

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* *Zeit. f. Phys. Chem.* XIII. 191 (1894).

No.	Ion	Ionic Formula	Formula of compound used for molecular volume	Molecular weight, M	Density (gms. per c.c.)/temp. deg. C.	Molecular volume, M/ρ	Ionic volume, $(M/\rho) \pm k$	Cube root of ionic volume	Ionic velocity, cm./sec., volt/cm. at 25° C. × 10^4	Ionic velocity × cube root of ionic volume × 10^5
	Fatty acid anions:									
1	Formic acid . .	$H.COO'$ . .	$H.COOH$ . .	46	1·22 /20	37·7	32·4	3·19	5·63	179
2	Acetic ,, . .	$CH_3.COO'$ . .	$CH_3.COOH$.	60	1·05 /20	57·2	51·9	3·73	4·21	157
3	Propionic ,, . .	$C_2H_5.COO'$ . .	$C_2H_5.COOH$.	74	·995/20	74·4	69·1	4·10	3·77	154
4	Butyric ,, . .	$C_3H_7.COO'$ . .	$C_3H_7.COOH$.	88·1	·960/20	91·8	86·5	4·42	3·38	149
5	Isobutyric ,, . .	$C_3H_7.COO'$ . .	$C_3H_7.COOH$.	88·1	·950/20	92·7	87·4	4·44	3·40	151
6	*n*-Valeric ,, . .	$C_4H_9.COO'$ . .	$C_4H_9.COOH$.	102·1	·943/20	108·2	102·9	4·69	3·17	149
7	*n*-Caproic ,, . .	$C_5H_{11}.COO'$.	$C_5H_{11}.COOH$.	116·1	·929/20	125·0	119·7	4·93	3·01	148
	Organic ammonium cathions of hydrochlorides:									
	Pyridines									
8	Pyridine . . .	$C_5H_5HN^{\cdot}$ . .	C_5H_5N . . .	79·0	·985/15	80·2	85·5	4·41	4·85	214
9	α-Picoline . . .	$C_6H_7HN^{\cdot}$ . .	$C_5H_4CH_3N$ . .	93·1	·933/22	99·8	105·1	4·72	4·30	203
10	α-Lutidine . . .	$C_7H_9HN^{\cdot}$ . .	$C_5H_3(CH_3)_2N$.	107·1	·945/22	113·3	118·6	4·91	4·03	198
11	Ethyl pyridine .	$C_7H_9HN^{\cdot}$ . .	$C_5H_4C_2H_5N$ . .	107·1	·936/20	114·4	119·7	4·93	4·05	200
12	α-Collidine . . .	$C_8H_{11}HN^{\cdot}$ . .	$C_5H_2(CH_3)_3N$.	121·1	·953/22	127·1	132·4	5·10	3·83	195

	cathions:									
	Amines									
13	Ethylamine	$C_2H_5H_3N^{\cdot}$	$C_2H_5H_2N$	45·1	·696/ 8	64·8	70·1	4·12	5·15	212
14	Allylamine	$C_3H_5H_3N^{\cdot}$	$C_3H_5H_2N$	57·1	·863/15	66·2	71·5	4·15	4·57	190
15	Propylamine	$C_3H_7H_3N^{\cdot}$	$C_3H_7H_2N$	59·1	·722/15	81·9	87·2	4·43	4·41	195
16	Isopropylamine	$C_3H_7H_3N^{\cdot}$	$C_3H_7H_2N$	59·1	·689/18	85·8	91·1	4·50	4·40	198
17	Diethylamine	$(C_2H_5)_2H_2N^{\cdot}$	$(C_2H_5)_2HN$	73·1	·706/20	103·5	108·8	4·77	3·97	190
18	Isobutylamine	$C_4H_9H_3N^{\cdot}$	$C_4H_9H_2N$	73·1	·735/15	99·5	104·8	4·71	3·99	188
19	Isoamylamine	$C_5H_{11}H_3N^{\cdot}$	$C_5H_{11}H_2N$	87·1	·749/18	116·2	121·5	4·95	3·73	185
20	Dipropylamine	$(C_3H_7)_2H_2N^{\cdot}$	$(C_3H_7)_2HN$	101·1	·710/15	142·3	147·6	5·28	3·34	177
21	Triethylamine	$(C_2H_5)_3HN^{\cdot}$	$(C_2H_5)_3N$	101·1	·735/15	137·5	142·8	5·23	3·59	188
29	Benzylamine	$C_6H_5CH_2H_3N^{\cdot}$	$C_6H_5CH_2H_2N$	107·1	·983/19	108·9	114·2	4·85	3·76	182
22	Diisobutylamine	$(C_4H_9)_2H_2N^{\cdot}$	$(C_4H_9)_2HN$	129·2	·743/25	173·9	179·2	5·64	2·96	167
23	Tripropylamine	$(C_3H_7)_3HN^{\cdot}$	$(C_3H_7)_3N$	143·2	·756/18	189·4	194·7	5·80	2·82	163
	Organic ammonium cathions of hydrochlorides:									
	Anilines									
24	Aniline	$C_6H_5H_3N^{\cdot}$	$C_6H_5H_2N$	93·1	1·023/15	91·0	96·3	4·58	3·95	181
25	Methylaniline	$C_6H_5CH_3H_2N^{\cdot}$	$C_6H_5CH_3HN$	107·1	·976/15	109·7	115·0	4·86	3·85	187
26	*o*-Toluidine	$C_6H_4CH_3H_3N^{\cdot}$	$C_6H_4CH_3H_2N$	107·1	·999/20	107·2	112·5	4·83	3·69	178
[27	*p*-Toluidine*	„	„	107·1	1·046/ –	102·4	107·7	4·76	3·63	173]
28	*m*-Toluidine	„	„	107·1	·998/25	107·4	112·7	4·83	3·61	174
29	Benzylamine	$C_6H_5CH_2H_3N^{\cdot}$	$C_6H_5CH_2H_2N$	107·1	·983/19	108·9	114·2	4·85	3·76	182
30	Ethylaniline	$C_6H_5C_2H_5H_2N^{\cdot}$	$C_6H_5C_2H_5HN$	121·1	·954/18	126·7	132·0	5·09	3·36	171
31	Diethylaniline	$C_6H_5(C_2H_5)_2HN^{\cdot}$	$C_6H_5(C_2H_5)_2N$	149·1	·936/18	159·3	164·6	5·48	3·15	173

* A solid.

In finding data for the table we have been restricted by the considerations that the ionic volumes may not be legitimately deduced for substances, which are gaseous (e.g. methylamine), or solid (e.g. morphine, oxalic acid), at the ordinary temperature, nor may substances whose densities have been determined only at temperatures considerably removed from the range 15°—20° C. be used. We propose, however, to attempt to obtain these ionic volumes later by indirect methods. Having regard to the above, all the values for ionic volumes that have been obtained for organic acids and bases are included in the table. (Considerable difficulties are met with in finding comparable values for the inorganic ions.)

Benzylamine from its preparation and properties—an ammoniacal liquid not forming diazonium compounds—would be classed with the primary fatty amines; from its graphical formula with the anilines. In the table it is put with both.

DISCUSSION OF TABLE.

The last three columns of the table show that the velocity of the ion of a member of a homologous series of the organic acids and bases examined varies inversely as the cube root of its volume, for the product of these is sensibly constant. Thus there is considerable evidence that for a homologous series the velocity of an ion is inversely proportional to its linear dimensions. The average value of the product, ionic velocity × cube root of ionic volume, for the acetic*—*n*-caproic acid series is 151, for the pyridines 202, for the amines 186, for the anilines 177. Thus, for ions of the same linear dimensions (cube root of ionic volume), the chemical properties influence their velocities as is shown by the following table in which the ions have closely the same volume.

	Ionic volume	$(\text{Ionic vol.})^{\frac{1}{3}}$	Ionic velocity	Product
A *n*-Caproic acid .	119·7	4·93	3·01	148
B Ethyl pyridine	119·7	4·93	4·05	200
C Isoamylamine .	121·5	4·95	3·73	185
a *n*-Valeric acid .	102·9	4·69	3·17	149
b α-Picoline . .	105·1	4·72	4·30	203
c Isobutylamine .	104·8	4·71	3·99	188

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* The value 179 for this product in the case of the formic acid anion is not surprising as this anion does not contain a CH_2 group, and so the shape of the ion (see later) is unlike that of the remaining members of this series.

The constancy of the product—ionic linear dimensions × velocity—for a homologous series of the organic ions in the table is in accordance with the hydrodynamical equation

$$va = \frac{F}{S\mu} \text{ (body of any shape)}$$

$$= \frac{F}{6\pi\mu} \text{ (sphere, no slipping)},$$

where F is the same for all the ions in the table, and μ is the viscosity of infinitely dilute aqueous solutions (i.e. of water) at 25° C. S is a constant for ions of the same shape *. Thus we find under the physical conditions mentioned in the table

$$va = \frac{1}{S} \times \text{constant},$$

S would be approximately constant for the members of a homologous series since their graphical formulae are similar; according to this view the chemical property that determines the velocity of ions of the same volume is the shape of the ion.

CONCLUSIONS.

1. The linear dimensions of thirty-one ions of the homologous series—formic-caproic acids, pyridines, amines, anilines—have been calculated from molecular and atomic volumes. The velocities of these ions in cm./sec., volt/cm. have been given. It is found that the velocity of an ion of these series is closely inversely proportional to its linear dimensions.

2. It is shown that these results may be accounted for by considering the motion of an ion through an electrolyte as similar to that of a body through a viscous medium.

We intend continuing work in this direction. We wish to thank Mr Whetham, F.R.S., for some kindly criticism.

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* Lamb, *Treatise on the Motion of Fluids*.

CAVENDISH LABORATORY,
March, 1906.

On the reduction of the general ternary quintic to Hilbert's canonical form. By H. W. RICHMOND, M.A., King's College.

[*Received* 12 March 1906.]

The theorem that a ternary quintic form may generally be uniquely expressed as the sum of the fifth powers of seven linear forms is due to Hilbert*: recently the theorem has been rediscovered in ignorance of Hilbert's result by myself, and again a year later by Palatini†. The possibility of reduction to this form being established, a method of effecting the reduction (depending upon the solution of a single irreducible equation of the seventh degree) which I obtained soon after the publication of my former paper is here indicated.

The ternary quintic form is the equation of a plane curve of the fifth order, C_5: if italic letters $x, y, \ldots$, with suffixes 0, 1, 2 be the coordinates of points, and Greek letters stand for coordinates of lines, the equation of C_5 may be written

$$a_x^5 \equiv (a_0x_0 + a_1x_1 + a_2x_2)^5 = 0,$$

while C_n and Γ_n will denote a curve of order n and of class n respectively. It is clear that

The Γ_3's apolar to C_5 form generally a linear system of four terms; in fact the only exceptions to this statement are when C_5 possesses an apolar Γ_2. Among the Γ_3's apolar to C_5 are included those which touch the seven lines representing each of the seven linear forms mentioned above. It will appear that it is always possible to find three Γ_3's apolar to C_5 whose equations $\Upsilon_0=0$, $\Upsilon_1=0$, $\Upsilon_2=0$ satisfy

$$\xi_0\Upsilon_0 + \xi_1\Upsilon_1 + \xi_2\Upsilon_2 \equiv 0 \quad\text{.....................}(1),$$

the solution being generally unique; if so, the three Γ_3's have seven common tangents which give the seven linear forms sought.

That the identity (1) should hold it is necessary and sufficient that

$$\Upsilon_0 \equiv \xi_1\Sigma_2 - \xi_2\Sigma_1; \quad \Upsilon_1 = \xi_2\Sigma_0 - \xi_0\Sigma_2; \quad \Upsilon_2 = \xi_0\Sigma_1 - \xi_1\Sigma_0;$$

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* *Liouville's Journal*, Series 4, Vol. IV, p. 256.

† *Quarterly Journal of Mathematics*, 1902, p. 337. *Atti d. R. Acc. d. Sc. di Torino*, 30 Nov. 1902; *Rendic. d. R. Acc. dei Lincei*, 17 May, 1903.

where Σ_0, Σ_1, Σ_2 are curves of class two; further, we may assume without loss of generality that Σ_0, Σ_1, Σ_2 lack respectively terms in ξ_0^2, ξ_1^2, ξ_2^2. Suppose

$$\Sigma_0 = \alpha_\xi^2, \quad \Sigma_1 = \beta_\xi^2, \quad \Sigma_2 = \gamma_\xi^2,$$

then the conditions of apolarity to C_5 impose the following conditions upon the 15 coefficients of Σ_0, Σ_1 and Σ_2:

$$\{a_1 (a_\gamma)^2 - a_2 (a_\beta)^2\} a_x^2 \equiv 0, \quad \{a_2 (a_\alpha)^2 - a_0 (a_\gamma)^2\} a_x^2 \equiv 0,$$
$$\{a_0 (a_\beta)^2 - a_1 (a_\alpha)^2\} a_x^2 = 0,$$

for all values of x_0, x_1, x_2; eighteen conditions, homogeneous in fifteen unknown coefficients, have to be satisfied. But clearly the conditions obtained by equating to zero the coefficients of $x_0 x_s$ in the first, of $x_1 x_s$ in the second, and of $x_2 x_s$ in the third are not independent, s standing for either 0, 1, or 2: thus the eighteen conditions are brought down to fifteen. The possibility of satisfying these depends on the vanishing of a determinant of 15 rows; but this determinant is obviously skew-symmetrical and so must vanish.

An Indicator for Strong Acids and Bases. By H. J. H. FENTON, Sc.D., F.R.S., Christ's College.

[*Read* 26 February 1906.]

In previous communications to the Society reference has been made to a new condensation-product, derived from bromomethylfurfural, which has the molecular formula $C_{11}H_8O_4$ and to which the constitution

$$CH_3C_4H_2O.CO.C_4H_2O.CHO \text{ or } C_4H_3O.CO.CO.C_4H_2O.CH_3$$

has been provisionally assigned.

It was mentioned that this substance may have useful applications in organic analysis since it gives highly characteristic colour-reactions with certain classes of compounds such as amines and ureas.

Later experiments have shewn that this reagent may conveniently be used in the form of test papers and in this way the application of the tests is greatly simplified although some of the reactions are more sensitive when brought about in solution in the manner previously suggested.

Test papers prepared from an aqueous-alcoholic solution of the reagent give, immediately, an intense green colour with primary amines in acetic acid solution; this colour is at once destroyed by strong mineral acids or by alkalis.

With urea in presence of strong hydrochloric acid a very brilliant blue colour is slowly developed; it appears more quickly in presence of dehydrating substances such as acetyl chloride or phosphorus oxychloride.

With caustic alkalis a beautiful violet-blue colour is immediately produced which is at once discharged by acids. The colour given with normal caustic soda or potash is very intense; as the concentration of the solution is diminished the colour becomes paler and is only just perceptible in centi-normal solutions. The gradual variation of the colour-intensity with concentration is so well marked that it affords at once a rough indication of the degree of alkalinity or concentration of hydroxyl ions.

The limit at which the colour is just barely visible varies, as might be expected, greatly with different bases. Rough preliminary observations indicated that the limit of concentration for sodium hydroxide, potassium hydroxide, or tetramethylammonium hydroxide is about 0·01 normal; for dimethylamine 0·04 normal and for ammonia 1·8 normal. With sodium carbonate the alkalinity is only perceptible in very strong solutions and with borax or sodium acetate no effect at all can be observed at the ordinary temperature in solutions of any concentration; on warming the strong solutions however the colour becomes apparent.

If the above-mentioned reagent is fused at about 120° with urea a colourless base is obtained which immediately becomes blue when treated with acids [*Trans. Chem. Soc.*, 1903, 187]. Test-papers prepared from an aqueous-alcoholic solution of this base may be used therefore as an indicator for acids. Normal solutions of hydrochloric or nitric acids give a brilliant Prussian blue colour, the intensity becoming less and fading to a pale sky-blue as the solution becomes more dilute. The limit of concentration in this case at which the colour is just perceptible is for hydrochloric acid about deci-normal.

Half normal solutions of oxalic acid give a bright sky-blue colour whereas solutions of tartaric acid of the same concentration give no colour; solutions of boric acid of any concentration give a negative result. The limit for hydrofluoric acid appears to be about 1·3 normal and with acetic acid the colour is not perceptible until the solution reaches a strength of 12 or 13 normal.

An acid-indicator of a different character may be prepared by boiling together the first-named reagent with a primary amine, such as β naphthylamine, in alcoholic solution. Test-papers prepared with this solution give, at once, an intense green colour with *weak* acids. The limits of sensitiveness in this case are, with hydrochloric acid, between about 0·005 normal and deci-normal; with greater concentrations only a yellow colour is produced.

Notes on Cycads: with exhibition of a rare species (Cycas Micholitzii) *acquired by the Botanic Garden.* By A. C. SEWARD, M.A., F.R.S., Fellow of Emmanuel College.

[*Read* 12 February 1906.]

The Author exhibited a plant of *Cycas Micholitzii* recently obtained by the Curator of the University Botanic Garden from Messrs Sander and Sons. This species was discovered by Mr W. Micholitz, one of Messrs Sander and Sons' collectors, in Annam, and described last year by Sir William Thiselton-Dyer in the *Gardeners' Chronicle**. The illustrations accompanying the description were drawn by an artist on the staff of Mr Ridley, the Director of the Botanic Garden, Singapore, who sent specimens of the plant to the former Director of the Royal Gardens, Kew. The following is the diagnosis of the new species:—"Cycas Micholitzii, sp. n. Caudex subterraneus, 1½ ped. longus 1½ poll. crassus. Folia pauca, ad 10 ped. longa, basi ad 4 ped. aculeis armata, pinnulis 8 poll. longis, fere 2-dichotomis, segmentis ¾ poll. latis. Strobilus mas (juvenilis) ad 6 poll. longus, squamis antheriferis brevissime acuminatis. Carpophylli lamina terminalis obovato-rhombea superne profunde pectinato-lacera. Annam. Legit cl. Micholitz. *W. T. Thiselton-Dyer.*"

The most striking character of this species is the repeated dichotomy of most of the pinnae. The two lowest pinnae on one of the fronds represented by the Singapore artist are divided into six and seven linear ultimate segments respectively: the branching appears to be dichotomous; the lobed pinnae are divided by a median sinus, which reaches almost or quite to the rachis, into two equal branches, and these are again subdivided. The terminal pinnae are simple and similar to those of *Cycas revoluta* and other species of the genus: these are succeeded by pinnae with two equal segments, and in the lower pinnae the ultimate segments exhibit further dichotomy. The Cambridge plant is clearly a young specimen and differs from that figured by the author of the species in the simple character of most of the pinnae, while others are divided almost to the base into two linear segments. None of the pinnae are subdivided into more than two lobes. The appearance of the slender stem of the young Cambridge

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* *Gardeners' Chronicle*, August 19, 1905, p. 142.

specimen, having a diameter at the broadest part of 4 cm. and rising to a height of 15 cm. above the surface of the soil, does not appear consistent with the statement quoted by Sir William Thiselton-Dyer from Mr Micholitz that the stem is subterranean. The following extract from a letter written by Mr Micholitz* expresses the views of the discoverer of the species on this point. "The plant has its stem in the ground, that is to say the stems grow downwards instead of upwards; the stem of young plants is covered with 3—4 inches of soil (precaution against heat and bush-fires?) and the older the plant the longer or deeper the stem grows into the ground, so that the stem is never visible above ground, and the flowering cones appear just level with the ground." It would seem, therefore, that the species, as seen in its native habitat, is characterised by the subterranean position of the stem and that the aerial stem of the cultivated plant does not represent the normal condition. The surface of the young stem is covered by contiguous transversely elongated bases of scale- and foliage-leaves, which do not exhibit that marked contrast in size characteristic of some other species of *Cycas*. A few apogeotropic roots project above the surface of the ground close to the base of the stem. The tip of the stem bears a few short brown scale-leaves and two fronds. The petioles of the fronds are long and slender, armed with a few short spines and terminating in swollen bases: the margins of the enlarged bases are winged and the wings extend a short distance up the sides of the petiole as membranous extensions. The largest frond has a length of 130 cm., measured from the tip of the uppermost pinna to the base of the petiole; the pinnae, reaching a length of 35 cm., are either simple or divided almost to the base into two equal segments.

The vascular strand in the axis of a pinna exhibits the usual Cycadean structure, but, as shown in Fig. 5, there is a considerable development of centrifugal xylem. The main portion of the metaxylem, which consists of reticulate elements, has the form of a slightly curved band (Fig. 5, *cp*): the somewhat crushed and swollen protoxylem tracheids (*px*) form the pointed extremity of a group of elements projecting upwards from the middle of the upper face of the centripetal xylem. The space between the centripetal and the centrifugal xylem is occupied by conjunctive tissue composed of fairly large parenchymatous cells. The strand is enclosed by a sheath (*s*) of crystal-containing cells with thick lignified inner walls; between the centripetal xylem and the sheath occur 2—3 layers of parenchyma. The phloem shows the usual Cycadean features. A complete section of the axis of a pinna exhibits a dorsiventral differentiation: the upper half is distinguished by a more compact band of hypodermal fibres and by the occurrence of a few layers of chlorophyll-containing cells internal to the stereome. The hypoderm next the lower epidermis forms a less clearly defined band and passes gradually into scattered fibres interspersed with the parenchymatous ground-tissue. No secretory canals were observed.

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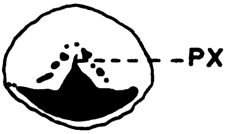
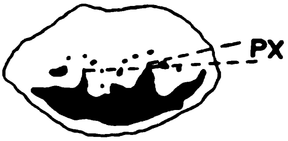
* I am indebted to Mr Lynch, the Curator of the Cambridge Botanic Garden, for this extract, which he obtained through the courtesy of Messrs Sander and Sons.

Figs. 1—4* illustrate the gradual change in the form of a vascular strand in the region of forking. The single protoxylem group (*px*) becomes double (Fig. 2), and the transverse elongation of the centripetal xylem and separation of the two protoxylem points are followed by an equal division into two strands. The forking of the pinna is thus seen to be accompanied by a dichotomy of the vascular bundle.

1



3



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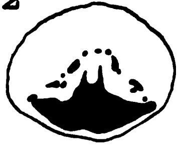

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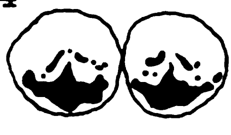

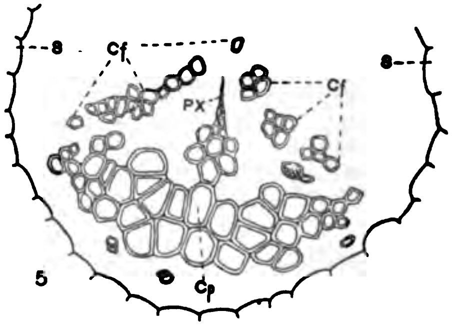

5

The occurrence of forked pinnae of Cycadean fronds was first

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* The sections were prepared by Miss Sykes of Girton College.

noticed by Moore in the Australian species *Macrozamia heteromera*[*], in some varieties of which the narrow pinnae may be divided into six or seven ultimate segments. In the dichotomous method of branching there is the closest resemblance between Moore's species and *Cycas Micholitzii.* It is now generally admitted that Cycads and Ferns were probably derived from common ancestors, a conclusion based in part upon certain resemblances between existing members of these two groups, but founded mainly on the discovery of several Palaeozoic types of a generalised type in which Cycadean and Filicinean characters are intimately associated. In view of this relationship it is interesting to find another common feature in the dichotomous branching of the pinnae. The fronds of *Matonia pectinata*[†], as recently shown by the authors of an exhaustive paper on the young plants of this ancient type, exhibit both monopodial and dichotomous branching.

Similarly, Mr Boodle[‡] speaks of the dichotomy of the primary pinnae in *Gleichenia* as a primitive character which occurs also in *Lygodium.*

In *Matonia sarmentosa,* Baker[§], the forked linear pinnae bear a close resemblance to those of *Cycas Micholitzii* in their method of branching. Numerous other instances among fossil and recent plants might be quoted of the close correspondence in the possession of dichotomously branched pinnae between Cycads and Ferns.

The discovery of a species of *Cycas* with dichotomously branched pinnae suggests the possibility that the simple pinnate type of Cycadean frond, with its unbranched pinnae, may be the result of reduction from an older type characterised by the more primitive dichotomous habit.

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[*] Moore, *Journ. Proc. R. Soc. New South Wales*, Vol. XVIII. 1884, p. 115. Seward, *The Wealden Flora*, Part II. (Catalogue of the Mesozoic Plants in the Department of Geology, British Museum) 1895, p. 5, pl. xiii.

[†] Tansley and Lulham, *Annals Botany*, Vol. XIX. 1905, p. 477.

[‡] Boodle, *Annals Botany*, Vol. XV. 1901, p. 705.

[§] Baker, *Annals Botany*, Vol. V. 1890, p. 191, pl. xiv.

Experiments on the Hybridisation of Barleys. By R. H. Biffen, M.A., Emmanuel College.

[*Read* 12 February 1906.]

The cultivated barleys form a group of elementary species known collectively as *Hordeum sativum.* The most important of these are *H. hexastichum*, *H. vulgare*, *H. zeocriton*, *H. distichum*, and *H. decipiens.* In the first pair each of the spikelets in the triple groups characteristic of the genus *Hordeum* is fertile, they differ from one another in the fact that the former has dense ears and the latter lax ones. Both may correctly be described as six-row barleys. *H. vulgare* is the name given to the group known on the Continent as *H. tetrastichum.* The use of this latter term is open to the serious objection that it leads to the erroneous impression that the barley is four-rowed owing to the suppression of the median floret of the triplet. *H. zeocriton* and *H. distichum* form a similar pair of dense and lax groups differing from the former in the lateral florets, which are staminate only and consequently set no grain. These are popularly known as two-row barleys. In the group *H. decipiens* the reduction of the lateral florets is carried still further, and they are altogether sexless and practically wanting.

The six- and two-row groups are connected together by the little known *H. intermedium*, in which the lateral florets, though fertile, are reduced in size and awnless.

The varieties within the elementary species are distinguished by a number of characters. The most important of these are as follows:

1. The paleae may or may not adhere to the grain proper. The first condition is described as naked, the second I have called "trapped."
2. The colour of the paleae may be "white," that is, various shades of yellow; or it may be black, brown, or purple.
3. The grain may be white, bluish-grey, or purple when stripped of its paleae.
4. The awns may be normal or trifurcate. This latter term is used to denote the presence of a secondary floret borne on the awn either at its base or some two or three cms. up.
5. The glumes at the base of each spikelet may be narrow or ovate-lanceolate in shape.

In addition there are a number of minor characters, so that it is obvious that there may be an almost indefinite number of combinations.

In order to simplify another problem we have in hand it has been necessary to trace in some detail the inheritance of these various characters. For the most part this has not proved a difficult matter, and few complications have been met with. Most of the various characters segregate sharply from one another, and form good examples of the allelomorphic pairs we are now so familiar with. These pairs are as follows, the dominant character being placed first in each:

1. Trapped and naked grain.
2. Brown, black, or purple paleae and white.
3. Dark and light coloured grain.
4. Trifurcate and normal awns.
5. Narrow and broad glumes.
6. Two-row and six-row.
7. The *decipiens* type and two-row.
8. The *decipiens* type and six-row give an intermediate.
9. Lax and dense ears give an intermediate in the F 1.

For the present attention need only be called to the more complex cases. The first of these is provided by crosses between the awned and trifurcate varieties. Tschermak has described one example of such a cross, and he states that the awned form is dominant over the trifurcate. In the four cases I have examined the supernumerary flowers have always been present in the F 1, so that it would appear as if the hooded or trifurcate form was normally dominant over the awned. The F 1's, however, differ a good deal among themselves with regard to the position of the extra florets. In one cross-bred they were practically sessile, as in the parent, but in the other three they were borne on long awns which were prolonged beyond them. In the subsequent generation most of the hooded forms bore the extra floret on awns some three or four centimetres in length. In many of these plants few of the awns carried such fully developed extra florets as the parent. If all of these partially hooded forms are to be reckoned as trifurcate then there is good reason to consider the trifurcate condition as dominant, for they occur in the proportion of three to one of the normally awned forms. On the other hand, it may be that the F 1 should be described as an intermediate, in which case the partially hooded types should occur in the proportion of two to one of the awned, but the difficulty of deciding between fully and partially hooded ears is too great for one to separate the forms satisfactorily. This can, however, be determined in the following generation. One other cross between a hooded variety may be mentioned here. In this case an awnless variety was used as the other parent. The F 1 was awnless, showing that this condition, as in the case of the wheats, is dominant, whilst the F 2 consisted of hooded and awnless individuals. The awnless parent is a variety in the sense of the word as used by de Vries, and it is one of very recent origin, yet it is dominant over the older type.

The crosses between varieties in which one parent has dense the other lax ears have now to be considered. In the case of wheat, where these characters are also present, Spillman showed that the F 1 was intermediate between the parents in this respect, and that in the second generation the normal segregation occurred, resulting in the production of lax, intermediate, and dense ears in the proportion of 1 : 2 : 1. In the barleys I have examined up to the present the F 1 is also intermediate between the parents in the density of the ears. When, however, one attempts to separate the three expected types one can readily pick out many of the two extremes, but numerous cases occur where one cannot say whether any given ear should be classed as an intermediate or not. On shuffling the ears and again attempting to separate them, a result totally at variance with the former one was generally obtained. At first I was under the impression that this was due to the difficulty of correctly gauging the relative laxness and denseness by eye, so I measured the length of the internodes, but the figures so obtained did not fall into the three expected groups. On plotting curves for a number of separate crosses with these figures on the base line the explanation of these apparently non-conformable cases was found. Where there were extremely lax and dense forms crossed it was clear that the three separate curves existed, but the ends of the curves corresponding to the lax and dense types were covered by the curve of the intermediates. In cases where the difference between the parents was less pronounced the overlapping was so great that the three curves fused into one, showing, however, on either side a distinct secondary apex. The F 2 generation of the cross between *H. pyramidatum* and *H. decipiens* may be taken to illustrate this point further. The average internode length of the parents was found to be 1·8 and 3·4 mms. respectively, that of the F 1 3·2 mms. The measurements for the F 2 generation were as follows:

1·4	1·6	1·8	2·0	2·2	2·4	2·6	2·8	3·0	3·2	3·4	3·6	3·8	4·0	4·2	4·4	mms.
0	3	7	14	19	12	42	41	28	15	6	4	2	2	1	0	individuals.

An individual ear measuring 24 mms. then may be either a true dense form or a heterozygote, and the distinction can only be made by sowing its grains and observing whether the progeny are constant or whether segregation occurs. This test has been carried out in the example quoted, and by separating the individuals in this way it was found that the splitting was normal, as it resulted in the production of two intermediates to one each of the lax and dense types. One other point has to be briefly noted here—that is, that, knowing the range of the variability of the parents, one should be able to calculate from the F 2 curve the number of individuals in the three groups of which it is composed. This assumes that the fluctuating variability of the extracted lax and dense forms is the same as that of the parents.

This does not appear to be the case, and a considerable amount of evidence, not only among the barleys but also among the wheats, has accumulated, which shows that the range of variability of the extracted characters is widened to an appreciable extent. Thus amongst the lax individuals are some in which the ears show as much as a 10 per cent. increase in this character. It is also doubtful whether the average internode length of the extracted types is the same as that of the lax or the dense parents. What little evidence we have on this point leads me to suspect that the average length itself is altered. To obtain the necessary statistics will be tedious work, but, in view of the importance of the question to those who are attempting to apply Mendelian principles to the improvement of our crops, it will have to be undertaken.

As a proof of the value of these observations to the plant-breeder it may be stated that the extremes selected from the F 2 have in the cases tested bred true to this particular character.

The crosses between varieties showing different degrees of development of the lateral florets have given some interesting results. In a previous communication I pointed out that where a variety with no lateral florets was crossed with one possessing male florets the F 1 resembled the former parent in this respect, inasmuch as that, though very minute florets were produced, they were altogether sexless. Since then the F 2 of a number of such crosses has been grown, and also one F 3. In each case the F 2 consisted of individuals with either no lateral florets, or with minute and sexless laterals, or with well-developed male laterals. The three groups occur in the proportion of 1 : 2 : 1, and they are perfectly distinct from one another. In the single F 3 so far tested the individuals with the minute laterals have invariably proved to be heterozygotes, whilst those with no laterals and those with male laterals have proved to be homozygotes. The sexless condition of the laterals may therefore be described as dominant. Where one parent has black paleae in such crosses the lateral florets, both male and rudimentary, are white.

A number of hybrids between varieties of this *decipiens* type and varieties with fully fertile laterals have been raised. The F 1 in such cases bears small lateral florets containing either vestiges of stamens or fully developed ones. The heterozygote is thus a two-row barley in the popular sense of the word, or from the systematic point of view a *distichum* or a *zeocriton* according to the laxness or denseness of the ear. In the following generation three forms of ear occur, the six-row, two-row, and the *decipiens*, in the proportion of 1 : 2 : 1. The reciprocal crosses give precisely the same results. Only one F 3 of this combination of characters has yet been grown, and that unfortunately a small one. It serves, however, to show that the forms with the male laterals produced in the F 2 are true heterozygotes, since they give rise to individuals with hermaphrodite or with sexless laterals as well as the form with staminate lateral florets only. These latter are indistinguishable from the two-row barleys in general cultivation as far as external appearances go. Summarizing this part of the story, then, we may say that the result of crossing varieties with sexless and with hermaphrodite laterals is to suppress the production of the female portion, so giving a unisexual male heterozygote.

It remains now to consider the results of crossing together varieties with male and hermaphrodite lateral florets. My first experiments in this direction led me to conclude that the heterozygote in such cases was a true two-rowed individual, and that its progeny consisted of three of the former to one of the latter type. More recently I have found cases in which the heterozygote bore a few undoubtedly fertile florets in the lateral rows, and such occur amongst its offspring in the following generation. If such are to be included in the group with male laterals and looked upon as abnormalities, which as a matter of fact occur from time to time in the two-rowed varieties, then the segregation is normal, for there are three two-rowed individuals to each two-rowed. It is more probable though that we again have to deal with an intermediate heterozygote in which the formation of small but normally fertile florets is the rule, but, as there are complications occurring, very possibly owing to differences of nutrition, the further description of this case must be deferred until the individuals from the F 2, assumed to be heterozygotes, have been tested in the following generation.

A Preliminary Note on the Maiotic Phenomena in the Eggs of the Hermaphrodite "Angiostomum nigrovenosum" ("Ascaris nigrovenosa"). By S. A. McDowall, B.A., Trinity College.

[*Received* 23 February 1906.]

It was suggested to me by Mr Punnett, of Caius College, that an investigation of the maturation processes in the sex cells of *Angiostomum nigrovenosum*, Rud. (*Angiostomum*, Duj.; *Ascaris*, Linn.; *Rhabdonema*, Kütz.) might lead to interesting results. As is well known this nematode presents the unique phenomenon of a protandrous hermaphrodite generation, alternating with a dioecious*; and it seemed very possible that there might be corresponding peculiarities in the maturing sex cells of the worm.

The technical difficulties of the examination proved unexpectedly great, owing to the imperviousness of the cuticle, and also to the minute size of the dioecious or *Rhabditis* form. The work was much delayed by illness, and I have only been able to obtain satisfactory results in the hermaphrodite generation.

As, however, the *maiotic*† phenomena in the oocytes of this generation present features of some interest, it seemed worth while to publish a preliminary account now, in the hope of supplementing the description later with a fuller account of them in both the hermaphrodite form and the *Rhabditis*. The preservative used was Petrunkewitsch's fluid‡, but special methods of application were resorted to in order to ensure satisfactory results. The sections were stained with iron-haematoxylin and orange G. The onset of the maiotic phase is well marked. The nucleoli are large; they are generally not vacuolated, and only in very rare instances is more than one present.

The first sign of activity manifests itself in an increase in the linin network, which becomes very dense. "Chromatic clouds" appear in the nodal aggregations of linin, and these gradually become denser, spreading along the linin threads, and forming the distinct chromatin network, with nodal chromatin-masses.

The reticulum next splits into definite chromosomes, apparently twelve in number.

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* E. Mecznikow, *Archiv für Anat. u. Phys.* (*Müller's Archiv*), 1865.

† I.e. *Reduction*, v. Farmer and Moore, *Q.J.M.S.*, Vol. 48, Part IV. 1905, p. 489.

‡ A. Petrunkewitsch, *Zool. Jährb.*, Abtheilung für Anat. u. Ont. der Thiere, Bd. XIV. p. 576.

They are rather elongated, sometimes rod-like, sometimes more irregular in form. These chromosomes, however, do not remain long in this condition. Even from their earliest appearance they tend to lie in pairs, parallel to one another, though there seems to be no actual union; and very shortly they are seen to be no longer parallel, but at an angle one to the other.

Fusion between the approximating ends takes place, so that six V-shaped double chromosomes are formed (Fig. 1). (This figure is purely diagrammatic, and the chromosomes are somewhat reduced in size. In no individual nucleus can the pairing be seen in more than one or two cases.)

A transitory contraction-phase seems to ensue.

The V-shaped chromosomes then elongate considerably, and form what at first sight appears to be a segmented spireme. Sometimes an indistinct longitudinal split is visible in the chromosomes at this time (Fig. 2).

Next, the chromosomes shorten and thicken, and again become V-shaped; a second *contraction-phase* seems to occur[*].

The angle of each V now points inwards towards the centre of the nucleus, while the tips of the arms touch the nuclear membrane. This gives the appearance of a secondary reticulum, with a very dim network and twelve rounded nodal bodies, intensely chromatic, of approximately equal size (Fig. 3).

The next stage is interesting. The chromatin appears to run more and more towards these false nodes, the arms of the V projecting inwards towards the angle becoming more and more contracted, and the angle enclosed by them wider and wider; so that at last only the two nodal spots are left, now very close together and enlarged, joined by a band of "Zwischen substanz[†]."

We have thus twelve darkly-staining chromosomes of very characteristic shape, fused together in pairs (Fig. 4). Frequently there appears to be one very large pair, one slightly smaller, and four a little smaller still. Often before the breaking-down of the nuclear membrane a second split is visible, dividing each chromosome—probably the remains of the longitudinal split seen in the pseudo-spireme stage—so that a tetrad appearance is found. Meantime the nucleolus (*N*, diagrams 1—3), which has been gradually diminishing in size, shrinks rapidly, and is finally extruded into the cytoplasm just before the breaking down of the nuclear membrane, and there it is immediately disintegrated. The maturation-divisions themselves are difficult to follow.

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[*] Cf. Farmer and Moore, *Q.J.M.S.* Vol. 48, Part IV. 1905, p. 546.

[†] K. Bonnevie (*Anat. Anzeiger*, Bd. XXVI. p. 501), describes this "Zwischensubstanz" as chromatic in Enteroxenos: "Die beiden chromatinfäden der Doppelchromosoma sind, wie schon oben erwähnt, durch eine *chromatische* Zwischensubstanz verbunden." But in the case of *Angiostomum nigrovenosum* it is distinctly not chromatic, and apparently stains lightly with orange G. Perhaps, however, "*chromatische*" is a misprint for "*elastiche*," which word is used in the summary given at the end of the paper.

The deutoplasm is exceedingly dense[*] at this time, and often appears to contain fragments of the degenerated nucleolus, so that the details are much obscured. Additional difficulties arise, owing to the fact that the first maturation-division only takes place at or just before the time of the entrance of the spermatozoon, the second immediately succeeding it; while definite polar bodies are not formed, the polar nuclei remaining in the deutoplasm. The divisions, however, appear to be of the normal type[†], the double chromosomes splitting into their components and six going into the first polar body, which may again divide subsequently, the other six at once splitting along the already-formed line of fission. Thus the first division is a reduction. I have not been able to distinguish any definite spindle-fibres. This may be due to the density of the deutoplasm, or possibly to imperfect preservation, although the earlier stages are perfectly preserved. By using an oblique illumination, distinct lines of stress in the deutoplasm are sometimes made visible, so that it is quite possible that no actual fibres are formed.

The male nucleus is small, but very rapidly enlarges in its passage through the ovum. The chromosomes of the male and female nuclei perhaps unite in pairs, temporarily. Distinct indications of such a pairing are to be found.

The nucleus then enters on a densely-chromatic resting-stage. It is extremely large.

A large centrally-placed nucleolus appears, and the chromatin arranges itself round this in a very characteristic manner. The yolk rapidly increases, and acquires a different reaction, spherules being present which stain well with haematoxylin; whereas before fertilisation spherules were not recognisable, and the deutoplasm stained with orange G.

It may be worth noting here what very different results are obtained by using unsatisfactory preservatives. A series of fertilised ova, preserved with Flemming's fluid and stained with iron-haematoxylin and eosin, show a very distinct nuclear membrane, surrounding a clear space which stains rather unevenly a yellowish colour, in which lie two definite structures of large size, one curved and dark, the other a cloudy reflection of the first, staining a brownish colour.

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[*] Cf. C. Neuhaus, *Jenaische Zeitschr. für Naturwiss.*, Bd. xxxvii. p. 653.

[†] I can find no trace of the persistence of the double-chromosomes described by K. Bonnevie in Enteroxenos (*op. cit.*). Rather, the evidence distinctly points to a separation of the conjugated chromosomes, though whether this actually occurs in the *first* or the *second* maturation-division is difficult to determine, owing to the symmetrical tetrad form of the double-chromosomes.

In properly-preserved examples I find nothing resembling these structures, which however are the more curious, as they appear throughout one whole series of sections.

Although all the phases described, with the exception of the last stage of the synopsis, are complex and difficult to observe, yet in favourable instances they are sufficiently clear to make their interpretation certain, and the occurrence of these peculiar appearances in badly-preserved material does not alter my views as to the real meaning of the phenomena observed.

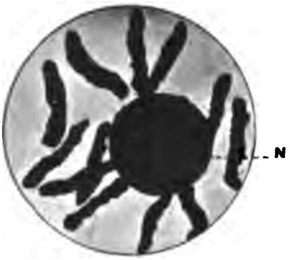


Fig. 1. Diagrammatic.

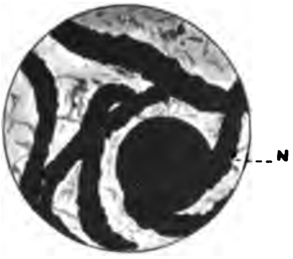


Fig. 2. Semi-diagrammatic.

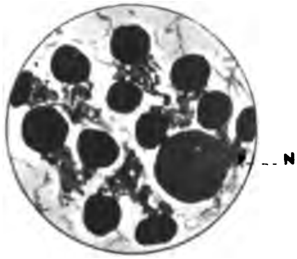


Fig. 3. Semi-diagrammatic.

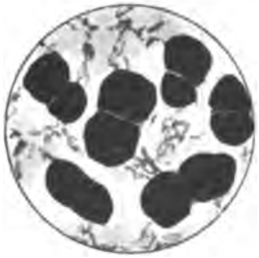
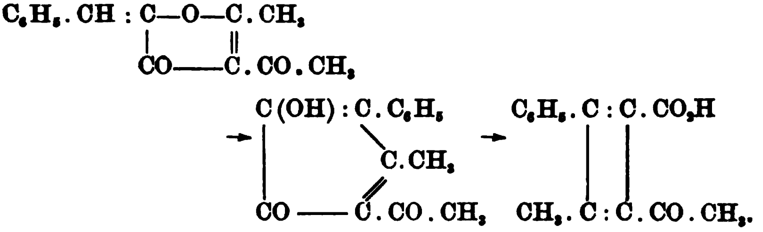


Fig. 4. Semi-diagrammatic.

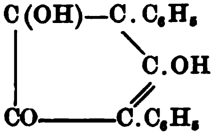
The Action of Acidchlorides of Acetylenic Acids on Ketonic Compounds. By S. Ruhemann, M.A., Caius College.

[*Read* 26 February 1906.]

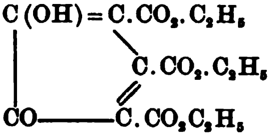
Some months ago Ruhemann and Merriman (*Trans. Chem. Soc.*, 1905, LXXXVII. 1383) published the results of an investigation on the action of phenylpropiolyl chloride on sodioacetylacetone, and showed that a yellow compound $C_{14}H_{12}O_3$ was formed which, under the influence of piperidine, changed into a red isomeride. This was found to dissolve in alkalis, their carbonates or organic bases to yield blue solutions and, on boiling with sodium carbonate, to suffer a transformation into a colourless acid which had the same molecular composition as the former two compounds. These changes were symbolised thus:



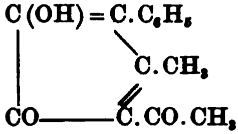
I have now undertaken the further study of these substances with the view of supporting the above constitutional formulae. My attention has been directed especially to the red isomeride $C_{14}H_{12}O_3$ the properties of which closely resemble the colourless triethyl oxalylaconitate (see Ruhemann and Hemmy, *Trans. Chem. Soc.*, 1897, LXXI, 34), and Claisen and Ewan's oxalyldibenzylketone (*Annalen*, 1895, CCLXXXIV, 245). The analogy of these three compounds is indicated, also, by the similarity of their structural formulae:



Oxalyldibenzylketone

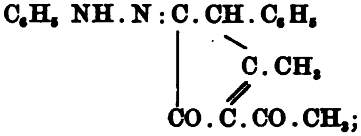


Triethyl oxalylaconitate

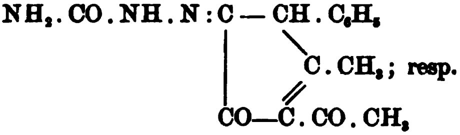


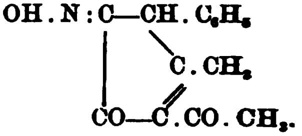
Oxalylphenylmethyl acetylpropene.

This fact, already, supports the constitution of the red isomeride $C_{14}H_{12}O_3$, and the following behaviour points to the same conclusion. Under the influence of phenylhydrazine, the compound yields a phenylhydrazone which, on account of its insolubility in alkalis, is to be represented thus:

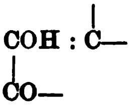


its formation, therefore, is accompanied by the change of the enolic into the ketonic grouping. Analogous compounds are formed by the action of semicarbazide and hydroxylamine on the red substance $C_{14}H_{12}O_3$; these are to be expressed by the symbols:



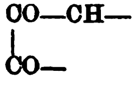


The circumstance that the latter two compounds are colourless leads to the conclusion that the red colour of $C_{14}H_{12}O_3$ is due to the enolic group,



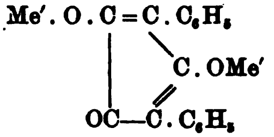
$$\begin{array}{ll} & | \\ \mathrm{COH} : & \mathrm{C}— \\ | & \\ \mathrm{CO}— & \end{array}$$

and disappears with the change into the ketonic grouping,

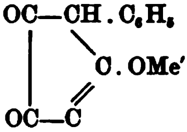


$$\begin{array}{l} \mathrm{CO—CH—} \\ | \\ \mathrm{CO—} \end{array}$$

This behaviour of the red isomeride allows of a ready explanation of the results at which Claisen and Ewan arrived in the course of their study of oxalyldibenzylketone. This substance forms yellow as well as blue salts. From the properties of the red compound, $C_{14}H_{12}O_3$, and triethyl oxalylaconitate it follows that the latter salts are to be formulated thus:



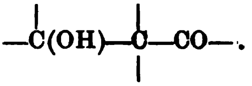
and the yellow salts:



This view leads to the conclusion that the arrangement

$$—CO . C(OH)=$$

gives rise to the formation of blue salts, whilst yellow salts are produced, if the carbinol group occupies the β-position towards the ketonic group:



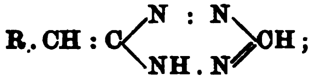
$$—\overset{|}{C}(OH)—\underset{|}{\overset{|}{C}}—CO—.$$

A number of compounds with the latter arrangement are known, and they dissolve in alkalis or their carbonates to yield yellow solutions. In the light of this view concerning the constitution of the blue and yellow salts of oxalyldibenzylketone, the various derivatives of this compound which Claisen and Ewan obtained, can readily be formulated, and these formulae are found to be in close agreement with the behaviour of those substances.

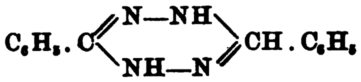
The Dihydrotetrazines. By S. RUHEMANN, M.A., Caius College.

[*Read* 26 February 1906.]

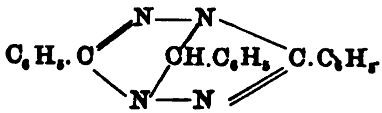
Lately Ruhemann and Merriman (*Camb. Ph. Soc.*, VIII. 209; *Trans. Chem. Soc.*, 1905, LXXXVII. 1768) showed that, in the presence of piperidine, tetrazoline condensed with aldehydes to form compounds which were represented thus:



it was, therefore, assumed that their formation was accompanied by the change of tetrazoline into its isomeride. This view seemed to be supported by the properties of the iodide $C_3H_7N_4I$ which is formed by the action of methyl iodide on tetrazoline, and by the behaviour of the hydrochloride of the base towards platinic chloride. Recently Stollé (*Ber.* 1906, XXXIX. 826) expressed the view that since diphenyltetrazoline,



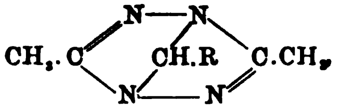
yielded with benzaldehyde a compound which on account of its synthesis had the formula,



an analogous constitution had to be assigned to the products which were produced by the interaction of tetrazoline with aldehydes. In a note which, shortly, will appear in the *Berichte* I have pointed out that such a conclusion is not justified since the behaviour of tetrazoline is markedly different from dimethyltetrazoline. This difference is also manifested by the following facts:

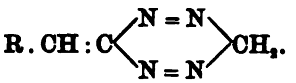
Dimethyltetrazoline reacts with methyl iodide to yield almost exclusively the iodide $C_5H_{11}N_4I$ which may be transformed into the periodide $C_5H_{11}N_4I_3$. This is brown whereas the corresponding compound from tetrazoline is dark blue. Again, the iodide $C_3H_7N_4I$ produced from tetrazoline, in the presence of an alkali, is oxidised by the oxygen of the air to yield a blue solution, but the iodide $C_4H_{11}N_4I$ which is formed from dimethyltetrazoline does not give such a reaction.

Ruhemann and Merriman (*loc. cit.*) have stated that the alcoholic solution of the mixture of dimethyltetrazoline and benzaldehyde does not deposit a solid even on standing with piperidine for many days. I have since found that a condensation-product is indeed formed which is very soluble in alcohol, but the yield is so small that I have not yet examined this compound. More promptly dimethyltetrazoline condenses with salicylic aldehyde, but the compound, which is formed, differs from the one which is produced by the action of salicylic aldehyde on tetrazoline. It has the composition $C_{11}H_{12}ON_4, H_2O$, and therefore possesses one mol. of water of crystallisation which it loses at 100°; this substance, moreover, is sparingly soluble in cold dilute hydrochloric acid, and this solution does not give a precipitate with platinic chloride, whereas *o*-hydroxybenzylidenetetrazoline as well as the other condensation-products of tetrazoline with aldehydes, are readily soluble in hydrochloric acid, and with platinic chloride, form chloroplatinates. These facts therefore lead to the conclusion that, although the condensation-products of dimethyltetrazoline with aldehydes are most probably to be expressed by the formula



the corresponding compounds from tetrazoline have a different constitution which is probably the one suggested by Ruhemann and Merriman.

Bisdiazomethane, like its isomeride tetrazoline, reacts with aldehydes, i.e. *m*-nitrobenzaldehyde or salicylic aldehyde, to yield compounds which, no doubt, are to be represented by the symbol



These substances are hydrolysed readily by boiling water and differ from the condensation-products of tetrazoline with aldehydes inasmuch as they do not yield precipitates with platinic chloride.

A Theory of the Widening of Lines in Spectra. By J. J THOMSON, M.A., F.R.S., Cavendish Professor of Experimental Physics.

[*Read* 12 March 1906.]

The well-known widening which occurs under suitable conditions in the lines of the spectra of many elements is regarded in this paper as due to the effect of resonance between systems which if free from each other's influence would vibrate in the same period. If two systems whose free periods are p_1, p_2 are brought near together so as to exert small forces on each other, the actual periods of the compound system thus formed will in general differ from p_1, p_2, the periods of the constituent systems. If p_1 is not equal to p_2 this difference however will be very small as the changes in the periods will be proportional to the squares of the small forces which the one system exerts on the other. If, however, p_1 and p_2 are equal the changes in the periods will be much larger as they will be proportional to the first power instead of to the square of the forces between the systems. We shall confine our attention to the influence on each other of systems whose free periods are the same; unless this influence is very large that of systems with unequal periods will be quite inappreciable.

We shall assume that the lines in the spectra are due to the vibrations of negatively electrified corpuscles: the forces exerted by one vibrating corpuscle on another are of two kinds, (1) the electrostatic repulsion between the corpuscles, (2) those due to electromagnetic induction between the moving corpuscles. We shall consider these separately, beginning with the electrostatic forces. Consider two corpuscles (1) and (2) vibrating parallel to the axis of x; let x_1, x_2 be the displacements from equilibrium of these corpuscles respectively, m the mass of either corpuscle, μ the force of restitution for unit displacement, then if the corpuscles were vibrating freely and not influencing each other we should have

$$m\frac{d^2x_1}{dt^2}+\mu x_1=0,\quad m\frac{d^2x_2}{dt^2}+\mu x_2=0$$

When we take the electrostatic forces between the two corpuscles into account these equations become

$$\left.\begin{aligned} m\frac{d^2x_1}{dt^2}+\mu x_1&=\frac{e^2}{r^3}(1-3\cos^2\theta)(x_1-x_2)\\ m\frac{d^2x_2}{dt^2}+\mu x_2&=\frac{e^2}{r^3}(1-3\cos^2\theta)(x_2-x_1)\end{aligned}\right\}\text{ ....(1)},$$

where e is the electrostatic measure of the charge on a corpuscle, r the distance between the corpuscles and θ the angle the line joining the corpuscles makes with the axis of x. If p_1 and p_2 are now the times of vibration of the system of two corpuscles, we see from equation (1) that p_1 and p_2 are the roots of the equation

$$(\mu - a - mp^2)^2 = a^2$$

where a is written for

$$\frac{e^2}{r^3}(1 - 3\cos^2\theta);$$

thus
$$\mu - a - mp^2 = \pm a.$$

If p_0 is the undisturbed period $\mu - mp_0^2 = 0$, then

$$p_1^2 = p_0^2,$$
$$p_2^2 = p_0^2 - \frac{2a}{m}.$$

One period is thus the same as the undisturbed period and if δp is the small change in the other period we have

$$\delta p = -\frac{a}{mp_0} = -\frac{e^2}{mp_0r^3}(1 - 3\cos^2\theta).$$

Thus if $3\cos^2\theta$ is less than unity the period will be diminished, i.e. the line will be displaced towards the red end of the spectrum, while if $3\cos^2\theta$ is greater than unity the period will be increased, i.e. the line will be displaced towards the violet end. The greatest negative value of $1 - 3\cos^2\theta$ is -2, the greatest positive value 1, thus the maximum increase in the period is twice the maximum decrease.

Thus if we have a crowd of vibrating corpuscles symmetrically distributed round one vibrating with the same period, the sharp line corresponding to this period will be broadened out into a band, the maximum displacement towards the violet end will be twice that towards the red end, but inasmuch as $\Sigma(1 - 3\cos^2\theta)$ vanishes for a symmetrical distribution, the amount of light in the band on the red side of the original line will be the same as that on the blue side.

We see that the maximum amount of widening varies as $1/r^3$, i.e. is proportional to the density of the synchronously vibrating corpuscles; when the spectrum is produced by the electric discharge the number of these corpuscles will increase with the current density, hence we see the reason for the well-known widening of the lines when the current density is increased. A large number of luminous particles would not give widened lines unless the particles were vibrating synchronously, it is only the density of the synchronous particles that affects the widening and not the density of the whole collection of particles. Thus we might have a great density of luminous particles and yet a small density of the particles giving out some particular line, in this case we should not get a widening of the line: thus for example if we have a small quantity of one gas *A* mixed with a large quantity of another *B*, we might have a large current density through the mixture and yet a small density of the particles giving out the *A* lines—in such a case the lines of *A* would not widen: we see in this way the explanation of the well-known fact that the lines of an impurity do not widen.

Different lines in the same spectrum widen to very different amounts, some become quite broad whilst others hardly widen at all: the difference being far greater than could be explained by the factor $1/p$ in the expression for δp. May not this difference in the behaviour of the lines be due to different lines originating in different systems? We are accustomed to regard all the lines in, say, the iron spectrum as corresponding to possible periods of vibration in the normal atom of iron; it seems to me more probable that many of these lines originate from systems which are not the normal atom, but systems—corpusculates we might call them—formed by the combination of the atom with some of the crowd of corpuscles by which it is surrounded in the electric discharge. When a discharge passes the atoms are surrounded by a crowd of corpuscles and it seems probable that compounds, not permanent indeed, but lasting sufficiently long to give out their characteristic vibrations might be formed between the atom and the corpuscles. These compounds need not be all of the same kind, nor need all kinds be equally numerous, the lines given out by systems which were numerous would widen, while those given out by the scarce systems would not.

The absorption spectrum of the cold vapour would not show the lines due to these temporary compounds of atoms and corpuscles, but these lines might be reversed by vapours through which an electric discharge is passing or which are at such high temperatures that they are good conductors of electricity; in such cases corpuscles are present and could form compounds with the atoms.

The lines most likely to be reversed would be those corresponding to the most prevalent compound, as in the reversing layer the systems able to absorb these lines would be more numerous than the systems able to absorb any other lines; and since in the luminous gas the density of these systems would also be greater than that of the others, these lines would be the most easily widened. Thus we should expect lines easily widened to be easily reversed.

So far we have only considered the effect on the periods of the electrostatic repulsion between the corpuscles, we can easily see that similar effects are produced by the forces due to electromagnetic induction.

The effect of these forces can be represented by introducing a term

$$\frac{e^2}{V^2r}\left(\frac{dx_1}{dt}\frac{dx_2}{dt}+\frac{dy_1}{dt}\frac{dy_2}{dt}+\frac{dz_1}{dt}\frac{dz_2}{dt}\right),$$

in the expression for the kinetic energy of two corpuscles, V being the velocity of light, the rest of the notation being the same as before. The introduction of this term, if we neglect squares and products of the velocities, leads to the equations

$$m\frac{d^2x_1}{dt^2}+\mu x_1=\frac{e^2}{V^2r}\frac{d^2x_2}{dt^2},$$

$$m\frac{d^2x_2}{dt^2}+\mu x_2=\frac{e^2}{V^2r}\frac{d^2x_1}{dt^2}.$$

Hence p_1 and p_2, the two periods, are the roots of the equation

$$(\mu-mp^2)^2=\left(\frac{e^2}{V^2r}\right)^2p^4,$$

or

$$\mu-mp^2=\pm\frac{e^2}{V^2r}p^2,$$

or

$$\delta p=\pm\frac{e^2}{V^2mr}\frac{p}{2}.$$

Thus in the case of two corpuscles the shift due to the electromagnetic induction would be the same on the two sides of the original line; the effect in this case is proportional to p while for the electrostatic repulsion it is proportional to $1/p$. It is also proportional to $1/r$ instead of $1/r^3$ so that though the electromagnetic effect of a single corpuscle is less than the electrostatic effect, the effect of more distant corpuscles will be relatively more important and we have to integrate the effects over a larger volume. The general conclusions drawn from the consideration of the electrostatic effect apply also to the electromagnetic.

On Secondary Röntgen Radiation. By Prof. J. J. THOMSON, M.A., F.R.S.

[*Read* 12 March 1906.]

The greater part of the secondary radiation produced when Röntgen rays pass through a medium is absorbed in the medium itself, the part which emerges and which is all that can be measured is but a small fraction of the whole secondary radiation. We can find the relation between the energy of all the secondary radiation and of that which emerges in the following way. Suppose the absorbing medium is a plate of thickness h bounded by planes at right angles to the axis of x, and let the primary rays be travelling parallel to the axis of x.

Let I_0 be the amount of energy crossing unit area in unit time for the primary waves when they enter the plate, λ the coefficient of absorption of the primary radiation; I the intensity of the primary radiation when it has travelled a distance x in the plate will be equal to $I_0\epsilon^{-\lambda x}$; the amount of energy absorbed in a distance dx is $\lambda I_0\epsilon^{-\lambda x}dx$, let the amount of this energy emitted as secondary radiation be $\kappa\lambda I_0\epsilon^{-\lambda x}dx$. Half of this will travel forward in the direction of the primary radiation and half will travel backward. Let T, R be the amounts of energy in the secondary rays travelling backwards and forwards respectively which cross unit area in unit time at a distance x from the front of the plate, β the coefficient of absorption of the secondary rays; we have

$$dT = \tfrac{1}{2}\kappa\lambda I_0\epsilon^{-\lambda x}dx - \beta T dx,$$

$$-dR = \tfrac{1}{2}\kappa\lambda I_0\epsilon^{-\lambda x}dx - \beta R dx;$$

the solutions of these equations satisfying the conditions that $T=0$ when $x=0$, and $R=0$ when $x=h$, are

$$T=\frac{1}{2}\frac{\kappa\lambda I_0}{\beta-\lambda}(\epsilon^{-\lambda x}-\epsilon^{-\beta x}),$$

$$R=\frac{1}{2}\frac{\kappa\lambda I_0}{\beta+\lambda}(\epsilon^{-\lambda x}-\epsilon^{\beta x}\epsilon^{-(\beta+\lambda)h}).$$

To find the values when the secondary rays emerge from the plate we put $x=h$ in the expression for T and $x=0$ in the expression for R; doing this we find

$$T=\frac{1}{2}\frac{\kappa\lambda I_0}{\beta-\lambda}(\epsilon^{-\lambda h}-\epsilon^{-\beta h}),$$

$$R=\frac{1}{2}\frac{\kappa\lambda I_0}{\beta+\lambda}(1-\epsilon^{-(\beta+\lambda)h}).$$

If the plate is thick enough to make βh large, we have

$$T=\frac{1}{2}\frac{\kappa\lambda I_0}{\beta-\lambda}e^{-\lambda h},$$

$$R=\frac{1}{2}\frac{\kappa\lambda I_0}{\beta+\lambda}.$$

If S is the ionization produced by the secondary rays in a thin layer of gas close to the front layer of the plate $S=R\gamma t$, where t is the thickness of the layer, and γ the coefficient of absorption of the secondary rays in the gas. If P is the ionization produced by the primary rays in the same layer $P=I_0\delta t$, δ being the coefficient of absorption of the primary rays in the gas; hence

$$\frac{S}{P}=\frac{R}{T_0}\frac{\gamma}{\delta}=\frac{1}{2}\frac{\kappa.\lambda}{\beta+\lambda}\cdot\frac{\gamma}{\delta}.$$

As the secondary rays are, in general, much more easily absorbed than the primary, β will be large compared with λ; when this is the case, we have approximately

$$\frac{S}{P}=\frac{1}{2}\frac{\kappa.\lambda}{\beta}\cdot\frac{\gamma}{\delta}.$$

Now λ/δ is the ratio of the coefficients of absorption of the primary rays in the plate and gas respectively, while β/γ is the ratio of these coefficients for the secondary rays, and for Röntgen rays the softer the rays the greater is the absorption of dense media in comparison with gases; hence, as the secondary rays are softer than the primary rays, we should expect β/γ to be greater than λ/δ, and in this case

$$\frac{S}{P}<\tfrac{1}{2}\kappa,$$

or

$$\kappa>2S/P.$$

When the plate is made of a dense substance like lead S is much greater than P. I have lately measured S for a large number of different elements, and found that taking any two elements the one with the greater atomic weight gives the greater value of S, so that the values of S for the different elements are in the same order as their atomic weights. If, however, S is greater than P, κ is greater than unity; this implies that unless the proportion of energy spent in ionization is much larger in the case of the secondary radiation than in that of the primary, more energy is given out as secondary radiation than is supplied by the primary. We may regard the primary radiation as causing an explosion in some of the atoms resulting in the emission of secondary radiation, and suppose that part of the energy of the secondary radiation comes from the liberation of the internal energy of the atoms of the absorbing substance. This is consistent with the result recently obtained by Professor Bumstead, that when equal amounts of energy of primary radiation were absorbed by lead and zinc, the heat produced in the lead was greater than that produced in the zinc; this points to the conclusion that, at any rate in the case of lead, the absorption of primary Röntgen rays is accompanied by a change in some of the atoms.

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PROCEEDINGS

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Cambridge Philosophical Society.

The influence of a very strong magnetic field on the spark spectra of palladium, rhodium and ruthenium. By J. E. PURVIS, M.A., St John's College, Cambridge.

The author has previously given an account* of an investigation on the influence of a magnetic field on the spark spectra of vanadium, platinum and iridium. It is there pointed out that there are lines of vanadium which can be grouped together in having the same general appearance of the constituents of the divided lines, and essentially identical values of $d\lambda/\lambda^2$ for these constituents; and also that there are lines of platinum and iridium which can be classified in a similar manner.

The following account is to describe the results of similar experiments with palladium, rhodium and ruthenium, elements which are usually put in the same group in possessing similar general physical and chemical properties. The same apparatus was used as in the previous experiments. The strength of the magnetic field was 39,980 units; and only those lines were measured whose constituents left well marked images on the photographic plate in an exposure of 30 minutes. The column λ gives the wave length of the undisturbed line; the distances of the constituents of the divided lines (represented in the column $d\lambda$ in the previous paper) are omitted; but the values of $d\lambda/\lambda^2$ calculated from them are given, a + sign signifying constituents of greater wave length, and a − sign those of shorter wave length. As before, the letter *s* means that the line vibrates so that its electric-vector is perpendicular to the lines of force, and *p* that it is parallel thereto. The remark, "the middle constituent removed by calcite" means that the constituents were so close, or so intense, or so diffuse that they could only be seen separated by cutting out those parallel to the lines of force by introducing a calcite prism between the source of light and the quartz lens.

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* *Trans. Camb. Phil. Soc.* Vol. xx, No. viii, p. 193.

An attempt has been made to describe the intensities of the constituents of the divided lines, the lower numbers representing lines of greater, and the higher numbers lines of smaller, intensities, but they should only be read as approximate values.

Palladium.

The lines were identified by comparison with the measurements and descriptions of Exner and Haschek (*Sitz. d. k. Akad. Wien*, 1895, vol. CIV. II$_A$. abth. p. 950).

The following lines are divided into four constituents. Their general appearance is not the same and the differences are indicated by the intensity numbers as well as the values of $d\lambda/\lambda^2$. They may be compared with the constituents of the two rhodium lines described in p. 332.

λ	$d\lambda/\lambda^2$	Intensity	
3460·4	+ 2·19 *s*	4	**Compare with the quadruplet of Rh. 3502·7, p. 332. The constituents appear to be simple multiples of one another.**
	+ 1·05 *p*	2	
	0		
	− 1·05 *p*	2	
	− 2·19 *s*	4	
3258·7	+ 2·24 *s*	3	**Compare with the quadruplet of Rh. 3474·7, p. 332. The constituents appear to be equally intense.**
	+ 0·84 *p*	3	
	0		
	− 0·84 *p*	3	
	− 2·25 *s*	3	

The following lines are divided into triplets. Generally, the middle constituent is the strongest, but this is not always the case, and relative differences in intensities are marked by approximate numbers. Unless otherwise stated, the middle constituent vibrates parallel, and the two outside ones perpendicular, to the lines of force. The distances were measured from the centre of the middle constituent.

4213·0	+ 1·54	2	
	0	1	
	− 1·53	2	
4087·2	+ 1·22	5	**The middle constituent removed by calcite.**
	0	3	
	− 1·22	5	

λ	$d\lambda/\lambda^2$	Intensity	
3958·8	− 1·33	5	
	0	3	
	− 1·34	5	
3894·3	+ 2·10	4	
	0	2	
	− 2·12	4	
3719·2	+ 1·84	5	The constituents were very diffuse.
	0	3	
	− 1·85	5	
3634·8	+ 1·85	2	The 3 components appeared to be equally strong.
	0	2	
	− 1·86	2	
3609·8	+ 1·54	2	do.
	0	2	
	− 1·55	2	
3533·0	+ 1·70	2	do.
	0	2	
	− 1·68	2	
3517·0	+ 1·26	2	The middle constituent removed by calcite. The 3 components are equally strong.
	0	2	
	− 1·26	2	
3489·7	+ 1·63	4	
	0	2	
	− 1·64	4	
3481·1	+ 1·30	3	
	0	2	
	− 1·31	3	
3441·3	+ 1·50	3	
	0	2	
	− 1·53	3	
3433·3	+ 1·68	4	
	0	2	
	− 1·71	4	

λ	$d\lambda/\lambda^2$	Intensity	
3421·2	+ 1·49	2	The 3 components are equally strong.
	0	2	
	– 1·50	2	
3404·4	+ 1·74	1	*The 2 outside components are stronger than the middle one.*
	0	2	
	– 1·70	1	
3372·8	+ 1·98	3	
	0	2	
	– 2·00	3	
3267·1	....	...	Too weak to measure the constituents.
3242·8	+ 1·90	2	The 3 components are equally strong.
	0	2	
	– 1·89	2	
3114·1	+ 1·48	5	
	0	3	
	– 1·50	5	
3027·9	+ 1·53	5	
	0	3	
	– 1·54	5	
2854·5	+ 1·71	3	
	0	2	
	– 1·68	3	
2763·2	....	...	The constituents are too weak to measure.
2658·7	+ 2·20	4	
	0	2	
	– 2·22	4	
2636·0	....	...	The constituents of all these lines were too weak and diffuse to measure.
2628·3	....	...	
2593·3	....	...	
2584·1	....	...	
2569·7	....	...	

λ	$d\lambda/\lambda^2$	Intensity	
2552·0	+ 1·12	3	The middle constituent removed by calcite.
	0	2	
	− 1·12	3	
2551·0	....	...	Too weak to measure.
2534·5	+ 1·35	5	The middle constituent removed by calcite.
	0	3	
	− 1·35	5	
2506·2	+ 1·27	3	do.
	0	2	
	− 1·27	3	
2499·2	+ 1·54	3	
	0	2	
	− 1·57	3	
2496·7	....	...	Too weak to measure.
2489·5	+ 2·20	5	
	0	3	
	− 2·22	5	
2488·9	+ 1·60	3	
	0	2	
	− 1·60	3	
2486·4	+ 1·59	3	
	0	2	
	− 1·60	3	
2472·6	+ 1·16	6	The middle constituent removed by calcite.
	0	4	
	− 1·16	6	
2471·3	+ 1·15	5	do.
	0	3	
	− 1·15	5	
2470·0	+ 2·28	5	
	0	3	
	− 2·31	5	
2469·0	+ 1·86	5	
	0	3	
	− 1·93	5	

λ	$d\lambda/\lambda^2$	Intensity	
2457·4	………	…	Too weak and diffuse to measure.
2446·4	………	…	
2435·4	+ 1·72 0 − 1·72	5 3 5	The middle constituent removed by calcite.
2433·2	+ 1·06 0 − 1·06	5 3 5	do.
2430·7	………	…	Too weak and diffuse to measure.
2419·0	+ 1·47 0 − 1·47	5 3 5	The middle constituent removed by calcite.
2414·7	………	…	Too weak and diffuse to measure.
2406·9	………	…	
2388·4	………	…	
2362·4	+ 2·23 0 − 2·20	4 2 4	
2351·7	+ 1·63 0 − 1·63	4 2 4	The middle constituent removed by calcite.
2296·5	………	…	Too weak to measure.

The following lines become doublets, but they have not the same general appearance. The two constituents of several of them are equally strong in both edges; whilst, of others, the inside edges are sharper and stronger than the outside edges, which appear to be more diffuse. It is probable that other constituents overlap—but they are too weak and diffuse to separate distinctly by the calcite. The measurements were taken between the centres of these aggregates. The values of $d\lambda/\lambda^2$ for the constituents of 3690·3 and 2372·2 appear to be identical with those of the strong constituents of the quadruplet from 3460·4 (p. 326).

λ	$d\lambda/\lambda^2$	Intensity	
4473·4	.. ..	...	The constituents are too weak to measure; the outer edges of the constituents appear to be more diffuse and weaker than the inside edges.
3832·6	+ 1·31 0 − 1·31	3 3	The 2 constituents appear to be equally strong on both sides.
3690·3	+ 1·04 0 − 1·04	2 2	The 2 constituents are very strong and diffuse. Compare the quadruplet of 3460·4 (p. 326).
3382·6	....	...	Too weak to measure.
3301·9	+ 0·72 0 − 0·72	2 2	The constituents of both these lines are strong and diffuse.
3251·4	+ 0·65 0 − 0·65	3 3	
3059·4	....	...	Too weak to measure.
3032·2	....	...	
3002·6	+ 1·24 0 − 1·24	4 4	The outside edges of the constituents appear to be more diffuse than the inside edges.
2980·7	....	...	Too weak to measure.
2372·2	+ 1·05 0 − 1·05	3 3	Compare with 3690·3 and the quadruplet from 3460·4 (p. 326).

The lines 3571·1, 2424·5, 2357·7, are neither divided nor widened.

There are many other lines which are widened or divided, but they are too weak for analysis. Only those lines are described which distinctly affect the photographic plate during an exposure of 30 minutes.

Rhodium.

The lines were identified by comparison with the measurements and descriptions of Exner and Haschek (*Sitz. d. k. Akad. Wien*, 1895, vol. CIV. II$_A$. abth. p. 960).

The following lines are divided into four constituents. Their general appearance is not the same, as is indicated by the differences in the intensity numbers, as well as by the values of $d\lambda/\lambda^2$.

λ	$d\lambda/\lambda^2$	Intensity	
3502·7	$+2{\cdot}08\ s$	4	Compare with the quadruplet from Pd. 3460·4, p. 326.
	$+0{\cdot}95\ p$	2	
	0		
	$-0{\cdot}95\ p$	2	
	$-2{\cdot}08\ s$	4	
3474·7	$+2{\cdot}21\ s$	3	Compare with the quadruplet from Pd. 3258·7, p. 326.
	$+0{\cdot}82\ p$	3	
	0		
	$-0{\cdot}82\ p$	3	
	$-2{\cdot}22\ s$	3	
3461·9	$+1{\cdot}02\ s$	6	
	$+0{\cdot}87\ p$	3	
	0		
	$-0{\cdot}87\ p$	3	
	$-1{\cdot}02\ s$	6	
3280·4	$+1{\cdot}86\ s$	3	The constituents appear to be simple multiples of one another.
	$+0{\cdot}62\ p$	3	
	0		
	$-0{\cdot}62\ p$	3	
	$-1{\cdot}85\ s$	3	

The following lines are divided into triplets. The middle constituent is generally the strongest, and relative differences are noted by approximate numbers, as well as by different appearances in the diffuseness. The middle constituent vibrates parallel, and the two outside ones perpendicular, to the lines of force. The distances of the outside constituents were measured from the centre of the middle one.

λ	$d\lambda/\lambda^2$	Intensity	
4528·9	…….	…	Too weak to measure.
4374·9	+ 1·76	3	
	0	2	
	− 1·77	3	
4288·9	+ 1·70	5	
	0	3	
	− 1·73	5	
4211·3	+ 2·14	3	
	0	2	
	− 2·12	3	
4154·6	+ 1·85	6	
	0	4	
	− 1·84	6	
4135·4	+ 1·71	5	
	0	3	
	− 1·73	5	
4129·1	+ 1·92	5	
	0	3	
	− 1·90	5	
4121·9	+ 1·61	5	The constituents are very diffuse.
	0	3	
	− 1·59	5	
4083·0	+ 1·95	5	do.
	0	3	
	− 1·90	5	
3984·7	+ 1·89	6	
	0	4	
	− 1·86	6	
3975·6	+ 1·96	6	
	0	4	
	− 1·94	6	

λ	$d\lambda/\lambda^2$	Intensity	
3959·1	+ 1·57	3	
	0	2	
	− 1·60	3	
3942·8	+ 1·95	4	
	0	3	
	− 1·94	4	
3936·1	+ 1·91	4	
	0	3	
	− 1·96	4	
3877·6	....	...	The constituents are too weak to measure.
3870·4	+ 2·07	5	The 3 constituents are broad and diffuse like those from 3815·2.
	0	4	
	− 2·10	5	
3856·7	+ 1·78	2	The 3 constituents appear to be almost equally strong. Compare 3658·1.
	0	2	
	− 1·75	2	
3834·1	+ 1·38	5	The 3 constituents are diffuse.
	0	3	
	− 1·37	5	
3828·7	+ 2·10	3	The 3 constituents are sharp and similar to those of 3818·4.
	0	2	
	− 2·12	3	
3822·4	+ 1·40	3	The 3 constituents are diffuse.
	0	2	
	− 1·37	3	
3818·4	+ 2·12	3	
	0	2	
	− 2·14	3	
3816·7	+ 1·69	5	The 3 constituents are very diffuse.
	0	3	
	− 1·65	5	
3815·2	+ 2·12	5	The constituents are broad and diffuse, and impinge upon each other. Only by removing the middle constituent by calcite was it possible to measure the separation. Compare 3870·4.
	0	4	
	− 2·12	5	

λ	$d\lambda/\lambda^2$	Intensity	
3806·1	+ 1·66	5	
	0	3	
	– 1·69	5	
3793·4	+ 1·92	3	
	0	2	
	– 1·91	3	
3788·6	+ 1·91	5	
	0	3	
	– 1·88	5	
3778·3	+ 1·42	6	The middle constituent removed by calcite.
	0	4	
	– 1·42	6	
3770·1	+ 1·74	5	
	0	3	
	– 1·76	5	
3765·2	+ 2·04	3	
	0	2	
	– 2·01	3	
3754·4	+ 2·31	5	
	0	3	
	– 2·30	5	
3744·3	+ 2·00	6	
	0	4	
	– 2·03	6	
3737·5	+ 2·10	6	
	0	4	
	– 2·10	6	
3735·5	+ 1·74	3	
	0	2	
	– 1·76	3	
3702·1	+ 1·66	3	The middle constituent is very slightly stronger than the two outside ones.
	0	3	
	– 1·64	3	
3699·7	+ 1·60	6	
	0	4	
	– 1·57	6	

λ	$d\lambda/\lambda^2$	Intensity	
3695·7	+ 1·62	5	Compare 3690·9.
	0	3	
	− 1·61	5	
3690·9	+ 1·61	5	
	0	3	
	− 1·62	5	
3681·3	+ 2·15	6	
	0	4	
	− 2·14	6	
3666·4	+ 1·81	6	
	0	4	
	− 1·79	6	
3662·1	....	...	The constituents are too weak and diffuse to measure.
3658·1	+ 1·65	2	The 3 components appear to be almost equally strong. Compare 3396·7.
	0	2	
	− 1·70	2	
3626·8	+ 1·80	5	
	0	3	
	− 1·77	5	
3615·2	....	...	The constituents are too weak to measure.
3597·4	+ 1·27	5	
	0	3	
	− 1·25	5	
3596·4	+ 1·47	5	
	0	3	
	− 1·48	5	
3583·3	+ 1·85	5	
	0	3	
	− 1·82	5	
3538·3	+ 2·10	5	
	0	3	
	− 2·10	5	

λ	$d\lambda/\lambda^2$	Intensity	
3528·1	+ 1·77	3	
	0	2	
	− 1·75	3	
3507·4	+ 1·36	4	
	0	2	
	− 1·35	4	
3498·9	....	...	The constituents are too weak to measure.
3479·0	+ 1·47	5	
	0	3	
	− 1·49	5	
3477·9	....	...	do.
3470·5	+ 0·51	3	The middle number removed by calcite.
	0	2	
	− 0·51	3	
3440·4	+ 1·29	6	
	0	4	
	− 1·29	6	
3434·3	+ 1·73	3	The middle constituent is *not so strong* as the two outside ones. (Compare Pd. 3404·4, p. 328.)
	0	4	
	− 1·71	3	
3412·2	+ 1·63	6	
	0	4	
	− 1·64	6	
3399·7	+ 1·53	6	
	0	4	
	− 1·54	6	
3396·7	+ 1·76	2	The 3 constituents appear to be equally strong. Compare 3658·1.
	0	2	
	− 1·78	2	
Ir.? 3323·0	+ 1·53	3	The middle constituent is only a shade stronger than the two outside.
	0	2	
	− 1·53	3	

λ	$d\lambda/\lambda^2$	Intensity	
	+ 1·15	5	
3283·5	0	3	
	− 1·16	5	
	+ 1·64	6	
3271·5	0	4	
	− 1·65	6	
3267·5	………	…	The constituents are too weak to measure.
	+ 1·44	6	
3263·0	0	4	
	− 1·42	6	
3191·0	………	…	do.
3187·8	………	…	The 3 constituents are very broad and widen into each other. They are too weak to measure on separation by the calcite.
	+ 2·50	5	
3159·2	0	3	
	− 2·48	5	
3141·0	………	…	The constituents are too weak to measure.
	+ 1·52	6	
2988·9	0	4	
	− 1·48	6	
	+ 1·80	5	
2963·4	0	3	
	− 1·79	5	
2926·4	………	…	do.
	+ 2·01	6	
2923·8	0	4	
	− 2·01	6	
	+ 1·82	3	
2910·1	0	2	
	− 1·83	3	
	+ 2·84	5	
2845·5	0	3	
	− 2·85	5	
2802·1	………	…	do.

λ	$d\lambda/\lambda^2$	Intensity	
2781·6	+ 2·21	6	
	0	4	
	– 2·21	6	
2778·1	+ 1·65	5	The middle constituent removed by calcite.
	0	3	
	– 1·65	5	
2775·6	+ 1·64	6	
	0	4	
	– 1·66	6	
2766·4	....	...	The constituents are too weak to measure.
2739·8	+ 2·39	5	
	0	3	
	– 2·41	5	
2737·2	+ 1·42	3	
	0	2	
	– 1·42	3	
2728·9	+ 2·31	6	
	0	4	
	– 2·31	6	
2715·2	+ 1·76	3	
	0	2	
	– 1·75	3	
2705·5	+ 1·75	3	
	0	2	
	– 1·76	3	
2683·8	– 1·70	5	
	0	3	
	– 1·67	5	
2663·6	+ 2·19	5	
	0	3	
	– 2·19	5	
2659·1	....	...	The constituents are too weak and diffuse to measure.
2651·8	+ 1·78	5	
	0	3	
	– 1·77	5	

λ	$d\lambda/\lambda^2$	Intensity	
2639·0	....	...	The constituents are weak and widen into $\lambda\lambda$ 2638·8, 2639·3, lines not marked by Exner and Haschek as belonging to Rhodium.
2635·1	+ 1·69	5	
	0	3	
	− 1·69	5	
2630·3	+ 2·04	6	
	0	4	
	− 2·07	6	
2628·1	+ 1·55	5	
	0	3	
	− 1·60	5	
2625·3	+ 1·97	5	
	0	3	
	− 1·95	5	
2609·1	+ 1·76	5	
	0	3	
	− 1·76	5	
2596·8	+ 1·65	5	
	0	3	
	− 1·70	5	
2591·9	+ 2·25	6	
	0	4	
	− 2·23	6	
2587·1	....	...	The constituents are too weak to measure.
2581·4	+ 1·83	5	
	0	3	
	− 1·83	5	
2568·8	+ 2·50	5	
	0	3	
	− 2·47	5	
2559·8	....	...	do.
2557·1	+ 1·79	5	
	0	3	
	− 1·79	5	

λ	$d\lambda/\lambda^2$	Intensity	
	+ 2·32	5	
2545·3	0	3	
	– 2·32	5	
	+ 1·98	3	
2520·6	0	2	
	– 1·95	3	
2517·7	....	...	The constituents are too weak to measure.
	+ 1·95	3	
2510·7	0	2	
	– 1·96	3	
	+ 1·94	5	
2505·1	0	3	
	– 1·92	5	
	+ 2·09	5	
2503·8	0	3	
	– 2·08	5	
	+ 1·69	5	
2491·8	0	3	
	– 1·69	5	
	+ 1·86	3	
2490·8	0	2	
	– 1·88	3	
	+ 1·71	5	
2475·6	0	3	
	– 1·70	5	
	+ 1·93	3	
2461·1	0	2	
	– 1·96	3	
	+ 1·87	3	
2459·0	0	2	
	– 1·84	3	
	+ 2·02	5	
2456·0	0	3	
	– 2·02	5	
2429·8	....	...	The constituents are too weak and diffuse to measure.

λ	$d\lambda/\lambda^2$	Intensity	
2427·1	+ 1·31	3	The middle constituent removed by calcite.
	0	2	
	− 1·31	3	
2421·0	+ 1·37	3	do.
	0	2	
	− 1·37	3	
2415·8	+ 1·27	3	do.
	0	2	
	− 1·27	3	
2396·6	+ 1·62	5	do.
	0	3	
	− 1·62	5	
2392·3	....	...	The constituents are too weak to measure.
2366·8	+ 1·71	5	The middle constituent removed by calcite.
	0	3	
	− 1·71	5	
2346·6	+ 1·46	5	do.
	0	3	
	− 1·46	5	
2334·8	+ 1·95	3	
	0	2	
	− 1·95	3	
2327·6	....	...	The constituents are too weak to measure.

The following lines become doublets, but they have not the same appearance. The constituents of several of them seem to be equally sharp on both edges; whilst the inside edges of others are sharper and stronger than the outside edges. There are probably other constituents overlapping, but they are too weak and diffuse to measure on separation by the calcite. The measurements were taken between the centres of these aggregates. The values of $d\lambda/\lambda^2$ for the constituents of 3748·5 are comparable with those of the strong constituents of the quadruplet of 3280·4; and those of 2819·1 with the strong constituents of the quadruplet of 3461·9.

λ	$d\lambda/\lambda^2$	Intensity	
3934·4	+ 1·20	3	
	0		
	− 1·20	3	
3806·8	+ 1·36	3	
	0		
	− 1·36	3	
3748·5	+ 0·62	3	The constituents are very narrow and strong. Compare the quadruplet of 3280·4 (p. 332).
	0		
	− 0·62	3	
3713·1	+ 1·31	3	The 2 constituents are very diffuse.
	0		
	− 1·31	3	
3655·2	....	...	Too weak to measure.
3606·1	+ 1·12	4	The constituents appear to be more diffuse on the outer edges than on the inner edges, but they are too weak to analyse.
	0		
	− 1·12	4	
3549·6	+ 1·14	3	
	0		
	− 1·14	3	
3372·0	....	...	All too weak and diffuse to measure.
3207·2	....	...	
3047·2	....	...	
3009·3	....	...	
2819·1	+ 0·88	3	Compare the quadruplet of 3461·9 (p. 332).
	0		
	− 0·88	3	
2674·3	+ 1·13	3	
	0		
	− 1·13	3	

The lines 3612·7, 2635·1, 2537·6, are not affected when vibrating in the magnetic field: they are neither divided nor widened.

There are many lines which are widened, but they are too weak for analysis; and only those lines are described which distinctly affect the photographic plate in the time exposure of these experiments.

Ruthenium.

The lines were identified by comparison with the measurements and descriptions of Exner and Haschek (*Sits. d. k. Akad. Wien* 1896, vol. cv. II$_A$. abth. p. 707.

The following lines are divided into triplets. The middle constituent is generally the strongest; and relative differences of intensities are marked by approximate numbers as well as by different appearances in the diffuseness. The middle constituent vibrates parallel, and the two outside ones perpendicular, to the lines of force. The distances of the two outside constituents were measured from the centre of the middle one.

λ	$d\lambda/\lambda^2$	Intensity	
4710·0	+ 1·79	5	
	0	3	
	− 1·78	5	
4647·9	+ 2·45	5	
	0	3	
	− 2·45	5	
4599·4	....	...	Too weak to measure.
4554·9	+ 1·52	2	
	0	1	
	− 1·54	2	
4410·4	....	...	Too weak and diffuse to measure.
4307·9	....	...	
4298·1	+ 1·46	3	
	0	2	
	− 1·45	3	
4247·1	+ 1·40	5	
	0	3	
	− 1·39	5	
4243·3	+ 2·54	5	Compare 4214·8.
	0	3	
	− 2·53	5	
4241·4	....	...	Too weak to measure.

λ	$d\lambda/\lambda^2$	Intensity	
4214·8	+ 2·57	5	Compare 4243·3.
	0	3	
	– 2·57	5	
4212·5	+ 1·89	2	Compare 4200·3.
	0	1	
	– 1·88	2	
4200·3	+ 1·92	2	
	0	1	
	– 1·93	2	
4168·0	………	…	Too weak and diffuse to measure.
4146·1	………	…	
4144·5	+ 2·01	3	
	0	2	
	– 2·01	3	
4113·1	+ 1·88	5	
	0	3	
	– 1·89	5	
4098·2	………	…	do.
4081·0	+ 1·92	3	
	0	2	
	– 1·93	3	
4051·8	+ 2·05	5	
	0	3	
	– 2·05	5	
3945·7	+ 1·48	5	The middle constituent removed by calcite.
	0	3	
	– 1·48	5	
3923·7	+ 1·77	5	
	0	3	
	– 1·77	5	
3909·3	………	…	Too weak to measure.
3868·0	+ 2·10	5	
	0	3	
	– 2·13	5	

λ	$d\lambda/\lambda^2$	Intensity	
	+ 1·53	5	
3862·8	0	3	
	− 1·53	5	
	+ 2·30	3	
3857·6	0	2	
	− 2·30	3	
	+ 2·57	6	
3850·6	0	4	
	− 2·58	6	
	+ 1·97	5	
3839·8	0	3	
	− 1·95	5	
3822·2	………	…	Too weak to measure.
	+ 1·65	5	
3817·4	0	3	
	− 1·67	5	
3812·7	………	…	do.
	+ 1·75	3	The most refrangible constituent of ? Rh. 3799·5 widens into the least refrangible one of 3799·0.
? Rh. 3799·5	0	2	
	− 1·75 ?	?	
	+ 1·77 ?	?	
3799·0	0	2	
	− 1·77	3	
	+ 1·67	3	
3790·6	0	2	
	− 1·66	3	
	+ 1·52	3	
3786·2	0	2	
	− 1·53	3	
3781·3	………	…	Too weak to measure.
3756·0	………	…	
	+ 1·64	3	
3745·7	0	2	
	− 1·62	3	

λ	$d\lambda/\lambda^2$	Intensity	
3743·0	+ 1·71	5	The most refrangible constituent widens into the doublet of 3742·8.
	0	3	
	?	?	
3730·6	+ 1·89	2	
	0	1	
	– 1·87	2	
3728·3	+ 2·00	2	
	0	1	
	– 2·03	2	
3727·2	+ 1·87	2	
	0	1	
	– 1·90	2	
3669·7	+ 1·74	5	
	0	3	
	– 1·72	5	
3663·5	………	…	Too weak to measure.
3661·6	+ 1·28	3	The middle constituent removed by calcite.
	0	2	
	– 1·28	3	
3635·1	+ 2·23	3	
	0	2	
	– 2·20	3	
3600·0	+ 1·71	5	
	0	3	
	– 1·74	5	
3596·4	+ 1·89	3	
	0	2	
	– 1·92	3	
3593·2	+ 1·24	2	do.
	0	1	
	– 1·24	2	
3570·7	………	…	Too weak and diffuse to measure.
3499·1	+ 1·87	2	
	0	1	
	– 1·80	2	

λ	$d\lambda/\lambda^2$	Intensity	
	+ 1·56	3	
3437·0	0	2	
	− 1·53	3	
	+ 2·31	3	
3428·5	0	2	
	− 2·37	3	
	+ 1·25	3	
3417·4	0	2	
	− 1·25	3	
	+ 1·89	5	The most refrangible constituent of 3339·8 widens into the least refrangible one of 3339·6.
3339·8	0	3	
	?	?	
	?	?	
3339·6	0	3	
	− 2·16	5	
3294·2	....	...	Too weak to measure.
	+ 1·73	5	
3228·5	0	3	
	− 1·76	5	
	+ 1·97	5	
3177·0	0	3	
	− 2·00	5	
3093·8	....	...	The constituents are all too weak to measure.
3056·7	....	...	
3036·4	....	...	
	+ 1·14	3	
2991·5	0	2	
	− 1·14	3	
2988·8	....	...	Too weak to measure.
	+ 1·62	5	The most refrangible constituent of 2979·9 widens into the least refrangible one of 2979·6.
2979·9	0	3	
	?	?	
	?	?	
2979·6	0	3	
	− 1·65	5	

λ	$d\lambda/\lambda^2$	Intensity	
2978·5	+ 2·03	5	
	0	3	
	− 2·05	5	
2977·4	+ 1·85	6	The most refrangible constituent widens into 2977·2.
	0	4	
	?	?	
2977·2	+ 0·73	5	The middle constituent removed by calcite.
	0	3	
	− 0·73	5	
2976·5	+ 1·80	2	
	0	1	
	− 1·82	2	
2965·6	+ 1·97	2	
	0	1	
	− 1·99	2	
2945·6	+ 2·16	2	
	0	1	
	− 2·18	2	
2927·6	+ 2·12	3	
	0	2	
	− 2·09	3	
2916·4	....	...	Too weak to measure.
2882·4	+ 2·24	3	
	0	2	
	− 2·24	3	
2841·9	....	...	Too weak and diffuse to measure.
2813·5	....	...	
2810·9	+ 1·67	3	The middle constituent removed by calcite.
	0	2	
	− 1·67	3	
2788·0	+ 2·39	3	
	0	2	
	− 2·39	3	
2778·6	+ 2·13	3	
	0	2	
	− 2·09	3	

λ	$d\lambda/\lambda^2$	Intensity	
2752·9	+ 1·98	5	
	0	3	
	– 1·98	5	
2744·1	+ 2·49	3	
	0	2	
	– 2·51	3	
2734·5	+ 2·19	3	
	0	2	
	– 2·22	3	
2717·6	+ 1·95	5	The middle constituent removed by calcite.
	0	3	
	– 1·95	5	
2712·5	+ 2·28	3	
	0	2	
	– 2·26	3	
2692·3	+ 2·25	3	
	0	2	
	– 2·21	3	
2687·7	………	…	Too weak and too diffuse to measure.
2679·0	+ 2·16	2	
	0	1	
	– 2·20	2	
2662·0	+ 2·26	3	
	0	2	
	– 2·29	3	
2661·5	+ 1·81	5	
	0	3	
	– 1·78	5	
2656·5	………	…	Too weak and diffuse to measure.
2643·1	………	…	
2617·2	………	…	
2591·5	………	…	
2543·5	+ 1·38	3	The middle constituent removed by calcite.
	0	2	
	– 1·38	3	

λ	$d\lambda/\lambda^2$	Intensity	
2507·1	....	...	Too weak and diffuse to measure.
2493·8	....	...	
2481·3	....	...	
2455·7	+ 1·76	5	
	0	3	
	− 1·76	5	
2403·0	+ 1·80	3	The middle constituent removed by calcite.
	0	2	
	− 1·80	3	
2358·2	....	...	Too weak and diffuse to measure.

The following lines become doublets, but they have not the same appearance. The constituents of several of them seem to be equally sharp on both sides; whilst the inside edges of others are sharper and stronger than the outside edges. Probably there are other overlapping constituents, but they are too weak for analysis, as in the case of palladium and rhodium. The measurements were taken between the centres of these aggregates.

4584·7	+ 1·09	2	Compare λ 4372·7.
	0		
	− 1·09	2	
4460·4	....	...	Too weak and diffuse to measure.
4372·7	+ 1·00	2	The outside edge of each constituent is more diffuse and weaker than the inside edge. It is too weak to analyse.
	0		
	− 1·00	2	
4217·6	+ 1·41	3	do.
	0		
	− 1·41	3	
4206·5	+ 1·37	2	do.
	0		
	− 1·37	2	
3926·1	+ 0·79	3	The constituents seem to be equally sharp in both edges.
	0		
	− 0·79	3	

λ	$d\lambda/\lambda^2$	Intensity	
	+ 1·41	3	Compare 3514·6.
3831·8	0		
	− 1·41	3	
	+ 1·14	3	
3742·8	0		
	− 1·14	3	
	+ 1·32	3	The 2 constituents are very diffuse.
3539·4	0		
	− 1·32	3	
	+ 1·42	3	
3514·6	0		
	− 1·42	3	
2822·7	....	...	Too weak to measure.
2807·0	....	...	
	+ 1·24	3	The edges of each constituent seem to be equally sharp.
2769·2	0		
	− 1·24	3	
	+ 1·21	3	do.
2725·6	0		
	− 1·21	3	
	+ 2·31	3	
2513·4	0		
	− 2·31	3	
	+ 1·33	3	The edges of the constituents are very diffuse.
2479·1	0		
	− 1·33	3	

The lines 3777·7 and 3589·4 are neither separated nor widened when vibrating in the magnetic field.

There are many other lines widened or divided, but they are too weak for analysis. Only those lines are noted which clearly affect the photographic plate in 30 minutes exposure.

General Results.

The more important results to which attention may be drawn in the preceding observations of the spectra of palladium, rhodium and ruthenium are as follows:

(1) If the lines of each separate metal are compared with one another, it will be noticed that some of them can be grouped together as having the same general appearance of the constituents, the same polarisation and essentially identical values of $d\lambda/\lambda^2$: and this agreement is well seen amongst the lines which become triplets. Amongst the lines which become doublets, this comparison cannot be so rigid, because, as already stated, there are probably other overlapping constituents. But they may be compared with each other in the same way and also with the stronger constituents of the quadruplets.

(2) If the lines of the three metals are compared with one another, it will be noticed that there are some which have the same appearance, the same intensity, the same polarisation and the same value of $d\lambda/\lambda^2$. The most prominent examples are the quadruplets from palladium 3460·4 and rhodium 3502·7, and also the quadruplets from palladium 3258·7 and rhodium 3474·7. Amongst the lines which become triplets and doublets there are some which can be grouped together in a similar way.

(3) The displacements of the constituents of some of the divided lines are simple multiples of one another. Several instances are noticeable amongst the quadruplets of palladium and rhodium. And in some instances, the value of $d\lambda/\lambda^2$ for the constituents of some lines is a multiple of the value for the constituents of other lines.

The results of other metals will be published later.

I have again to thank Professor Liveing for the use of his spectroscope, and Professor Larmor for his interest in the work.

Experiments on the band-spectrum of nitrogen in a strong magnetic field.

By J. E. PURVIS, M.A., St John's College, Cambridge.

[*Read* 14 May 1906.]

Experiments have previously been recorded by Becquerel and Deslandres (*Comptes rendus*, vol. 127, 1898, p. 20) on the band-spectrum of nitrogen when vibrating in a magnetic field, and they shewed that there was neither appreciable division nor widening (ni dédoublement ni élargissement appréciables) of the bands when vibrating under the influence of the electro-magnetic field utilised by them. The strength of this field with a current of 9–10 ampères was 20,000 units.

The author has repeated the experiments when the discharge took place in a field of 41,000 units. The tubes containing the nitrogen were specially made with the capillary parts of 3·5 mm. external diameter, so that they could be introduced between the poles of a magnet 4 mm. apart. With a current of 25 ampères, a field strength of 41,000 units was obtained between these poles. The nitrogen was obtained from barium nitride kindly given to me by Dr Scott, and was dried and purified before entering the tubes. The parts of the capillary not within the sphere of influence of the magnetic field were screened off, and the light from the unscreened part was focussed on the slit of the spectroscope by a single lens. The discharge was obtained by means of a coil giving a 6-inch spark and without a jar. In the first experiments a plane grating was used of 14,438 lines to the inch, and the various bands in the third and fourth orders of the spectrum were respectively focussed on the photographic plate, so that each band could be investigated separately. In the later experiments, Professor Liveing's large 21-foot concave grating was used, whereby the bands could be photographed at the same time on the same plate, and the 1st and part of the 2nd orders observed. The second order ranged from about λ 3500, so that the bands in the ultra-violet could not be photographed in consequence of the glass tubes absorbing these vibrations.

Eye observations were made in the earlier experiments with the plane grating, and photographs in the later ones. But the range of the latter is obviously limited in consequence of the absorption by the glass of the rays of greatest refrangibility. As is well known, the colour of the discharge in nitrogen when vibrating in a magnetic field changes from a buff or pink to a bright steely blue colour. At the same time the spectral bands in the red, orange and yellow almost disappear, whilst those in the green, blue and violet become brighter and stronger, and this is especially noticeable in the bands in the blue parts of the spectrum. But there did not appear to be any shift of the bands either relatively to each other or to their respective previous positions. Nor did there appear to be any widening or division of the separate lines forming the bands. Particular attention was devoted to the three strong lines which form the "heads" of the bands, and which are much easier to observe in consequence of their great intensity as compared with the other lines. But no widening nor separation could be seen in any of them. The apparent widening may be explained by the increased intensification in the luminosity of the separate constituents of the bands in the green, blue and violet regions, when the discharge is taking place in the magnetic field.

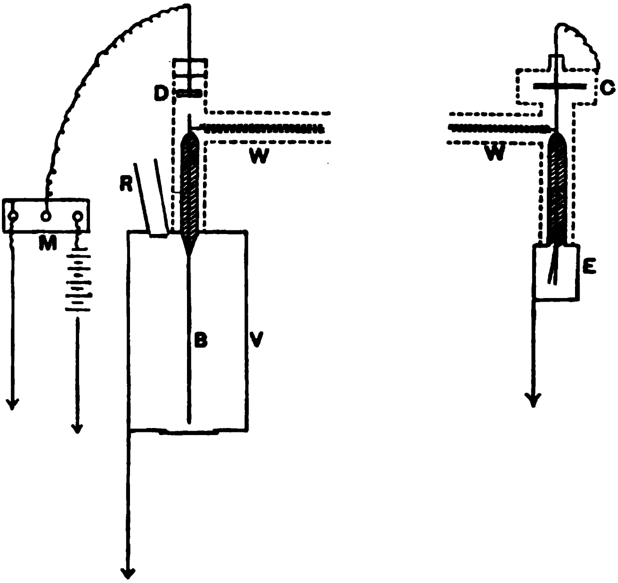
These experiments, therefore, confirm the previous observations. And whatever be the origin of the vibrations which produce the nitrogen bands, they seem to prove that these vibrations are different from those produced in the spark spectra of elements, whose lines, in a magnetic field of the same strength, are not only widened but are separated into several constituents.

On the ionization of gases exposed simultaneously to Röntgen rays and the radiation from radio-active substances.

By T. NODA, Tokio Higher Normal School, Japan. Communicated by Prof. J. J. THOMSON.

[*Read* 14 May 1906.]

§ 1. The following experiments were undertaken to see whether the ionization due to X-rays and different Becquerel rays acting simultaneously is equal to the sum of the ionizations due to the same rays acting separately.



The apparatus consists of an ionization vessel *V* and a small gold-leaf electrometer *E*, with the connecting wire *W* between them.

The ionization vessel is a vertical brass cylinder 12 cm. in diameter and 19 cm. in length. A brass rod 2 mm. thick and 31 cm. long passes through a small hole in the centre of the top of the cylinder. It is fixed in the cylinder and insulated from it by a plug of sulphur.

The bottom of the cylinder has a large hole 6 cm. in diameter, which is covered with a thin leaf of aluminium.

The electrometer is furnished with a small adjustable condenser *C*, consisting of two circular discs 5 cm. in diameter coated with sealing-wax. Both the cylinder and the box of the electrometer are earthed, while all the other parts including the connecting wire are enclosed in brass tubes which are also earthed, so that they are protected from any electrical disturbances. Moreover, the connecting wire is enclosed in a small glass tube to protect it against those rays which may enter the protecting tubes. The charging and discharging of the brass rod are effected by pushing a small disc *D*, which can be connected by means of a commutator *M* with the negative pole of a battery or with the earth at will, against the top of the rod, whose potential is indicated by the divergence of the gold-leaf of the electrometer, which is read by a microscope with a micrometer eye-piece. The leakage of the apparatus itself (that is, the ionization vessel, the electrometer and the connecting wire) when no ionizing agent is at work, is very small; it required about 40 minutes to decrease the divergence of the electrometer by one division, so that it can be entirely neglected in the following experiments, which treat of the potential fall corresponding to 20 divisions in 1 or 2 minutes.

§ 2. The amount of ionization due to rays is measured by the leakage thereby produced from the brass rod to the wall of the cylinder. If the current between the rod and the cylinder wall due to the ionization is always in the state of saturation, then its strength is independent of the potential difference between them, and the electricity which escapes, and therefore the fall of potential of the apparatus, is jointly proportional to the intensity of ionization and the time required. Consequently if we apply two ionizing agents separately and produce the same fall of potential of the rod every time, then the strength of ionization of the two agents is inversely proportional to the time required for it.

In order to see how strong the ionization can be while still maintaining the state of saturation, I calibrated the electrometer (with the ionization vessel and the connecting wire) with a Weston's voltmeter.

The following is the result.

EXPERIMENT 1.

Potential in volts	Divergence of electrometer in scale divisions
39·0	11·0
59·0	20·8
78·0	32·2
99·0	47·0
116·0	60·0
138·0	80·0

Therefore the fall of potential corresponding to the decrease of divergence from 80 to 47 of the scale is equal to that corresponding to the decrease from 47 to 21·4.

EXPERIMENTS 2, 3, 4.

	Time in sec. required by the electrometer in falling in potential		Ratio
	from 80 to 47	from 47 to 21·4	
Exp. 2.	19·0	22·0	1·16
	19·2	21·6	1·13
	19·6	22·9	1·17
	18·3	20·6	1·13
	17·8	20·5	1·15
			Mean value 1·15
Exp. 3.	31·5	32·7	1·04
	29·8	31·0	1·04
	30·0	31·5	1·05
	29·6	31·6	1·07
			Mean value 1·05
Exp. 4.	118·5	116·8	·986
	117·3	112·7	·961
	111·4	114·6	1·029
	110·0	114·0	1·036
			Mean value 1·003

A radium specimen was put in the lead tube *R* and various intensities of ionization were obtained by putting a number of thin aluminium plates at the bottom of the tube. The time required to reduce the divergence from 80 to 47 was compared with that required from 47 to 21·4.

The procedure is as follows: the disc was connected with the battery, pushed against the top of the discharging rod, drawn back a few centimetres and earthed, and then the transits of the gold-leaf across the micrometer scale divisions 80, 47, and 21·4 were observed by a stop-watch. The results are given on the opposite page.

Assuming the constancy of the intensity of the radiation from the radium we see that the current was saturated only in the last experiment, so that it was necessary to work always with about that or a smaller intensity of ionization. In reality I used only those intensities of ionization which produce the fall of potential from 65 to 45 of the scale in about one minute or a longer time.

§ 3. Firstly, I tried the combination of X-rays and uranium-rays. As before the radium was put in the lead tube *R* with a proper number of thin aluminium plates at the bottom. An X-ray tube was placed below the ionization vessel and enclosed in a thick-walled lead box, which has a hole 8 cm. in diameter just above the tube.

The hole is guarded by a lead cylinder through which the rays pass from the tube, they also pass through the aluminium leaf at the base of the ionization vessel.

For the purpose of preventing the ray which passed through the ionization vessel from attacking the conducting wire two large thick lead plates were placed on the vessel, so that the leakage from parts of the apparatus other than that of the discharging rod in the vessel may be taken as small.

It may here be remarked that the complete absence of this leakage, though desirable, is not absolutely necessary for these experiments, so long as we keep the conditions of ionization the same throughout one set of experiments.

Since the variation of the intensity of X-rays during one set of experiments was unavoidable, the leakage due to X-rays was measured before and after the measurement of the leakage under the combined effect of X-rays and radium-rays, and the mean of them taken. The results are as follows.

EXPERIMENT 5.

Time in sec. required by radium-rays	Time in sec. required by X-rays	Time in sec. required by radium-rays and X-rays	Leakage due to radium-rays	Leakage due to X-rays	Sum of the two	Leakage due to radium-rays and X-rays	Ratio
133·0	107·9	60·2	0·00752	0·00927	0·0168	0·0166	0·99
135·0	117·3	60·4	741	853	159	166	1·05
129·0	119·9	62·6	775	834	161	160	0·99
133·3	107·9	59·7	752	927	168	168	1·00
127·4	94·3	54·4	785	1065	185	184	0·99
127·7	130·4	65·0	783	767	155	154	0·99
126·5	143·8	66·7	791	695	149	150	1·01
130·4	127·2	66·4	767	787	155	151	0·98
131·5	122·8	66·2	760	815	157	151	0·96
134·4	121·1	63·7	744	827	157	157	1·00
						Mean value	1·00

From this table we see that the ionization due to radium-rays and X-rays acting simultaneously is equal to the sum of the ionizations due to their acting separately.

§ 4. Secondly, I tried the combination of radium-rays and uranium-rays. As the source of the latter I made use of uranium oxide. The aluminium leaf at the bottom of the ionization vessel was taken off and the hole was covered with a shallow circular brass dish 7·5 cm. in diameter and 5 mm. in depth containing uranium oxide, which was pressed against the bottom of the cylinder by a strong spring. When only the radium was acting, I put instead of this uranium dish another dish of exactly the same form and size, but without uranium oxide, so that the leakage by the radium took place in exactly the same conditions whether it acted singly or conjointly with the uranium oxide.

Since the radiations from the radium and uranium oxide can be considered as constant during the experiment, I adopted in this case a simpler procedure than in the previous one. The radium specimen was put in the lead tube *R* and the leakage measured.

Next, the uranium oxide dish was substituted for the empty one and the leakage measured. Then the radium was taken off and the leakage due to the uranium oxide only measured.

The results are as follows.

EXPERIMENT 6.

Time in sec. required by radium-rays	Time in sec. required by uranium-rays	Time in sec. required by radium- and uranium-rays	Leakage due to radium-rays	Leakage due to uranium-rays	Sum	Leakage due to radium- and uranium-rays	Ratio
123·6	137·5	64·2	0·00809	0·00727	0·0153	0·0156	1·02
122·7	138·8	63·0	815	721	154	159	1·03
121·4	138·4	63·6	824	723	155	157	1·01
124·3	134·2	65·5	805	745	155	153	0·99
124·8	132·6	63·4	801	754	156	158	1·01
123·2	143·2	65·8	812	698	151	152	1·01
122·4	138·7	65·5	817	721	154	153	0·99
124·5	139·5	66·4	803	717	152	151	0·99
122·0	139·5	63·0	820	717	154	159	1·03
129·8	132·7	68·0	770	754	152	147	0·97
						Mean value	1·00

From this table we see that the ionization due to radium and uranium oxide acting simultaneously is equal to the sum of the onizations due to them acting separately.

§ 5. Thirdly, I tried the combination of X-rays and uranium-rays. Since the experiment could not be conveniently carried out by the old arrangement, a new ionization vessel was constructed.

The vessel is a very short cylinder 14 cm. in diameter and 3 cm. in height.

Its top has a hole at the centre through which a brass rod passes into the vessel in the same manner as in the previous cases. The lower end of the rod carries a wire net in the form of a circle 10 cm. in diameter in a horizontal position.

The distance of the net from the base of the vessel is about 1 cm. The base itself is not soldered to the vessel, but only pressed against it by means of two brass springs, so that it can be taken away at will. The central part of the base is marked for the reception of a circular metallic disc 10 cm. in diameter, which is covered with uranium oxide leaving a blank margin.

In consequence the ionization current due to the uranium oxide mainly takes place between the oxide and the wire net.

When only X-rays were acting, I put instead of this disc another disc of exactly the same form and size, but without uranium oxide. The side wall of the vessel has an aluminium window wide enough to let X-rays from a tube by the side of the vessel pass through every part of the vessel.

The procedure of the experiment is the same as in the last experiment. It is only to be remarked that since the discharge was greater than in the previous cases, owing to a larger surface of the wire net and of the uranium oxide used, I let the discharge go on through 25 divisions (from 65 to 40) of the micrometer scale.

The condition of saturation of the current was tested and found complete.

The following are the results.

EXPERIMENT 7.

Time in sec. required by uranium-rays	Time in sec. required by X-rays	Time in sec. required by uranium-rays and X-rays	Leakage due to uranium-rays	Leakage due to X-rays	Sum	Leakage due to uranium-rays and X-rays	Ratio
139·5	159·3	73·1	0·00717	0·00628	0·0135	0·0137	1·01
135·2	159·5	73·4	740	627	137	136	0·99
136·7	162·7	73·8	732	615	135	136	1·01
133·5	176·8	74·7	749	566	132	134	1·02
135·2	131·6	67·3	740	760	150	149	0·99
140·9	131·0	66·4	710	763	147	151	1·03
141·3	126·5	69·0	708	791	150	145	0·97
142·5	122·9	67·5	702	814	152	148	0·98
133·0	144·5	70·0	752	692	144	143	0·99
138·2	134·0	69·7	724	746	147	144	0·98
						Mean value	1·00

From this table we see that the ionization due to uranium-rays and X-rays acting simultaneously is equal to the sum of the ionizations due to their acting separately.

Thus X-rays and all Becquerel rays have the same ionizing power whether they act separately or simultaneously.

In conclusion I have much pleasure in acknowledging the invaluable advice and kindly interest of Prof. Thomson.

On the Measurement of the Earth-Air Current and on the Origin of Atmospheric Electricity. By C. T. R. WILSON, M.A., Sidney Sussex College.

[*Read* 30 April 1906.]

In ordinary fine weather there is, as is well known, an electric field near the earth's surface, the direction of the field being downwards; in other words, the surface of the ground is negatively charged. We now know also that the atmospheric air contains free ions, which will move under the action of the electric force. Unless therefore there are other compensating processes at work, there must be a continuous flow of positive electricity in fine weather regions from the atmosphere into the ground.

In order that we should be able to judge of the sufficiency or otherwise of the various theories of atmospheric electricity, a knowledge of the magnitude of this current from the atmosphere into the ground is required. For this current is a measure of the rate at which the electric field is being destroyed, and therefore, if the field remains undiminished, of the rate of regeneration of the electric field that a theory of atmospheric electricity has to account for. A knowledge of the earth-air current at various places distributed over the earth's surface under different weather conditions and at different times of the day and year is much to be desired, especially if simultaneously measurements could also be made of the charge carried down to the earth's surface by precipitation. Before however an attempt is made to carry out systematically measurements of this kind, it is of importance that suitable methods of making such measurements should be devised and tested.

The methods at present in use for deducing the value of the earth-air current involve first a measurement of the potential gradient at the time and place of observation, and secondly a measurement of the leakage from a charged conductor under such conditions that the current is proportional to the charge—in other words, to the strength of the field at its surface. The latter measurement, which is equivalent to a measurement of the conductivity of the atmospheric air, is not an easy one to carry out satisfactorily. The *zerstreuungsapparat* of Elster and Geitel, with which the conducting power of the free atmospheric air was first proved and its variations studied, does not give a reliable measure of the true dissipation coefficient, the number of ions per c.c. in the air under test being less than in the free atmosphere (except when there is unusually efficient ventilation and rapid renewal of the air by the wind) owing to recombination of the ions and their removal by the electric field applied. In the improved methods, as used for example by Gerdien, the air is drawn through a tube with an axial rod, a sufficiently small difference of potential being maintained between the tube and the rod to ensure that only a very small proportion of the total number of ions is removed by the field from the air passing through, so that the charge acquired by the rod is proportional to the potential difference, *i.e.* there is no approach to saturation.

The present paper contains an account of experiments made to test a method suggested in a paper recently read before this Society (*Proc. Camb. Phil. Soc.* Vol. XIII. p. 184). An insulated conductor connected to an electrometer is initially at zero potential and under a metal cover. The earth connection is broken and the cover removed, the conductor being thus exposed to the earth's electrical field. The potential of the conductor is thus raised, but is at once brought back to zero by means of a compensator. When this adjustment has been made we know that the charge removed from the electrometer and its connections by the displacement of the compensator is equal and opposite to that held on the exposed part of the conductor when at zero potential by the action of the earth's field. When the compensator has once been standardised its readings measure the charge on the exposed conductor when kept at zero potential; this charge will be the same as if the conductor were earth connected. If now by means of the compensator the conductor be maintained at zero potential for a few minutes and the cover be then replaced, the new reading of the compensator, when again adjusted to bring the electrometer reading back to its zero, measures the charge which has entered the conductor from the atmosphere in the given time.

By this method both the charge on an exposed conductor and the current flowing into it from the atmosphere, when it is maintained under the same conditions as if it were connected to the earth, are directly determined. The method has considerable advantages over the more indirect methods.

We may be said to determine directly the actual charge which has passed from the atmosphere in a known time through a conductor into the earth. Under ideal conditions the conductor would take the form of a flat plate at the level of the earth's surface; we should then obtain directly the current per square cm. of a flat portion of the earth's surface. In practice it is not convenient to proceed in this way, but to measure the current through and charge upon a conductor of suitable form when kept at zero potential, and to assume the ratio of the current to the charge (the dissipation coefficient) to be the same for all exposed conductors; then if the potential gradient and hence the charge per square cm. of the ground be known the earth-air current may be deduced. As thus carried out the method is the same in principle as that hitherto adopted, the difference being in the measurement of the dissipation factor, which, according to the method of this paper, is obtained by observations on the current through a conductor exposed under normal conditions in the free atmosphere. There is the further advantage that if we use the same conductor exposed at the same height and otherwise under similar conditions in successive experiments, the charge on the conductor will be proportional to the potential gradient; and when once the appropriate reduction factor has been obtained, the potential gradient is measured concurrently with the other measurements, and without the disturbing influences of any of the various forms of collectors.

To make the method of measuring earth-air currents which is here suggested practicable, a portable electrometer, suitable for measurements in the neighbourhood of zero potential and provided with a compensator for maintaining the potential at zero, is required. Such an electrometer was described and its application to the present purpose suggested in the paper already mentioned. The main object of the present paper is to indicate how the method has worked in practice.

The figure (Fig. 1) shows the apparatus which was attached for the purpose of the present measurements to the electrometer already described. Supported on a vertical rod screwed into the

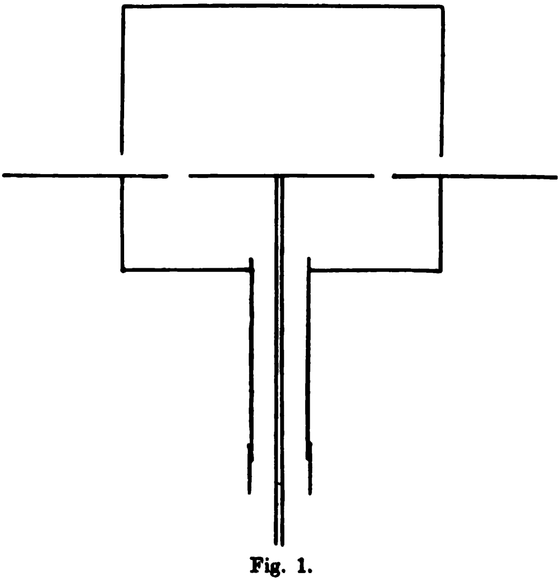


Fig. 1.

terminal of the electrometer is a horizontal circular brass "test-plate," 5 cms. in diameter. This is surrounded as shown by a guard ring 15 cms. in external diameter, the upper surfaces of test-plate and guard ring being in the same horizontal plane. The annular gap between the test-plate and the guard ring was half a cm. wide. The guard ring was supported on the top of a brass cylinder 2·5 cms. high and 8·5 cms. in diameter, the base of this being carried by a tube, about 5 cms. long and 1·5 cms. wide, which fitted on to the vertical tube attached to the top of the electrometer case and surrounding its terminal. On the guard ring could be placed the "cover" of brass, 4 cms. high and 8·5 cms. in diameter.

In making a measurement the electrometer is placed on a low tripod stand in an open space, the surface of the test-plate being in most cases from 60 to 100 cms. above the ground. The cover is placed on the top, and the system of test-plate, vertical supporting rod and gold-leaf earthed by means of the earthing rod. When it is desired to make a measurement the earth connection is broken and the cover removed. The observer with his eye at the observing microscope will see the gold-leaf deflected from its zero position when the cover is removed, and he at once brings it back to zero by sliding in the compensator; this adjustment can be made within a very few seconds. The reading of the compensator, if it has previously been standardised, will give at once the charge on the test-plate when exposed to the earth's field and kept at zero potential. By adjusting the compensator as may be required the observer now maintains the gold-leaf as nearly as possible at the reading corresponding to zero potential for a few minutes, and the cover is replaced after a definite number of minutes have elapsed since its removal. The compensator is now drawn out till the gold-leaf again indicates zero potential. If no current had entered the test-plate from the atmosphere the compensator would have had to be drawn right out to its initial position; the actual compensator reading when the zero potential is indicated by the gold-leaf enables the charge gained by the plate in the given time to be deduced. The test-plate is again earthed, the compensator drawn out to its standard position, the earth connection broken, the cover removed and the charge on the exposed test-plate when brought to zero potential again measured; under ordinary conditions the mean charge on the test-plate during the exposure may be taken as the mean of the results of the initial and final observations, which will of course be identical if the potential gradient has not meanwhile altered. Any rapid or irregular changes in the potential gradient during the exposure will be detected through the movements which have to be given to the compensator to keep the potential at zero.

We have obtained by these observations:

1. The charge on our test-plate when earth connected and exposed to the earth's electric field. This is directly proportional to the potential gradient, and if we always fix the instrument at the same height above the ground we may determine once for all the factor by which to obtain the potential gradient from the charge on the test-plate. This method of measuring the potential gradient has the advantage of being practically instantaneous and of not requiring the use of any flame, radio-active substance, water dropper or other collector.
2. The charge which has passed from the atmosphere into the test-plate in a definite number of minutes when the test-plate is kept at the same potential as if earth connected.
3. The ratio of the charge passing per minute into the test-plate from the atmosphere (obtained from 2) to the charge on the test-plate (1) both measured by the compensator may be taken as giving the dissipation constant under conditions of exposure identical with those to which natural earth-connected bodies are exposed.

It is only for a few seconds at the beginning and end of the measurement, in the interval between taking off or putting on the cover and bringing back the gold-leaf to its zero position by the compensator, that the potential of the test-plate and the connected system differs by more than a very small fraction of a volt from the potential when earth connected. The danger of error due to faulty insulation of the test-plate is therefore very small. The insulation can however be readily tested by putting on the cover and charging the test-plate system to any desired potential by means of the compensator; the insulation was always found to be high when thus tested. A failure of the insulation of the quartz Leyden jar supporting the inner case of the electrometer would be more serious; it was found however to remain practically perfect, the change of zero only amounting to a small fraction of a volt in 24 hours, and the system not requiring to have its charge replenished for some months when once charged.

There remains to be considered the working of the compensator. This consists of a brass tube, kept at a constant negative potential by being connected to a quartz Leyden jar, and forming with an inner concentric rod attached to the terminal of the gold-leaf an air condenser of variable capacity. The adjustment always being made in such a way as to bring the gold-leaf to zero potential, the charge held on the rod was proportional to the potential of the outer tube and to the capacity of the air condenser for a given position of the compensator. When the compensator was in its standard position, *i.e.* fully drawn out, the condenser tube was then some distance beyond the end of the rod, so that the potential of the tube might be varied largely without that of the rod being sensibly affected. With the compensator in its standard position there is thus no danger of an apparent gain of charge by the test-plate, merely due to defective insulation of the Leyden jar carrying the compensator tube.

Let us suppose the operations described above for the measurement of the charge on and current through the test-plate to be performed. When the cover has been removed and the compensator has been moved to bring the potential back to zero, the condition of all parts of the system which includes the gold-leaf test-plate and connecting rod will be the same as initially, except that there is a charge on the upper surface of the test-plate proportional to the potential gradient and an equal and opposite charge on the rod of the compensator, the total charge on the system remaining the same. The charge on the compensator rod can be deduced from the scale reading when the standardising measurements described below have been made. To maintain the gold-leaf at zero potential the compensator may have to be moved owing to either or both of two causes: (1) the potential gradient may change, and (2) electricity may flow from the air into the test-plate. An increase in the field (it is assumed to be of normal sign) will increase the negative charge on the surface of the test-plate, this requiring the compensator to be pushed further in till there is a further equal increase in the positive charge on the compensator rod. Under the action of the field, again, positive ions are likely to move up to the test-plate, and the positive charge entering the conducting system is prevented from raising the potential by again pushing in the compensator to increase by the corresponding amount the capacity of the sliding condenser, so that the whole increase of charge is held on the compensator rod. On putting on the cover the negative charge is no longer held on the upper surface of the test-plate, and to bring the potential of the system back to zero the compensator has to be drawn out till its capacity is such that the positive charge on the compensator rod when the potential is zero is equal to that which has been gained by the system from the atmosphere. This charge can then be determined from the position of the compensator.

We have to consider the question to what extent any failure of the insulation of the compensator may introduce error into the results. The charge measured is, for a given reading of the compensator, proportional to the potential of the compensator tube or of the inner coat of the Leyden jar to which it is attached. A change in the potential of the compensator tube, if not allowed for, would under the conditions of the experiments introduce a corresponding error into the measurement of both the charge on the test-plate and the current into it. It is however easy to test the constancy of the compensator constant from time to time by measuring the displacement required to cause a given change of potential of the gold-leaf system, the cover being on so that the gold-leaf system may have a definite capacity. In the instrument actually used there was rarely any sensible change during a series of observations.

To standardise the compensator the following method was used. A circular brass plate (Fig. 2), "the condenser plate," of the same diameter as the guard ring surrounding the test-plate, was supported by means of three insulating feet on the guard ring. The distance of the condenser plate from the guard ring and test-plate was 1·28 cms. The condenser plate could at will be earthed or connected to the positive terminal of a battery of one or more secondary cells of which the other terminal was connected to the electrometer stand. The compensator was first drawn out to its full extent, the test-plate and condenser plate both being earthed. The earth connections to both plates were now broken, and the condenser plate was brought to a definite potential by means of the cells. This raised the potential of the test-plate, causing a corresponding displacement of the gold-leaf. The compensator was then pushed in sufficiently to bring the potential of the test-plate as indicated by the gold-leaf back to zero. We now have the test-plate at the same potential, zero, as the guard ring, the condenser plate above it being also at a known potential V; we can thus readily calculate the charge on the test-plate. An equal and opposite charge has been removed from the system by the displacement of the compensator. By repeating this process for various values of the potential of the condenser plate the charge held on the compensator rod corresponding to any position of the compensator can be determined, and thus a calibration curve for

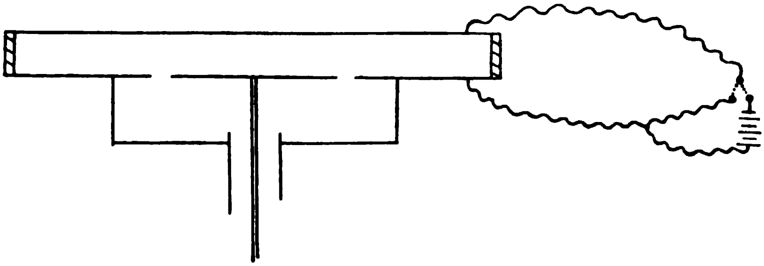


Fig. 2.

the compensator readings constructed. It is not however necessary to have available a whole series of cells, the whole calibration may in fact be made with a single cell by slightly modifying the method. The compensator reading corresponding to a charge equal to that induced on the test-plate by a single cell is determined as before. The compensator however, instead of being drawn back after this observation to its standard position, is left untouched; the test-plate and condenser plate are earthed and the operation described above repeated, the only difference being that the displacement of the compensator is from the point where it was left at the end of the previous determination. This process is repeated till the whole scale of the compensator has been traversed; at each displacement of the compensator an additional charge is drawn into the compensator rod equal to that induced on the test-plate when it is at zero potential and the condenser plate is connected to the positive terminal of the cell. A combination of the two methods was in practice found convenient, the first method being used till six cells had been inserted, and the second method with steps of six cells at a time being used for the rest of the calibration.

For a given compensator reading the charge held on the compensator rod (when kept at zero potential) is proportional to the potential of the compensator tube. The form of the calibration curve is thus the same whatever the potential of the compensator tube. When once a calibration curve has been constructed for one value of the potential of the compensator tube (chosen for its convenience in giving a suitable number of points on the curve with the cells available) we may adjust the charge of the Leyden jar attached to the compensator to suit the magnitude of the charge it is desired to measure. The ordinates of the calibration will merely have to be multiplied by a different factor, which can readily be found by comparing the displacements of the compensator required to raise the potential of the gold-leaf system by a definite amount as indicated by the displacement of the gold-leaf; the measurement in each case being made with the ordinary cover over the test-plate so that the capacity of the system may be the same.

The calibration curve for deducing from the compensator reading the charge which it was desired to measure was of the form shown in Fig. 3, which represents the results of observations made on April 20. The unit of charge of the table and curve is that held on the upper surface of the test-plate kept at zero potential when the condenser plate is at a potential of 2 volts. The reading 30·2 is that of the compensator when drawn fully out, the reading when it is pushed in against the stops being 0. It will be noticed that the displacement corresponding to a given increase of charge diminishes rapidly at first as the compensator is pushed in, but becomes constant before the total displacement has reached 10 mm. and remains uniform over the remaining

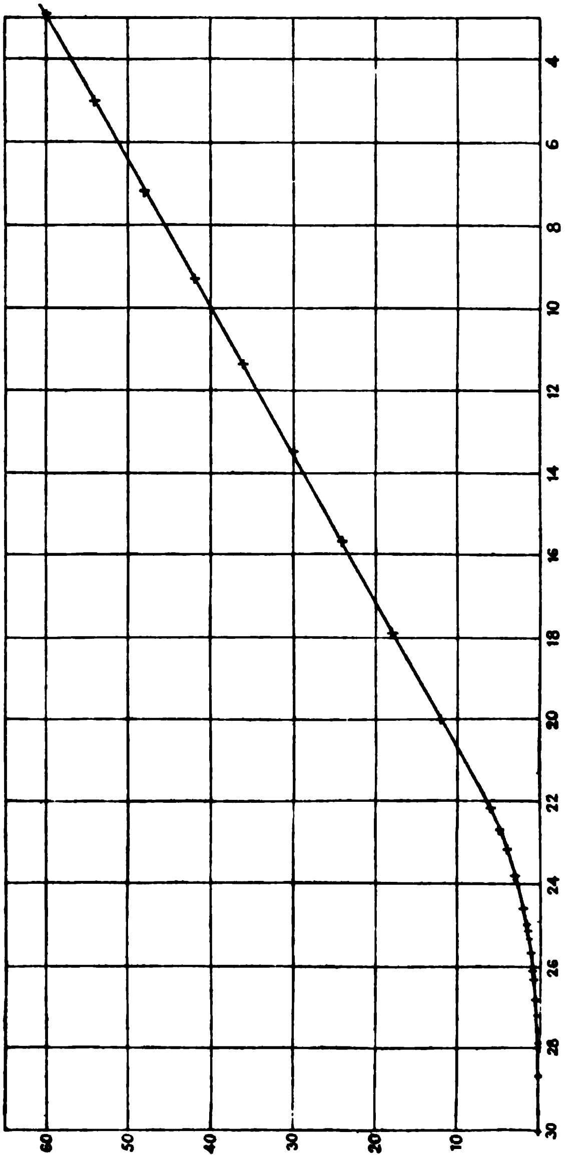


Fig. 3.

20 mm. of the scale. The first six observations* were made by the first method described above, the rest by the second method. The compensator was graduated in mm., and the tenths of a mm. were estimated.

Charge	Compensator reading	Charge	Compensator reading
0	30·2	24	15·7
1	26·1	30	13·5
2	24·6	36	11·4
3	23·8	42	9·3
4	23·2	48	7·2
5	22·7	54	5·0
6	22·2	60	2·9
0	30·2	66	0·8
6	22·2	0	30·2
12	20·0	6	22·2
18	17·9	1	26·1

The charge on the test-plate when at zero potential and with the condenser plate at a potential of two volts, the unit of charge in the above table, is given sufficiently nearly by

$$Q = \frac{r_1^2 + r_2^2}{2} \times \frac{V}{4d},$$

where r_1, r_2 are the radii of the test-plate and the aperture surrounding it, V is the potential difference between the plates in electrostatic measure and d is their distance apart. Inserting the actual values $r_1 = 2{\cdot}5$, $r_2 = 3$, $V = 2/300$, $d = 1{\cdot}28$, we obtain $Q = 0{\cdot}99 \times 10^{-2}$ electrostatic unit. We have thus the data for measuring the charge held on the compensator rod, kept at zero potential, for any given compensator reading, *i.e.* so long as the potential of the compensator tube remained the same. If this potential were altered the factor by which the ordinates of the calibration curve had to be multiplied to give the charge in electrostatic units would also be different. The potential of the compensator tube in the observations of the above table was such that with the ordinary cover over the test-plate the compensator had to be drawn out through 6·3 divisions to cause a movement of the gold-leaf through 30 scale divisions, the compensator being pushed in nearly to its full extent before the displacement. Thus the same condition of the compensator could be recovered at any time by suitably charging the Leyden jar attached to the compensator, or the new reduction factor to be applied to the calibration curve could be determined by an observation like that just described. Thus for example after the above calibration was performed the compensator potential was diminished so that to cause, under the standard conditions, a deflection of the gold-leaf amounting to 5 scale divisions the compensator, originally fully in, had to be drawn out through 9·8 mm. The reduction constant would thus be increased in the ratio $\frac{9·8}{5} : \frac{6·3}{30} = 9·3$. A direct determination of the compensator readings corresponding to known charges gave the results shown in the following table, the charges being given in the same units as in the preceding table.

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* In the curve some additional points have been inserted corresponding to observations made with the aid of a potentiometer.

Charge	Compensator reading
6	4·3
1	20·9
6	4·3
1	20·9
6	4·4
1	20·9
2	17·6

To give a reading 4·3 now given by a charge of 6 arbitrary units a charge of 56 units would before have been required, the ratio being 56/6 = 9·3, in agreement with that just obtained indirectly. For the compensator readings 17·6 and 20·9 the ratios are 9·4 and 9·5 respectively.

The sensitiveness of the electrometer was 6·0 eye-piece scale divisions per volt. The microscope was turned so that the rulings on the scale were parallel to the edge of the gold-leaf; under these conditions when a straight portion of the edge of the gold-leaf was used the coincidence of the edge of the gold-leaf with a line on the scale could be observed with great exactness; a departure from coincidence amounting to a small fraction of a scale division, certainly less than a twentieth, could readily be observed.

The capacity of the system which included the test-plate and gold-leaf was easily determined. For this purpose the condenser plate was placed on the guard ring as in standardising the compensator, and was raised to a known potential by means of a number of cells, the gold-leaf system being meanwhile earthed. The earth connection was then broken, the condenser plate removed and the cover put on in its place. The potential of the system can then be obtained from the gold-leaf reading, while the charge is known, since it is that which was held on the upper surface of the test-plate when at the same potential as the guard ring (*i.e.* zero), and with the condenser plate at a known potential above it. The capacity of the test-plate and gold-leaf system with the ordinary cover on and with the compensator drawn out was about 9·5 cms.; with the compensator in it was about 10 per cent. greater. A knowledge of the capacity of the system was not required in the measurements of potential gradient or earth-air currents. It is given for the purpose of indicating the order of magnitude of the smallest charge measurable with the instrument. The compensator could be adjusted to give coincidence between the edge of the gold-leaf and a scale division to within 1/20 of a scale division, *i.e.* to within 1/120 of a volt, corresponding to a quantity less than 1/120 × 1/300 × 9·5 = about 1/3000 E.U.

Most of the measurements of the charge upon and current through the test-plate when kept at zero potential have been carried out on the top of Hamildon Hill, near Peebles in Scotland. The hill is 1210 feet (370 metres) high, and has a flattened top. The electrometer stood upon a metal tripod adjusted to such a height that the test-plate was either 60 cms., 90 cms., or 130 cms. above the ground. I have not attempted to determine the factor by which the charge on the test-plate must be multiplied to give the undisturbed potential gradient or the charge per square cm. of the ground. The order of operations generally adopted was as follows. When the apparatus had been set up the compensator constant (the factor by which the ordinates of the calibration curve had to be multiplied to obtain the corresponding charge in electrostatic units) was first determined by finding the displacement of the compensator required to cause a movement of the gold-leaf through a definite number of scale divisions as described above. The test-plate system was then earthed by means of the earthing key and the electrometer zero observed; it was adjusted by a suitable displacement of the microscope so that the straight edge of the gold-leaf coincided exactly with one of the scale divisions of the eye-piece scale. The earth connection was then broken, and at a definite time the cover was removed and the compensator at once moved to bring the potential back to zero. The compensator reading was then read, this giving the charge on the plate. The compensator was moved as might be required to keep the potential at zero, and when a definite number of minutes, generally from two to five, had elapsed since its removal, the cover was replaced and the compensator drawn out to bring the potential back to zero, its reading then giving the charge gained by the plate in the known time. The system was then earthed to test the zero again. Finally the compensator was drawn out, the earth connection broken, and the first operation of finding the charge on the exposed test-plate at zero potential repeated, so that any change of the potential gradient might be detected. Generally the change in the potential gradient between the two measurements of charge was small, and the mean was taken as the charge for the time during which the current was measured, and this value was used in deducing the dissipation factor. A second measurement of charge and current could now be made as soon as might be desired. The testing of the compensator constant did not require as a rule to be made again except at the end of the whole series of observations; it generally showed no appreciable change in a few hours.

The results of the hill-top measurements are given in the table on page 376.

The time is that at which the cover was removed at the beginning of an observation. It was replaced from two to five minutes later.

The charge and the leak are those of the test-plate, and are given in electrostatic units; I have not attempted to deduce the corresponding charge and current per square cm. of the ground The last column, giving the ratio of the leak per minute to the charge, may be taken as the dissipation factor measured under conditions very like those holding at the surface of the ground. The observations were taken under very varied conditions. The smallest dissipation, of the order of one per cent. per minute, was observed on April 10th on a cloudless day with a somewhat thick haze. Values equal to or approaching 10 per cent. per minute were not unfrequent. Fairly high values exceeding 5 per cent. were obtained on the only occasion when such measurements were attempted at night (April 8th, between 11 p.m. and midnight). The night was perfectly clear and calm; the observations were made without the use of any artificial light which might have influenced the leakage; they were carried out entirely by the light of the full moon.

The mean value of the dissipation factor from these observations is 5·6 per cent. per minute.

Objections might with some justification be raised that the dissipation constant as deduced by the present or other methods can hardly be applied with confidence to the calculation of the actual earth-air current, the materials covering the earth's surface being very different from the metal test-plate.

The matter is very readily tested with the apparatus which has been described. For any conducting body may be placed on the test-plate, and the charge upon it and current through it, when exposed to the earth's electric field and kept at zero

Date	Time	Height of test-plate	Charge	Leak per minute	Leak per minute / Charge	
Dec. 12	11.20 a.m.	130 cms.	31·6 × 10^{-2}	1·16 × 10^{-2}	3·66 per cent.	Calm, bright sunshine, mist in valleys
Dec. 12	11.59 a.m.	90 „	32·8 „	·93 „	2·84 „	
Dec. 13	11.20 a.m.	90 „	25·0 „	1·12 „	4·5 „	Wind strong from W., covered with low stratus
Dec. 13	11.31 a.m.	90 „	24·3 „	1·07 „	4·4 „	
Dec. 16	11.13 a.m.	90 „	13·1 „	1·00 „	7·7 „	Sun hidden throughout observations, clear sky in places, wind N.W.
Dec. 16	11.18 a.m.	90 „	14·2 „	1·00 „	7·0 „	
Dec. 16	11.25 a.m.	90 „	15·2 „	·99 „	6·5 „	
Dec. 16	11.33 a.m.	90 „	15·2 „	1·06 „	7·0 „	
Dec. 16	12.6 p.m.	90 „	15·2 „	1·15 „	7·6 „	
Dec. 19	3.34 p.m.	90 „	10·9 „	·65 „	6·0 „	Clear, strong wind from W.
Dec. 26	11.36 a.m.	90 „	21·1 „	·70 „	3·35 „	Calm, mist cleared from top just before observations began
Dec. 26	11.47 a.m.	90 „	19·4 „	1·06 „	5·5 „	
Dec. 26	11.58 a.m.	90 „	16·1 „	·53 „	3·3 „	
Dec. 26	12.6 p.m.	90 „	13·3 „	1·26 „	9·5 „	
Dec. 26	12.36 p.m.	90 „	11·7 „	·95 „	8·1 „	
April 8	11.9 p.m.	90 „	9·9 „	·51 „	5·1 „	Calm, cloudless
April 8	11.17 p.m.	90 „	7·5 „	·33 „	4·4 „	
April 8	11.31 p.m.	130 „	8·2 „	·49 „	6·0 „	
April 8	11.43 p.m.	130 „	8·3 „	·57 „	6·9 „	
April 10	3.47 p.m.	90 „	23·6 „	·36 „	1·52 „	Sun shining all the time, nearly cloudless, haze
April 10	4.2 p.m.	90 „	16·4 „	·15 „	·91 „	
April 10	4.36 p.m.	60 „	15·2 „	·19 „	1·24 „	
April 17	3.15 p.m.	60 „	2·45 „	·26 „	10·6 „	Slight breeze from E., sky nearly covered with cumulus clouds, sun hidden most of the time
April 17	3.23 p.m.	60 „	3·8 „	·26 „	6·8 „	
April 17	3.50 p.m.	60 „	3·7 „	·33 „	8·9 „	
April 17	4.3 p.m.	60 „	4·45 „	·35 „	7·9 „	

potential, measured exactly as before. For the measurements of charges being all made in terms of the compensator readings, they are independent of the capacity of the rest of the system.

The test-plate may for example be covered with turf and the dissipation factor determined for this. Measurements of this kind were made on the hill-top on April 17th, observations with and without the turf being taken alternately. The test-plate was about 60 cms. above the ground.

	Time	Charge	Leak per minute	Dissipation factor
With turf	3.44	11·8	1·05	8·9
Without turf	3.50	3·7	·33	8·9
With turf	3.56	15·3	1·35	8·9
Without turf	4.3	4·4	·35	7·9

The charge with the turf on was considerably greater than without it, owing to the fact that it projected considerably above the level of the guard plate; the current was however increased in approximately the same ratio, so that the dissipation factor is not sensibly altered.

I have also made measurements by the same method of the charge upon and current through a growing plant exposed to the earth's field and kept at zero potential. To make a measurement of this kind the plant growing in a flower-pot is placed on the test-plate, and a metal cover large enough to admit of being removed and replaced without danger of contact with the plant was put over it, resting on the guard plate. The measurements were then made as before. The plant which I used to test this application of the method to such measurements was a young wall-flower, 14 cms. high and 8 cms. across at the widest part, the whole height with the pot being about 20 cms.; the cover was cylindrical, 22 cms. high and 12·5 cms. in diameter. The measurements were made in a garden, in which the electric field was very much weakened by the shielding action of buildings, trees and telegraph wires. The charge on the plant was about 20 times that on the test-plate, as found on making alternate exposures with and without the plant on the test-plate, reaching even in the weak electric field of the garden to as much as one electrostatic unit. The observations did not show any evidence that the dissipation factor was different for the plant and test-plate, but a large series of comparisons would be required to test to what extent, if at all, it differs. The experiments tried however were sufficient to prove the practicability of the method.

It is possible to test by a slight modification of the conditions of the experiments just described a whole class of theories of the origin of atmospheric electricity; those namely which suppose the electric field in fine weather regions to be maintained or regenerated by the action of the air on earth-connected bodies in these regions. Such theories would imply that air in passing over the earth's surface becomes positively charged by friction, by a difference in the mobilities of the positive and negative ions, or otherwise. Let the usual observations of the charge on the test-plate, or on a plant or other body placed upon it, while the potential is kept at zero, and of the charge gained by the system under these conditions in a given time, be made under the shelter of a tree. Under these conditions the electric field and the charge on the body under test will almost vanish, so that there is nothing to prevent the negative electrification of the body, which according to such theories would result.

Experiments of this kind have been tried both with the ordinary brass test-plate and with a plant. In no case was there any evidence of any tendency for a negative charge to be acquired, the current remaining in the normal direction even in exceedingly weak fields.

NOTE ON THE ORIGIN OF ATMOSPHERIC ELECTRICITY.

I take this opportunity of modifying an opinion expressed in an article in *Nature* (June, 1903), in which I discussed some possible sources of atmospheric electricity. I there spoke of the difficulty which the recently discovered conducting power of the air introduced into the theories which explain the fine weather electric field by the action of precipitation in wet weather regions. Assuming the electrical current, which according to this view must travel in the atmosphere from wet weather to fine weather regions, to be entirely one of convection, the positive charge being carried along by upper air currents, I pointed out that the greater part of the positive charge of the air would have disappeared by conduction to the earth before the air had travelled any considerable distance. Thus it would be impossible to explain the existence of an electric field at great distances from areas of precipitation. I therefore suggested that we might be driven to look to a cosmical source for the maintenance of the earth's charge, *e.g.* to negatively charged particles of great penetrating power traversing our atmosphere and being absorbed by the earth.

The possibility of the current from the wet regions to the regions of fine weather being, like the vertical electric current at the surface, not due to convection but to conduction, *i.e.* to movement of the free ions under the influence of an electric field, was there ignored. For this I now think there is no justification.

It may be desirable in the first place to make more clear the argument against the sufficiency of the condensation theory when conduction in the upper atmosphere is ignored.

Let us assume the total positive charge in the atmosphere over an area at a distance from the disturbed region where precipitation is going on to be equal in amount to the negative charge on the ground in that area. Let us suppose also that the rate of dissipation of the charge on the earth's surface is one per cent. per minute, a somewhat low estimate. Then this is also the positive charge lost per minute by the air overlying this area. Thus the total charge in the atmosphere overlying each square metre of the ground, and therefore also the corresponding charge on the ground, will diminish with increasing distance from the disturbed region at a rate which will be greater the smaller the horizontal convection current. It would be impossible to explain the strong electric fields which may persist in calm weather at great distances from any disturbed regions; for even if the air took only an hour to travel from the region of precipitation it would have lost about half of its charge. In the calm regions the lower layers, which carry nearly the whole positive charge, have a negligible horizontal motion. A quite impossible velocity would have then to be assigned to the upper layers (which carry only a very small fraction of the total charge) if the convection current carried by them is to make good the loss of the lower layers in each region over which they pass.

The only way in which the condensation theory can be made to explain the maintenance of the electrical field in fine weather areas at distance from the regions where precipitation is going on, is to suppose that the current in the atmosphere from the wet to the dry regions is one of conduction in the upper atmosphere. The question to be considered is, then, what grounds have we for supposing that the upper atmosphere has sufficient conducting power?

At a height of about 18 kilometres the pressure will only be about one-tenth of that near the ground; at twice this height it will only be about one-hundredth of an atmosphere. Over this range of pressures the mobility of the ions is inversely proportional to the pressure. Over the same range the coefficient of recombination, α, is approximately proportional to the pressure (Langevin, *C. R.* 137, p. 177, 1903). Let us suppose that the atmosphere at different heights is exposed to ionising radiation of the same intensity; the rate of production of ions per c.c., q, will be proportional to the pressure. When a steady state is reached $q = \alpha n^2$ or $n = \sqrt{\frac{q}{\alpha}}$, n being the number of ions of each sign per c.c. Now according to the assumptions made both q and α are proportional to the pressure; thus the number of ions per c.c. is the same at all heights. The conductivity which is proportional to the mobility and to the number of ions will thus be inversely proportional to the pressure. At 18 kilometres it will be 10 times and at 36 kilometres 100 times the conductivity near the ground. The conductivity near the ground is moreover much diminished by the presence of dust particles to which the ions attach themselves, the average life of the free ion being very much less than would be deduced from the ordinary recombination constant. The conduction of the higher layers may thus be very much greater than the above estimate. At greater heights than those considered the conductivity might be expected to increase at first at an even more rapid rate than would be given by the above law, as the size of the ions is less at these low pressures. It is of course a somewhat arbitrary assumption which has been made, *i.e.* that the atmosphere is exposed at all levels to ionising radiation of the same intensity; but it is likely that the uppermost layers are exposed to the action of various ionising radiations at the least as intense as those to which the lower layers are exposed. (As pointed out by Nordmann (*C. R.* 136, p. 1430, 1903), the diminution of the recombination constant with increasing height is of great importance in another way, for at great heights the ionisation would take some hours to die away after the ionising radiation, *e.g.* sunlight, had been cut off.) There may thus be above the lower layers a great thickness of air having a conductivity enormously greater than that of the lower atmosphere. The total variations of potential throughout this upper atmosphere even over great distances in a lateral direction may be comparatively small. Since the potential gradient in fine weather regions is of the order of 100 volts per metre near the ground and dies away in the course of a few thousand metres above the ground, the potential of the upper atmosphere in such regions is of the order of 100,000 volts; a very small lateral potential gradient in the conducting layers may be sufficient to maintain the necessary current. According to the condensation theory showers of the appropriate kind act as the batteries or dynamos by which the atmospheric electrical current is maintained. We may suppose that by the falling to the ground of a preponderance of negatively electrified rain there is maintained in the ascending air above the shower a very high positive potential. We know, from the strength of the field developed in the immediate neighbourhood of the shower and from the production of lightning in extreme cases, that the differences of potential developed may be very high, probably many millions of volts between a point above the shower and the ground. If we suppose that owing to the conductivity of the upper layers the potential differences within these layers remain small, we shall also have a potential fall in rising from the point considered into the conducting upper layers only a few hundred thousand volts less than in travelling from the point to the earth. There will thus be a flow of positive electricity into the upper atmosphere, helping to maintain its positive potential. Travelling with the current from the source we should have a very large fall of potential in rising to the conducting layers, a comparatively small fall in travelling laterally in the conducting layers, and a fall of a few hundred thousand volts in passing through the lowest few thousand metres to the ground.

It is a matter of fundamental importance to determine whether the negative charge carried down by rain is sufficient to counterbalance the conduction current from the atmosphere to the earth's surface over the whole world. We are of course a very long way from the solution of this question; according to Gerdien* however, to whom with Elster and Geitel what knowledge we possess as to the electrical charges brought down by precipitation is mainly due, there is at least agreement in the order of magnitude of the quantities.

The variations of the potential gradient near the ground, *e.g.* the daily variations, are mainly to be attributed to variations in the conductivity of the different parts of the atmosphere. The increase in the potential gradient due to fog or haze near the ground has long been known, and the explanation was given by Elster and Geitel in one of the first papers in which our knowledge of the existence of free ions in the atmosphere was applied to the explanation of the phenomena of atmospheric electricity. It has been pointed out by O. W. Richardson (*Nature*, April 26, 1906) that an increase in the conductivity at a height ought, according to the view that I have been supporting here, to cause also an increase in the potential gradient at the ground. For an increase in the conductivity above will increase the current reaching the surface of the earth, and if the conductivity of the lower layers is unaltered the potential gradient in the lower layers must be proportionately increased when a steady condition is established. A change of conductivity of the lowest layers extending to only a few metres above the ground will not alter to any considerable extent the earth-air current in that region, and the potential gradient when a steady condition is reached will be in this case inversely proportional to the conductivity.

Experiments have recently been described by Campbell and Wood showing a daily variation of the ionisation within a closed vessel resembling that of the atmospheric potential gradient, showing for example an early morning minimum. Dr Richardson has pointed out that if these results are interpreted as indicating the existence of penetrating radiations reaching the earth's surface from cosmical sources, the coexistence of low values of the radiation with low values of potential gradient is what might be expected, as the upper layers would be exposed to more intense radiation than the lower (for we must suppose that there is considerable absorption in traversing the atmosphere); the variation in ionisation would thus be greater above than below, and the potential gradient and radiation would thus vary together.

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* Gerdien, *Physikalische Zeitschr.* Jahrg. VI. 1905.

If the existence of a penetrating radiation from cosmical sources were established it would be of the greatest importance in connection with atmospheric electricity. For it would open the question as to whether the negative charge of the earth might not be supplied by these rays. At present I think it is much more likely that precipitation will prove to be a sufficient source.

A suggestion as to the nature of the Horny Teeth of the Marsipobranchii.

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[*Received* 30 April 1906.]

The morphology of the teeth of the Marsipobranchii has more than a passing interest, since it is intimately bound up with the question of the phylogeny of the group.

The view that the Cyclostomes are degenerate gnathostomatous vertebrates, degenerate as the result of parasitism, is supported by all the authority of Huxley, Dohrn, Howes, Beard and others. The opposite conclusion, that they are primitive vertebrates in which true jaws have not been developed, has been advocated by Balfour, Gegenbaur, Haeckel, and more recently by Ayres. Further, while Huxley regarded the Petromyzontidae as the most primitive of the Cyclostomes, Parker, Beard and others would place the Hags in that position.

Howes [2] has pointed out that whether or not the Cyclostomes are to be looked upon as gnathostomatous depends upon the interpretation placed upon the term, since it is used indifferently to "express either the mere possession of jaws, apart from any consideration of teeth or [more generally] that of a dentigerous apparatus."

The disposition and gross anatomy of the teeth have been described by J. Müller, W. K. Parker and others, but Beard was the first to investigate their histological characters [1]. Since the publication of his paper others have appeared by Behrend, Ayres and Warren, which, while agreeing in the main with Beard's results, differ in several important details.

While working recently at the morphology and development of the scales of the Teleostei my attention was directed to the subject of the teeth of the Cyclostomes, and it appears to me that the results of those investigations throw some light upon the nature and evolution of the cyclostome teeth, and to some extent harmonise the apparent discrepancies.

If one examines the scales of one of the Gadidae they are found to consist of a number of calcified scalelets arranged excentrically around a somewhat thickened and calcified "centrum" or "foyer," the surface of which is smooth. All these calcified structures lie upon a fibrous basis, the periphery of which is turned upwards to become continuous with the scale-pocket or "schuppen tache." The scale increases in size at its margin, that is to say, that the formation of additional calcified structures takes place in the groove where the scale-pocket and the fibrous basis become continuous. Comparison of the scales of different genera and species and with those of the Clupeoids show that there is a tendency to a fusion of the scalelets together with a disappearance of their projecting points, tending to form a continuous and smooth-surfaced calcified plate. Moreover, the development of the scale is seen to take place in the more superficial layer of the dermis, which is composed of elongated nucleated cells with ill-defined boundaries, and so closely set that I have spoken of it [4] as the "nuclear layer." Beneath this is a more fibrous-looking stratum with but few elongated nuclei. Lying superficially to these two layers is the epidermis with its mucin-secreting glands. The epidermis is readily detached and usually absent in preserved specimens.

If this account of the structure and development of the teleostean scale be compared with that given by Beard of the genesis of the horny teeth of Petromyzon, I think it will be seen that there is more than a superficial resemblance. Beard says that "the mouth is lined by a thick stratified epithelium. Lying in the deeper layers of this epithelium, that is forming its basis, one notices a somewhat symmetrical row of what, at first sight, looks like true tooth-sacs." "Though they possess a dental papilla they have no odontoblasts." And again, "The papilla is mesoblastic [of reticulate connective tissue], and upon it there rests an 'enamel epithelium' which passes over as in other cases, into the outer epithelium. The curious circumstance is that from the start the enamel epithelium and the circular horn groove into which it passes externally produce horn, and beyond the imitation of a true tooth-sac in which, if ever present, the odontoblasts never become functionally active, there is no trace of true tooth structure ever produced in *Petromyzon planeri*."

This description applies only to the teeth near the anterior margin of the mouth, those situated more inwardly "are formed simply in the basal layers of the stratified epithelium, without the intervention of a tooth-sac." Now if we substitute for "stratified epithelium" what I have termed the "nuclear layer," the conditions appear to me to be almost identical. The true epidermal covering is extremely liable to be removed, indeed by merely placing the freshly killed animal fish into a preservative fluid, this layer becomes detached, and I have not yet seen it present in any preserved stock material. It seems therefore not unlikely that this layer, even if present in the mouth, when the animal is alive, may be similarly liable to become detached, and therefore that the true epidermal layer is absent in the specimens examined by both Beard and Warren.

If then the nuclear layer be taken as homologous with the stratified epithelial layer, we find that the calcified scales are developed in the deeper layers just as are the horny plates. In the former case the cells immediately above the scale give rise to the scale-pocket, while those immediately below, to the fibrous basis of the scale. Both the scale and the horny plate continue to grow at the periphery, *i.e.*, where the subjacent and superficial layers of cells become continuous, the horn-groove of Beard. This gives precisely the picture of the development of the more inwardly placed teeth of Petromyzon "without the intervention of a tooth-sac." Towards the edges of the mouth the skin becomes corrugated, involving the cellular layer and giving rise to the appearance of a tooth-sac, in the interior of which would be included the deeper or more fibrous layer of the dermis forming the mesoblastic papilla of reticulate connective tissue. Such a buckling up of the cellular layers would give rise to the conical horny tooth, the centrum of the scale corresponding to the apex of the tooth. Just as the centrum is the first part of the scale to be formed and continues to grow centrifugally, so Warren [3] states that the process of cornification proceeds from the apex downwards.

If the above suggestion be correct then the horny teeth must be regarded as purely dermal structures and not epidermal as Beard thought. The scales of the Teleostei I believe to be dermal in origin, and therefore the horny plates would be homologous with the calcified portion of the scale and, if the homology may be pushed, possibly also to the dentinal layer of true teeth.

Upon this assumption as to the primitive origin of the tooth-sac it follows that the downgrowth of the oral epithelium in those cases in which it persists, for example in mammals, to form the enamel organ is a secondary feature. This is a point of some practical importance. The difficulty in deciding whether to rely on the shape of the dental papilla or on the contour of the enamel organ, where, as is so frequently the case the two do not coincide, has presented itself to everyone who has worked at the evolution of the cusps in the mammalian molars. From these considerations therefore the form of the dental papilla would appear to be the more reliable criterion.

With regard to the chemical nature of the materials composing the horny teeth and the scalelets I know of no exact analysis, but it may be pointed out that after the latter have been decalcified with an acid, there yet remains an organic basis probably of the nature of Collagen which is an albuminoid, as is also Keratin.

The peculiar character of the tooth succession in Petromyzon seems to me to present no very great difficulty. The imbrication of the scales of the Teleostei, each enclosed in its own scale-pocket, causes the wall of the pocket of one scale to lie partially subjacent to the superposed scale. If by a little crowding, such as would be caused by the involution into the mouth, the superposition became complete instead of partial, then there would be precisely the same relative position of parts as is shown by Beard to exist in Petromyzon.

The above hypothesis seems to me to be also quite compatible with the main facts, as given by Beard and Warren, of the structure and disposition of the teeth in Myxine and Bdellostoma. In these animals the conditions have become somewhat more specialised, the dental papilla better marked and its cells more differentiated to form the so-called "odontoblast cone." If such be the case, it furnishes further evidence in favour of the correctness of Huxley's view that the Petromyzontidae are the most primitive among the Cyclostomes.

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1. BEARD, J. "The nature of the teeth of the Marsipobranch fishes." *Morph. Jahr.* III. 1889, pp. 727—753.
2. HOWES, G. B. "On the affinities, inter-relationships and systematic position of the Marsipobranchii." *Proc. and Trans. Liverpool Biological Soc.* Vol. 6, 1892, pp. 122—148.
3. WARREN, E. "On the Teeth of Petromyzon and Myxine." *Quart. Journ. Micr. Sci.* Vol. 45, 1902, pp. 631—637.
4. TIMS, H. W. MARETT. "The development, structure and morphology of the Scales in some Teleostean fish." *Quart. Journ. Micr. Sci.* Vol. 49, 1905, pp. 39—68.

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1905—1906.

ANNUAL GENERAL MEETING.

October 30th, 1905.

In the Cavendish Laboratory.

Professor Marshall Ward, President, in the Chair.

The following were elected officers for the ensuing year:

President:

Professor Marshall Ward.

Vice-Presidents:

Professor Thomson.
Dr Baker.
Dr H. J. H. Fenton.

Treasurer:

Mr H. F. Newall.

Secretaries:

Mr A. E. Shipley.
Mr E. W. Barnes.
Mr P. V. Bevan.

Other Members of the Council:

Mr D. Sharp.
Professor Larmor.
Mr W. C. D. Whetham.
Mr G. H. F. Nuttall.
Mr F. H. Neville.
Mr S. Ruhemann.
Dr Anderson.
Mr A. Hutchinson.
Mr F. F. Blackman.
Mr H. W. Richmond.
Mr F. G. Hopkins.
Mr A. Harker.

The following was elected an Associate of the Society:

J. Laby, Non-Collegiate.

The following Communications were made:

1. On a well-sinking at Graveley near Huntingdon. By O. **Fisher**, M.A., Jesus College.

2. On a portable Gold-leaf Electrometer for low or high potentials, and its application to measurements in atmospheric electricity. By C. T. R. **Wilson**, M.A., Sidney Sussex College.

3. Contributions to the knowledge of the Tetrazoline-Group. By S. **Ruhemann**, M.A., Gonville and Caius College.

4. The action of Radium and other Salts on Gelatin. By W. A. D. **Rudge**, M.A., St John's College.

5. A suggestion as to the nature of the "walnut" comb in Fowls. By W. **Bateson**, M.A., St John's College and R. C. **Punnett**, M.A., Gonville and Caius College.

6. The absence of Isomerism in substituted Ammonium Compounds. By H. O. **Jones**, M.A., Clare College.

November 13*th*, 1905.

In the Cavendish Laboratory.

PROFESSOR LIVEING IN THE CHAIR.

The following was elected a Fellow of the Society:

F. J. M. Stratton, M.A., Gonville and Caius College.

The following Communications were made:

1. Polarization phenomena in the Eclipse, 1905, August 30. By H. F. **Newall**, M.A., Trinity College.

2. Suggestions for a theory of the Milky Way and the Clouds of Magellan. By A. R. **Hinks**, M.A., Trinity College.

3. The Effect of the Lunar Deflection of the Vertical on Latitude Observations. By B. **Cookson**, M.A., Trinity College.

November 27*th*, 1905.

In the Optical Lecture Room.

PROFESSOR MARSHALL WARD, PRESIDENT, IN THE CHAIR.

The following was elected an Associate of the Society:

F. C. Blake.

The following were elected Fellows of the Society:

R. F. D'Arcy, M.A., Gonville and Caius College.
R. H. Lock, M.A., Gonville and Caius College.
A. A. Robb, M.A., St John's College.

The following Communications were made:

1. On some effects produced by Kanalstrahlen. By J. J. THOMSON, M.A., Cavendish Professor of Experimental Physics.

2. Experiments on the retention of an electric charge by gases. By W. A. D. RUDGE, M.A., St John's College.

3. Note on the effect of hydrogen on the discharge of Electricity from hot Platinum. By O. W. RICHARDSON, M.A., Trinity College.

4. On Colour-inheritance in Rats. By L. DONCASTER, M.A., King's College.

5. A preliminary Communication on the life-history of *Pleistophora periplanetae* (Lutz and Splendore). By W. S. PERRIN, B.A., Gonville and Caius College. (Communicated by Mr A. E. Shipley.)

6. On the Osmotic Pressure of Alcoholic Solutions. By P. S. BARLOW, B.A., St John's College. (Communicated by Professor Thomson.)

7. Two wheels connected by an axle rolling on a rough horizontal plane. By G. M. K. LEGGETT, B.A. (Communicated by Mr G. Birtwistle.)

8. A series of optically active Nitrogen compounds containing the Allyl group. By Miss M. B. THOMAS, and H. O. JONES, M.A., Clare College.

January 29*th*, 1906.

In the Cavendish Laboratory.

PROFESSOR THOMSON, VICE-PRESIDENT, IN THE CHAIR.

The following was elected an Associate of the Society:

Jan Hubrecht, Christ's College.

The following was elected a Fellow of the Society:

L. Humphry, M.D., Trinity College.

The following Communications were made:

1. On the expansion of a gas into a vacuum and on the determination of the specific heat at constant pressure for gases. By G. F. C. SEARLE, M.A., Peterhouse.

2. On the action of Radium and other Salts on Gelatin. By W. A. D. RUDGE, M.A., St John's College.

3. A novel instrument for illustrating the magnetic properties of Iron. By A. H. PEAKE, M.A., St John's College.

4. The Permeability of Iron in the colloidal state. By E. F. BURTON and P. PHILLIPS.

February 12*th*, 1906.

In the Botany School.

MR FRANCIS DARWIN IN THE CHAIR.

The following were elected Fellows of the Society:

Baron A. von Hügel, M.A., Trinity College.
H. A. Webb, M.A., Trinity College.

The following Communications were made:

1. Notes on Cycads: with exhibition of a rare species acquired by the Botanic Garden. By A. C. SEWARD, M.A., Emmanuel College.

2. Respiration and Vitality. By F. F. BLACKMAN, M.A., St John's College.

3. Hybridization of Barleys. By R. H. BIFFEN, M.A., Emmanuel College.

4. A comparison of the results from the Falmouth Declination and Horizontal Force Magnetographs on quiet days in years of sunspot maximum and minimum. By C. CHREE, Sc.D., King's College.

February 26*th*, 1906.

In the University Chemical Laboratory.

Mr F. H. Neville in the Chair.

The following Communications were made:

1. An indicator for strong acids and bases. By H. J. H. **Fenton**, Sc.D., Christ's College.

2. (1) The action of acid chlorides of Acetylenic acids on Ketonic compounds.

(2) The Dihydrotetrazines. By S. **Ruhemann**, M.A., Gonville and Caius College.

3. The velocity of transformation of Sugars by alkalies. By R. S. **Morrell**, M.A., Gonville and Caius College, and A. E. **Bellars**, B.A., Magdalene College.

4. The influence of very strong electro-magnetic fields on the spark spectra of (*a*) vanadium, (*b*) platinum and iridium. By J. E. **Purvis**, M.A., St John's College.

March 12*th*, 1906.

In the Cavendish Laboratory.

Dr Fenton, Vice-President, in the Chair.

The following were elected Fellows of the Society:

H. Bateman, M.A., Trinity College.
P. C. Gaul, M.A., Trinity Hall.
W. S. Perrin, B.A., Gonville and Caius College.
P. W. Wood, M.A., Emmanuel College.

The following Communications were made:

1. (1) A theory of the widening of lines in the spectrum.

(2) On the reflexion and transmission of the radiation from radio-active substances. By J. J. **Thomson**, M.A., Cavendish Professor of Experimental Physics.

2. (1) On the asymptotic expansion of the integral functions

$$\sum_{n=0}^{\infty} \frac{x^n \Gamma(1+an)}{\Gamma(1+n)} \text{ and } \sum_{n=0}^{\infty} \frac{x^n \Gamma(1+n\theta)}{\Gamma(1+n+n\theta)}.$$

(2) On the asymptotic expansion of integral functions defined by generalised hypergeometric series. By E. W. **Barnes**, M.A., Trinity College.

3. A method of following the course of certain chemical actions, and a period of induction in the action of water on monochloracetic acid. By P. V. **Bevan**, M.A., Trinity College.

4. The Radio-activity of Metals and their Salts. By N. R. **Campbell**, M.A., Trinity College.

5. On a relation between the velocity and the volume of the ions of certain organic acids and bases. By T. H. LABY and G. A. CARSE. (Communicated by Professor Thomson.)

6. A preliminary note on the maiotic phenomena in the eggs of the Hermaphrodite *Angiostomum nigrovenosum* (*Ascaris nigrovenosa*). By S. A. MCDOWALL, B.A., Trinity College.

7. On the reduction of the general ternary quintic to Hibbert's canonical form. By H. W. RICHMOND, M.A., King's College.

April 30*th*, 1906.

In the Optical Lecture Room.

MR J. J. LISTER IN THE CHAIR.

The following Communications were made:

1. Demonstration of new apparatus for Psychological Tests. By W. H. R. RIVERS, M.A., St John's College.

2. On the measurement of the earth-air current and on the origin of atmospheric electricity. By C. T. R. WILSON, M.A., Sidney Sussex College.

3. A class of Integral Equations. By H. BATEMAN, M.A., Trinity College.

4. A suggestion as to the nature of the horny teeth of the *Marsipobranchii*. By H. W. MARETT TIMS, M.A., King's College.

May 14*th*, 1906.

In the Optical Lecture Room.

DR FENTON IN THE CHAIR.

The following Communications were made:

1. (1) The influence of a strong electro-magnetic field on the spark spectra of palladium, rhodium and ruthenium.

(2) Experiments on the band spectrum of nitrogen in a strong electro-magnetic field. By J. E. PURVIS, M.A., St John's College.

2. On the ionization of gases exposed simultaneously to Röntgen rays and the radiation from radio-active substances. By T. NODA. (Communicated by Professor Thomson.)

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